Calc III - Ch 13 - Extra Practice

#1b. Find the domain of the vector function $\vec{r}(t)=\left\langle \dfrac{t-2}{t+2},\ \sqrt{t+3},\ \ln\left(9-t^2\right)\right\rangle$.

#2b. Find the limit: $\lim\limits_{t\to\infty}\left\langle \arctan(t),\ e^{-2t},\ \dfrac{\ln(t)}{t}\right\rangle$.

#3b. Sketch the curve with the given vector equation. Indicate with an arrow the direction in which t increases: $\vec{r}(t)=\left\langle t^3,\ t^2\right\rangle$.

#4b. Find a vector equation and parametric equations for the line segment that joins P to Q: $P(1,-1,2),\ \ Q(4,1,7)$.

#5b. Sketch in 3D the parametric curve given by the parametric equations:
$x=e^{-t}\cos 10t,\quad y=e^{-t}\sin 10t,\quad z=e^{-t}$.

#6b. At what points does the helix $\vec{r}(t) = \langle \sin t,\ \cos t,\ t \rangle$ intersect the sphere $x^2 + y^2 + z^2 = 5$?

13.2

#1b. For $\vec{r}(t) = \langle t-2,\ t^2+1 \rangle$

(i) Sketch the plane curve.

(ii) Find $\vec{r}'(t)$

(iii) On your plane curve sketch, add sketches for $\vec{r}(-1)$ and $\vec{r}'(-1)$

#2b. Find the derivative of $\vec{r}(t) = \left\langle e^{t^2},\ -1,\ \ln(1+3t) \right\rangle$

#3b. Find the derivative of

$\vec{r}(t) = \left\langle t\cos t,\ \ t^3 + 2t,\ \ \sqrt{t^3 - 5t} \right\rangle$

#5b. If $\vec{r}(t) = \left\langle t^3,\ 2t^2,\ t \right\rangle$, find

$\vec{r}'(t), \vec{T}(1), \vec{r}''(t)$ and $\vec{r}'(t) x \vec{r}''(t)$

#4b. Find the unit tangent vector $\vec{T}(t)$ at the point with the given value of the parameter *t*.

$\vec{r}(t) = \left\langle te^{-t},\ 2\arctan t,\ 2e^t \right\rangle,\quad t = 0$

#6b. Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point.

$x = t, \quad y = e^{-t}, \quad z = 2t - t^2 \quad (0,1,0)$

#7b. At what point do the curves $\vec{r}_1(t) = \langle t,\ 1-t,\ 3+t^2 \rangle$ and $\vec{r}_2(s) = \langle 3-s,\ s-2,\ s^2 \rangle$ intersect? Find their angle of intersection correct to the nearest degree.

#8b. Evaluate the integral: $\int_1^2 \langle 9t^3,\ 6t^2,\ 3 \rangle \, dt$

13.3

#1b. Find the length of the curve
$\vec{r}(t)=\left\langle 2t,\ t^2,\ \frac{1}{3}t^3\right\rangle,\quad 0\le t\le 1$

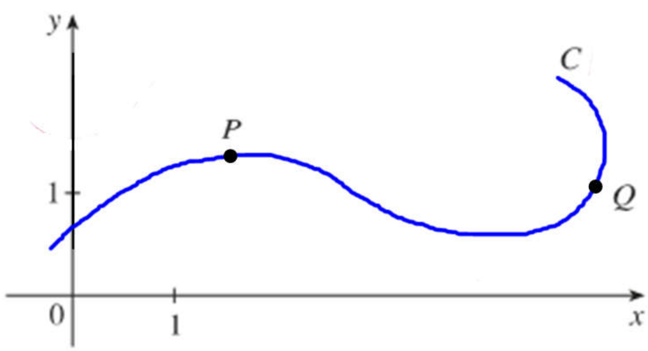
#2b. Reparametrize the curve
$\vec{r}(t)=\left\langle e^{2t}\cos 2t,\quad 2,\quad e^{2t}\sin 2t\right\rangle$ with respect to arc length measured from the point where $t=0$ in the direction of increasing t .

#3b. Find the unit tangent vector $\vec{T}(t)$, the unit normal vector $\vec{N}(t)$, and the curvature κ for
$\vec{r}(t)=\left\langle t^2,\quad \sin t - t\cos t,\quad \cos t + t\sin t\right\rangle,\ t>0$.

#4b. Find the curvature κ for
$\vec{r}(t)=\langle 3t,\quad 4\sin t,\quad 4\cos t\rangle$.

#5b. Find the curvature κ for $y = 4x^{5/2}$.

#6b. Given the curve:



(i) Is the curvature of the curve C shown in the figure greater at P or at Q? Explain.

(ii) Estimate the curvature at P and at Q by sketching the osculating circles at those points.

#7b. Find the vectors $\vec{T}(t)$, $\vec{N}(t)$, and $\vec{B}(t)$ for $\vec{r}(t) = \left\langle t^2, \frac{2}{3}t^3, t \right\rangle$ at $\left(1, \frac{2}{3}, 1\right)$.

#8b. Find equations of the normal plane and the osculating plane of the curve at the given point:
$x = t,\ \ y = t^2,\ \ z = t^3$ at $(1,1,1)$.

13.4

#1b. Find the velocity, acceleration, and speed of a particle with the given position function. Sketch the path of the particle and draw the velocity and acceleration vectors for the specified value of *t*.

$\vec{r}(t) = \langle 3\cos t,\ 2\sin t \rangle, \quad t = \frac{\pi}{3}$

#2b. Find the velocity, acceleration, and speed of a particle with the given position function.

$\vec{r}(t) = \left\langle \sqrt{2}t,\ e^{t},\ e^{-t} \right\rangle$

#3b. Find the position vector of a particle that has the given acceleration and the specified initial velocity and position:

$\vec{a}(t) = \left\langle t, e^{t}, e^{-t} \right\rangle, \ \vec{v}(0) = \langle 0,0,1 \rangle, \ \vec{r}(0) = \langle 0,1,1 \rangle$

#4b. The position function of a particle is given by $\vec{r}(t) = \langle 3t^2, 4t, t^2 - 9t \rangle$. When is the speed a minimum?

#5b. A projectile is fired from a position 200 m above the ground with an initial speed of 500 m/s and angle of elevation of 30°. Find (i) the range of the projectile, (ii) the maximum height reached, and (iii) the speed at impact.

#6b. A gun is fired from the ground with angle of elevation 30^o. What is the muzzle speed (initial projectile speed) if the maximum height of the shell is 500 m?

#7b. (Very challenging) A medieval city has the shape of a square and is protected by walls with length 500 m and height 15 m. You are the commander of an attacking army and the closest you can get to the wall is 100 m. Your plan is to set fire to the city by catapulting heated rocks over the wall (with and initial speed of 80 m/s). At what range of angles should you tell your army to set the catapult? (Assume the path of the rocks is perpendicular to the wall).