Calc III - Ch 13 - Extra Practice SOLUTIONS

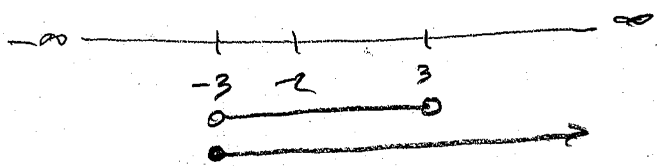
#1b. Find the domain of the vector function

$$\vec{r}(t) = \left\langle \frac{t-2}{t+2},\ \sqrt{t+3},\ \ln(9-t^2) \right\rangle.$$

$\frac{t-2}{t+2}$ $\quad$ $t \neq -2$

$\sqrt{t+3}$ $\quad$ $t+3 \geq 0$ $\quad$ $t \geq -3$

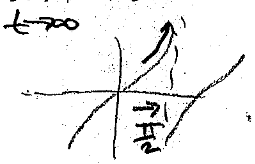
$\ln(9-t^2)$ $\quad$ $9-t^2>0$ $\quad$ $-t^2>-9$ $\quad$ $t^2<9$ $\quad$ $-3<t<3$



domain is intersection: $\boxed{(-3,-2)\cup(-2,0)}$

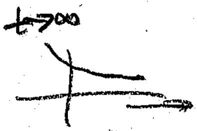
#2b. Find the limit: $\lim\limits_{t\to\infty} \left\langle \arctan(t),\ e^{-2t},\ \frac{\ln(t)}{t} \right\rangle.$

$\lim\limits_{t\to\infty} \arctan t$



$\frac{\pi}{2}$

$\lim\limits_{t\to\infty} e^{-2t}$



0

$\lim\limits_{t\to\infty} \frac{\ln t}{t}$ $\quad \frac{\infty}{\infty}$

L'Hôpital:

$= \lim\limits_{t\to\infty} \frac{\left(\frac{1}{t}\right)}{1}$

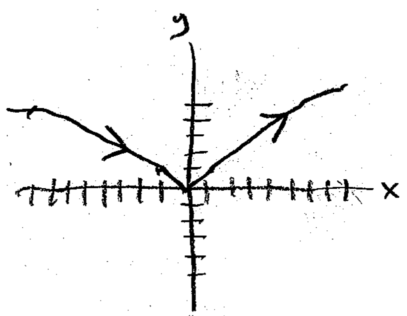
$\frac{\frac{1}{\infty}}{1} = \frac{0}{1}$

0

$= \boxed{\left\langle \frac{\pi}{2}, 0, 0 \right\rangle}$

#3b. Sketch the curve with the given vector equation. Indicate with an arrow the direction in which t increases: $\vec{r}(t) = \langle t^3, t^2 \rangle$.

t	$\langle t^3, t^2 \rangle$
-2	$\langle -8, 4 \rangle$
-1	$\langle -1, 1 \rangle$
0	$\langle 0, 0 \rangle$
1	$\langle 1, 1 \rangle$
2	$\langle 8, 4 \rangle$



#4b. Find a vector equation and parametric equations for the line segment that joins P to Q: $P(1,-1,2)$, $Q(4,1,7)$.

$\vec{r}(t) = (1-t)\vec{r_0} + t\vec{r_1}$

$= (1-t)\langle 1,-1,2 \rangle + t\langle 4,1,7 \rangle$

$= \langle 1-t, -1+t, 2-2t \rangle + \langle 4t, t, 7t \rangle$

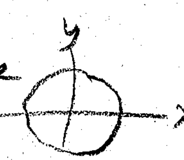
$\boxed{\vec{r}(t) = \langle 1+3t, -1+2t, 2+5t \rangle \quad 0 \leq t \leq 1}$

$$\boxed{\begin{aligned} x &= 1+3t \\ y &= -1+2t \quad 0 \leq t \leq 1 \\ z &= 2+5t \end{aligned}}$$

#5b. Sketch in 3D the parametric curve given by the parametric equations:

$x = e^{-t}\cos 10t$, $y = e^{-t}\sin 10t$, $z = e^{-t}$.

xy: $\langle \cos 10t, \sin 10t \rangle$ is circle

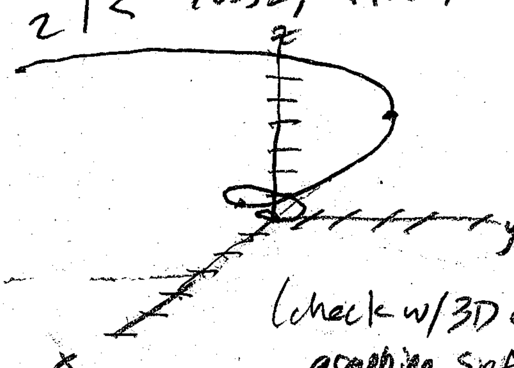


the e^{-t} multiplier changes the radius and if $z = e^{-t}$ means: $x = z\cos 10t$

$y = z\sin 10t$

get a few points to start:

t	$\langle e^{-t}\cos 10t, e^{-t}\sin 10t, e^{-t} \rangle$
-2	$\langle 3.0153, -6.746, 7.3891 \rangle$
-1	$\langle -2.281, 1.478, 2.7183 \rangle$
0	$\langle 1, 0, 1 \rangle$
1	$\langle -.3087, -.2001, .3679 \rangle$
2	$\langle .0552, .1236, .1353 \rangle$



(check w/ 3D online graphing software)

#6b. At what points does the helix $\vec{r}(t)=\langle \sin t,\ \cos t,\ t\rangle$ intersect the sphere $x^2+y^2+z^2=5$?

$\vec{r}(t)=\langle \sin t, \cos t, t\rangle$

means $x=\sin t$

$y=\cos t$

$z=t$

Substitute into...

$x^2+y^2+z^2=5$

$\underbrace{(\sin t)^2+(\cos t)^2}+(t)^2=5$

$1+t^2=5$

$t^2=4$

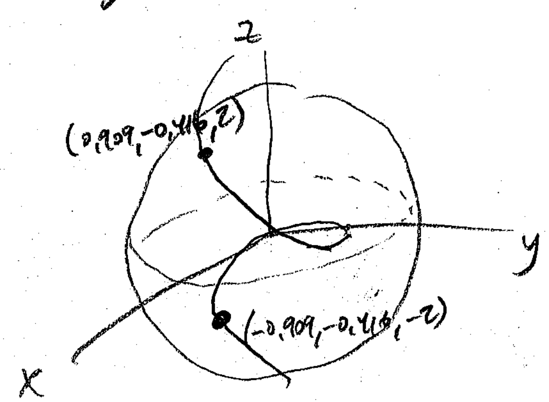
$t=2,\ t=-2$ (parameter values)

then, plug these into the parametric equations to get the coordinates!

$t=2$	$t=-2$
$x=\sin(2)\approx 0.909$	$x=\sin(-2)=-0.909$
$y=\cos(2)\approx -0.416$	$y=\cos(-2)=-0.416$
$z=2$	$z=-2$

$\boxed{(0.909,-0.416,2)\quad (-0.909,-0.416,-2)}$

(check w/ 3D graphing software if you like...)



13.2

#1b. For $\vec{r}(t)=\langle t-2,\ t^2+1\rangle$

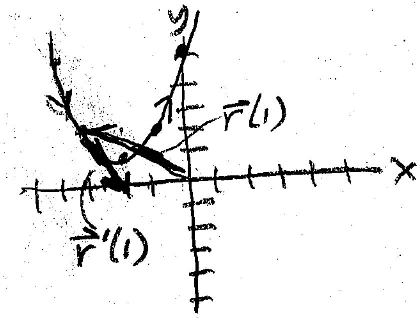
(i) Sketch the plane curve.

(ii) Find $\vec{r}'(t)$

(iii) On your plane curve sketch, add sketches for $\vec{r}(-1)$ and $\vec{r}'(-1)$

(i)

t	$\langle t-2, t^2+1\rangle$
-2	$\langle -4, 5\rangle$
-1	$\langle -3, 2\rangle$
0	$\langle -2, 1\rangle$
1	$\langle -1, 2\rangle$
2	$\langle 0, 5\rangle$



(ii) $\vec{r}'(t)=\langle 1, 2t\rangle$

(iii) $\vec{r}(-1)=\langle -1-2, (-1)^2+1\rangle=\langle -3, 2\rangle$

$\vec{r}'(-1)=\langle 1, 2(-1)\rangle=\langle 1, -2\rangle$

#2b. Find the derivative of $\vec{r}(t)=\langle e^{t^2},\ -1,\ \ln(1+3t)\rangle$

$\vec{r}'(t)=\langle \underbrace{e^{(t^2)}(2t)}_{\text{chain rule}}, 0, \underbrace{\frac{1}{1+3t}(3)}_{\text{chain rule}}\rangle$

$=\boxed{\langle 2te^{(t^2)}, 0, \frac{3}{1+3t}\rangle}$

#3b. Find the derivative of

$\vec{r}(t) = \langle t\cos t,\ t^3+2t,\ \sqrt{t^3-5t} \rangle$

product rule $\quad (t^3-5t)^{1/2}$ chain rule

$\vec{r}\,'(t) = \langle t(-\sin t)+\cos t(1),\ 3t^2+2,\ \frac{1}{2}(t^3-5t)^{-1/2} \cdot (3t^2-5) \rangle$

$= \boxed{\langle -t\sin t+\cos t,\ 3t^2+2,\ \frac{3t^2-5}{2\sqrt{t^3-5t}} \rangle}$

#4b. Find the unit tangent vector $\vec{T}(t)$ at the point with the given value of the parameter t.

$\vec{r}(t) = \langle te^{-t},\ 2\arctan t,\ 2e^t \rangle,\quad t=0$

$\vec{T}(t) = \frac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|}$

$\vec{r}(t) = \langle te^{-t},\ 2\arctan t,\ 2e^t \rangle$

product rule

$\vec{r}\,'(t) = \langle te^{-t}(-1)+e^{-t}(1),\ 2\frac{1}{1+t^2},\ 2e^t \rangle$

$\vec{r}\,'(0) = \langle -0e^{-0}+e^{-0},\ \frac{2}{1+0^2},\ 2e^0 \rangle$

$= \langle 1, 2, 2 \rangle$

$|\vec{r}\,'(0)| = \sqrt{1^2+2^2+2^2} = 3$

$\vec{T}(0) = \frac{\langle 1,2,2 \rangle}{3} = \boxed{\langle \frac{1}{3}, \frac{2}{3}, \frac{2}{3} \rangle}$

#5b. If $\vec{r}(t) = \langle t^3,\ 2t^2,\ t \rangle$, find $\vec{r}\,'(t), \vec{T}(1), \vec{r}\,''(t)$ and $\vec{r}\,'(t) x \vec{r}\,''(t)$

$\boxed{\vec{r}\,'(t) = \langle 3t^2, 4t, 1 \rangle}$

$\vec{r}\,'(1) = \langle 3, 4, 1 \rangle$

$\vec{T}(1) = \frac{\langle 3,4,1 \rangle}{\sqrt{3^2+4^2+1^2}} = \boxed{\langle \frac{3}{\sqrt{26}}, \frac{4}{\sqrt{26}}, \frac{1}{\sqrt{26}} \rangle}$

$\boxed{\vec{r}\,''(t) = \langle 6t, 4, 0 \rangle}$

$\vec{r}\,'(t) \times \vec{r}\,''(t)$

$$= \begin{vmatrix} + & - & + \\ 3t^2 & 4t & 1 \\ 6t & 4 & 0 \end{vmatrix}$$

$= \langle 0-4,\ -(0-6t),\ 12t^2-24t^2 \rangle$

$= \boxed{\langle -4,\ 6t,\ -12t^2 \rangle}$

#6b. Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point.

$x = t, \quad y = e^{-t}, \quad z = 2t - t^2 \quad (0,1,0)$

$\vec{r}(t) = \vec{r}_0 + t\vec{v}$

$\vec{r}_0 = \langle 0,1,0\rangle$

$\vec{v} = \vec{r}\,'(t)$

$\vec{r}(t) = \langle t, e^{-t}, 2t - t^2\rangle$

$\vec{r}\,'(t) = \langle 1, -e^{-t}, 2-2t\rangle$

find parameter at point:

$(0,1,0) \Leftrightarrow \vec{r}(t) = \langle t, e^{-t}, 2t-t^2\rangle$

$\underline{t=0}$ check y, z:

$y = e^{-t} = e^{-0} = 1$ ✓

$z = 2t - t^2 = 2(0) - (0) = 0$ ✓

$\vec{v} = \vec{r}\,'(0) = \langle 1, -e^{-(0)}, 2-2(0)\rangle$ at t=0

$= \langle 1, -1, 2\rangle$

So tangent line is:

new $\vec{r}_{tan}(t) = \langle 0,1,0\rangle + t\langle 1,-1,2\rangle$

$= \langle t, 1-t, 2t\rangle$

or
$$\boxed{\begin{array}{l} x = t \\ y = 1-t \quad -\infty < t < \infty \\ z = 2t \end{array}}$$

#7b. At what point do the curves

$\vec{r}_1(t) = \langle t, 1-t, 3+t^2\rangle$ and

$\vec{r}_2(s) = \langle 3-s, s-2, s^2\rangle$ intersect? Find their angle of intersection correct to the nearest degree.

intersections are systems, here xs, ys, zs must be equal:

$\begin{cases} t = 3-s \\ 1-t = s-2 \\ 3+t^2 = s^2 \end{cases}$ solve for s: $1-t = s-2$, $s = 1-t+2$, $s = 3-t$

subst. into: $3+t^2 = s^2$

$3+t^2 = (3-t)^2$

$3 + \cancel{t^2} = 9 - 6t + \cancel{t^2}$

$6t = 6$, $\underline{t=1}$ and $s = 3-t = 3-1$, $\underline{s=2}$

check $t=1, s=2$ in last equation:

$t \stackrel{?}{=} 3-s$

$1 = 3-2$ ✓

So $t=1, s=2$ at intersection.

curve 'direction' is given by their tangent vectors:

$\vec{r}_1\,'(t) = \langle 1, -1, 2t\rangle$, $\vec{r}_2\,'(s) = \langle -1, 1, 2s\rangle$

at intersection:

$\vec{r}_1\,'(1) = \langle 1, -1, 2(1)\rangle = \langle 1,-1,2\rangle$

$\vec{r}_2\,'(2) = \langle -1, 1, 2(2)\rangle = \langle -1,1,4\rangle$

use dot product to find angle:

$$\cos\theta = \frac{\vec{r}_1\,' \cdot \vec{r}_2\,'}{|r_1||r_2|} = \frac{\langle 1,-1,2\rangle \cdot \langle -1,1,4\rangle}{\sqrt{1^2+1^2+2^2}\sqrt{1^2+1^2+4^2}}$$

$$= \frac{(1)(-1)+(-1)(1)+(2)(4)}{(\sqrt{6})(\sqrt{18})}$$

$$\cos\theta = \frac{6}{\sqrt{108}}$$

$$\theta = \cos^{-1}\left(\frac{6}{\sqrt{108}}\right) = 54.7 \approx \boxed{55^\circ}$$

#8b. Evaluate the integral: $\int_1^2 \langle 9t^3, 6t^2, 3\rangle\, dt$

$\langle \int_1^2 9t^3 dt, \int_1^2 6t^2 dt, \int_1^2 3 dt\rangle$

$\langle \left[\frac{9}{4}t^4\right]_1^2, \left[2t^3\right]_1^2, \left[3t\right]_1^2\rangle$

$\langle \frac{9}{4}(2)^4 - \frac{9}{4}(1)^4, 2(2)^3 - 2(1)^3, 3(2) - 3(1)\rangle$

$\boxed{\langle \frac{135}{4}, 14, 3\rangle}$

13.3

#1b. Find the length of the curve $\vec{r}(t)=\left\langle 2t,\ t^2,\ \frac{1}{3}t^3\right\rangle,\quad 0\le t\le 1$

$L=\int_a^b |\vec{r}'(t)|\,dt$

$\vec{r}'(t)=\langle 2,2t,t^2\rangle$

$|\vec{r}'(t)|=\sqrt{2^2+(2t)^2+(t^2)^2}=\sqrt{t^4+4t^2+4}$

$=\sqrt{(t^2+2)^2}=t^2+2$

$L=\int_0^1 (t^2+2)\,dt=\left[\frac{1}{3}t^3+2t\right]_0^1$

$=\left(\frac{1}{3}(1)^3+2(1)\right)-\left(\frac{1}{3}(0)^3+2(0)\right)$

$=\frac{1}{3}+2=\boxed{\frac{7}{3}}$

#2b. Reparametrize the curve $\vec{r}(t)=\langle e^{2t}\cos 2t,\ 2,\ e^{2t}\sin 2t\rangle$ with respect to arc length measured from the point where $t=0$ in the direction of increasing t.

(product rule)

$\vec{r}'(t)=\langle e^{2t}(-2\sin 2t)+\cos 2t\cdot 2e^{2t},\ 0,$
$e^{2t}(2\cos 2t)+\sin 2t\cdot 2e^{2t}\rangle$

$=\langle 2e^{2t}(\cos 2t-\sin 2t),\ 0,\ 2e^{2t}(\cos 2t+\sin 2t)\rangle$

$|\vec{r}'(t)|=\sqrt{(2e^{2t}(\cos 2t-\sin 2t))^2+(2e^{2t}(\cos 2t+\sin 2t))^2}$

$=\sqrt{(2e^{2t})^2[\cos^2 2t-2\sin 2t\cos 2t+\sin^2 2t+\cos^2 2t+2\sin 2t\cos 2t+\sin^2 2t]}$

$=\sqrt{(2e^{2t})^2(2\cos^2 2t+2\sin^2 2t)}=\sqrt{(2e^{2t})^2\cdot 2}=2\sqrt{2}e^{2t}$

$\frac{ds}{dt}=|\vec{r}'(t)|=2\sqrt{2}e^{2t}$, so $ds=2\sqrt{2}e^{2t}\,dt$

$s=\int ds=2\sqrt{2}\int_0^t e^{2u}\,du=2\sqrt{2}\left[\frac{1}{2}e^{2u}\right]_0^t$

$s=\sqrt{2}\left[e^{2t}-e^{0}\right]=\sqrt{2}(e^{2t}-1)$

solve for t: $e^{2t}-1=\frac{s}{\sqrt{2}},\ e^{2t}=\frac{s}{\sqrt{2}}+1$

$t=\frac{1}{2}\ln\left(\frac{s}{\sqrt{2}}+1\right)$ sub this in $\vec{r}(t)$...

every 2t becomes $2\left(\frac{1}{2}\ln\left(\frac{s}{\sqrt{2}}+1\right)\right)=\ln\left(\frac{s}{\sqrt{2}}+1\right)$

$\vec{r}(s)=\left\langle e^{\ln(\frac{s}{\sqrt{2}}+1)}\cos\left(\ln\left(\frac{s}{\sqrt{2}}+1\right)\right),\ 2,\ e^{\ln(\frac{s}{\sqrt{2}}+1)}\sin\left(\ln\left(\frac{s}{\sqrt{2}}+1\right)\right)\right\rangle$

$\boxed{\vec{r}(s)=\left\langle\left(\frac{s}{\sqrt{2}}+1\right)\cos\left(\ln\left(\frac{s}{\sqrt{2}}+1\right)\right),\ 2,\ \left(\frac{s}{\sqrt{2}}+1\right)\sin\left(\ln\left(\frac{s}{\sqrt{2}}+1\right)\right)\right\rangle}$

#3b. Find the unit tangent vector $\vec{T}(t)$, the unit normal vector $\vec{N}(t)$, and the curvature κ for $\vec{r}(t)=\langle t^2,\ \sin t-t\cos t,\ \cos t+t\sin t\rangle,\ t>0$.

$\vec{r}'(t)=\langle 2t,\ \cos t-(t(-\sin t)+\cos t(1)),\ -\sin t+(t\cos t+\sin t(1))\rangle$

$=\langle 2t,\ t\sin t,\ t\cos t\rangle$

$|\vec{r}'(t)|=\sqrt{4t^2+t^2\sin^2 t+t^2\cos^2 t}=\sqrt{4t^2+t^2(\sin^2 t+\cos^2 t)}$

$=\sqrt{5t^2}=\sqrt{5}t$

$\vec{T}(t)=\frac{\vec{r}'(t)}{|\vec{r}'(t)|}=\frac{\langle 2t,t\sin t,t\cos t\rangle}{\sqrt{5}t}=\boxed{\frac{1}{\sqrt{5}}\langle 2,\sin t,\cos t\rangle}$

$\vec{T}'(t)=\frac{1}{\sqrt{5}}\langle 0,\cos t,-\sin t\rangle$

$|\vec{T}'(t)|=\frac{1}{\sqrt{5}}\sqrt{0^2+\cos^2 t+\sin^2 t}=\frac{1}{\sqrt{5}}\sqrt{1}=\frac{1}{\sqrt{5}}$

$\vec{N}(t)=\frac{\vec{T}'(t)}{|\vec{T}'(t)|}=\frac{\frac{1}{\sqrt{5}}\langle 0,\cos t,-\sin t\rangle}{\frac{1}{\sqrt{5}}}=\boxed{\langle 0,\cos t,-\sin t\rangle}$

$\kappa=\frac{|\vec{T}'(t)|}{|\vec{r}'(t)|}=\frac{\left(\frac{1}{\sqrt{5}}\right)}{(\sqrt{5}t)}=\boxed{\frac{1}{5t}}$

#4b. Find the curvature κ for $\vec{r}(t)=\langle 3t,\ 4\sin t,\ 4\cos t\rangle$.

$\vec{r}'(t)=\langle 3,4\cos t,-4\sin t\rangle$

$|\vec{r}'(t)|=\sqrt{3^2+(4\cos t)^2+(4\sin t)^2}$

$=\sqrt{9+16(\cos^2 t+\sin^2 t)}=\sqrt{25}=5$

$\vec{r}''(t)=\langle 0,-4\sin t,-4\cos t\rangle$

$\vec{r}'\times\vec{r}''=\begin{vmatrix} + & - & + \\ 3 & 4\cos t & -4\sin t \\ 0 & -4\sin t & -4\cos t\end{vmatrix}$

$=\langle -16\cos^2 t-16\sin^2 t,\ -(-12\cos t-0),\ -12\sin t-0\rangle$

$=\langle -16,\ 12\cos t,\ -12\sin t\rangle$

$|\vec{r}'\times\vec{r}''|=\sqrt{16^2+(12\cos t)^2+(12\sin t)^2}=\sqrt{16^2+12^2}=20$

$\kappa=\frac{|\vec{r}'\times\vec{r}''|}{|\vec{r}'|^3}=\frac{20}{5^3}=\boxed{\frac{4}{25}}$

(constant b/c this is a circular helix)

#5b. Find the curvature κ for $y = 4x^{5/2}$.

parametrize: $t = x$, $\vec{r}(t) = \langle t, 4t^{5/2} \rangle$

$\vec{r}'(t) = \langle 1, 10t^{3/2} \rangle$

$|\vec{r}'(t)| = \sqrt{1^2 + (10t^{3/2})^2} = \sqrt{1 + 100t^{9/4}}$

$\vec{r}''(t) = \langle 0, 15t^{1/2} \rangle$

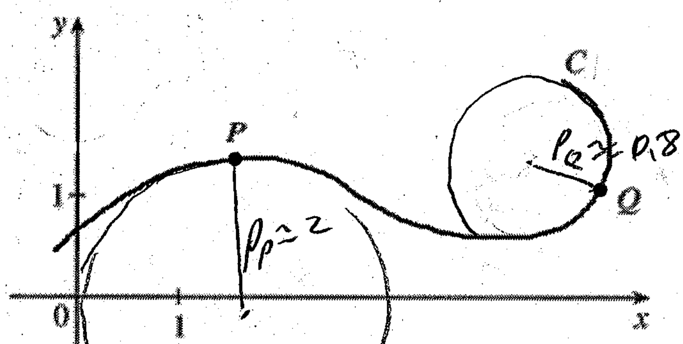
$$\vec{r}' \times \vec{r}'' = \begin{vmatrix} + & - & + \\ 1 & 10t^{3/2} & 0 \\ 0 & 15t^{1/2} & 0 \end{vmatrix}$$

$= \langle 0-0, -(0-0), 15t^{1/2} - 0 \rangle$

$= 15\sqrt{t}$

$$K = \frac{|\vec{r}' \times \vec{r}''|}{|\vec{r}'|^3} = \boxed{\frac{15\sqrt{t}}{(1 + 100t^{9/4})^{3/2}}}$$

#6b. Given the curve:



(i) Is the curvature of the curve C shown in the figure great at P or at Q? Explain.

K greater at Q b/c more curved

(ii) Estimate the curvature at P and at Q by sketching the osculating circles at those points.

$K = \frac{1}{\rho}$

$\boxed{K_P = \frac{1}{2}}$ $\quad \boxed{K_Q = \frac{1}{0.8} = \frac{5}{4}}$

#7b. Find the vectors $\vec{T}(t)$, $\vec{N}(t)$, and $\vec{B}(t)$ for $\vec{r}(t) = \left\langle t^2, \frac{2}{3}t^3, t \right\rangle$ at $\left(1, \frac{2}{3}, 1\right)$. $t = 1$

$\vec{r}'(t) = \langle 2t, 2t^2, 1 \rangle$

$|\vec{r}'(t)| = \sqrt{4t^2 + 4t^4 + 1} = \sqrt{4t^4 + 4t^2 + 1}$

$= \sqrt{(2t+1)^2} = 2t+1$

$\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} = \frac{\langle 2t, 2t^2, 1 \rangle}{2t+1}$

$\vec{T}(1) = \frac{\langle 2, 2, 1 \rangle}{3} = \boxed{\left\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \right\rangle}$

$\vec{T}(t) = \frac{\langle 2t, 2t^2, 1 \rangle}{2t+1} = (2t+1)^{-1} \langle 2t, 2t^2, 1 \rangle$

(product rule)

$\vec{T}'(t) = (2t+1)^{-1} \frac{d}{dt} \langle 2t, 2t^2, 1 \rangle + \langle 2t, 2t^2, 1 \rangle \frac{d}{dt} [(2t+1)^{-1}]$

$= (2t+1)^{-1} \langle 2, 4t, 0 \rangle + \langle 2t, 2t^2, 1 \rangle (-(2t+1)^{-2}(2))$

$= \frac{\langle 2, 4t, 0 \rangle}{2t+1} + \langle 2t, 2t^2, 1 \rangle \left(\frac{-2}{(2t+1)^2} \right)$

$\vec{T}'(1) = \frac{\langle 2, 4, 0 \rangle}{3} + \langle 2, 2, 1 \rangle \left(\frac{-2}{9} \right)$

$= \left\langle \frac{2}{3}, \frac{4}{3}, 0 \right\rangle + \left\langle \frac{-4}{9}, \frac{-4}{9}, \frac{-2}{9} \right\rangle = \left\langle \frac{2}{9}, \frac{8}{9}, \frac{-2}{9} \right\rangle$

$\vec{N}(1) = \frac{\vec{T}'(1)}{|\vec{T}'(1)|} = \frac{\left\langle \frac{2}{9}, \frac{8}{9}, \frac{1}{9} \right\rangle}{\sqrt{\frac{4}{81} + \frac{64}{81} + \frac{1}{81}}} = \boxed{\frac{1}{\sqrt{69}} \left\langle \frac{2}{9}, \frac{8}{9}, \frac{1}{9} \right\rangle}$

$\vec{B}(1) = \vec{T}(1) \times \vec{N}(1)$

$$= \begin{vmatrix} + & - & + \\ \frac{2}{3} & \frac{2}{3} & \frac{1}{3} \\ \frac{2}{9\sqrt{69}} & \frac{8}{9\sqrt{69}} & \frac{1}{9\sqrt{69}} \end{vmatrix}$$

$= \left\langle \frac{2}{27\sqrt{69}} - \frac{8}{27\sqrt{69}}, -\left(\frac{2}{27\sqrt{69}} - \frac{2}{27\sqrt{69}} \right), \frac{16}{27\sqrt{69}} - \frac{4}{27\sqrt{69}} \right\rangle$

$= \boxed{\left\langle \frac{-2}{9\sqrt{69}}, 0, \frac{4}{9\sqrt{69}} \right\rangle}$

#8b. Find equations of the normal plane and the osculating plane of the curve at the given point:
$x = t,\ y = t^2,\ z = t^3$ at $(1,1,1)$. $\underline{\underline{t=1}}$

$\vec{r}(t) = \langle t, t^2, t^3 \rangle$

$\vec{r}\,'(t) = \langle 1, 2t, 3t^2 \rangle$

$|\vec{r}\,'(t)| = \sqrt{1^2 + 4t^2 + 9t^4} = \sqrt{9t^4 + 4t^2 + 1}$

$\vec{T}(t) = \dfrac{\vec{r}\,'(t)}{|\vec{r}\,'(t)|} = \dfrac{\langle 1, 2t, 3t^2 \rangle}{\sqrt{9t^4+4t^2+1}} = (9t^4+4t^2+1)^{-1} \langle 1, 2t, 3t^2 \rangle$

at $\underline{t=1}$: $\vec{T}(1) = \dfrac{\langle 1, 2, 3 \rangle}{\sqrt{14}}$

$\vec{T}\,'(t) =$ use product rule on $= (9t^4+4t^2+1)^{-1} \frac{d}{dt}[\langle 1, 2t, 3t^2 \rangle] + \langle 1, 2t, 3t^2 \rangle \frac{d}{dt}[(9t^4+4t^2+1)^{-1}]$

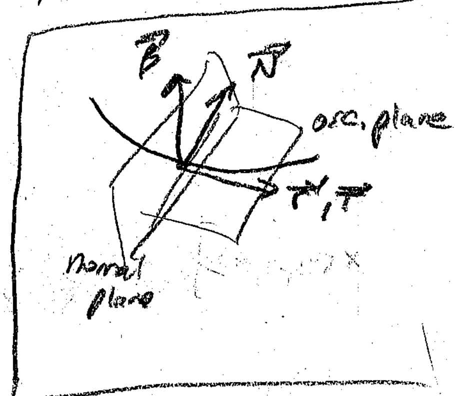
$\vec{T}\,'(t) = (9t^4+4t^2+1)^{-1} \langle 0, 2, 6t \rangle + \langle 1, 2t, 3t^2 \rangle \left(-(9t^4+4t^2+1)^{-2} (36t^3 + 8t) \right)$

at $\underline{t=1}$: $\vec{T}\,'(1) = \dfrac{1}{9+4+1} \langle 0, 2, 6 \rangle + \langle 1, 2, 3 \rangle \left(\dfrac{-(36+8)}{(9+4+1)^2} \right)$

$\vec{T}\,'(1) = \dfrac{\langle 0, 2, 6 \rangle}{14} - \dfrac{11}{49} \langle 1, 2, 3 \rangle = \langle -\frac{11}{49}, -\frac{15}{49}, -\frac{12}{49} \rangle$

$|\vec{T}\,'(1)| = \sqrt{\left(\frac{11}{49}\right)^2 + \left(\frac{15}{49}\right)^2 + \left(\frac{12}{49}\right)^2} = \sqrt{\frac{10}{49}} = \frac{\sqrt{10}}{7}$

$\vec{N}(1) = \dfrac{\vec{T}\,'(1)}{|\vec{T}\,'(1)|} = \dfrac{-\frac{1}{49} \langle 11, 15, 12 \rangle}{\frac{\sqrt{10}}{7}} = -\dfrac{1}{7\sqrt{10}} \langle 11, 15, 12 \rangle$



$\vec{B}(1) = \vec{T}(1) \times \vec{N}(1) = \left(\frac{1}{\sqrt{14}}\right) \langle 1, 2, 3 \rangle \times \left(-\frac{1}{7\sqrt{10}}\right) \langle 11, 15, 12 \rangle$

$= \left(\frac{1}{\sqrt{14}}\right)\left(-\frac{1}{7\sqrt{10}}\right) [\langle 1, 2, 3 \rangle \times \langle 11, 15, 12 \rangle]$

$= \dfrac{-1}{7\sqrt{140}} \begin{vmatrix} + & - & + \\ 1 & 2 & 3 \\ 11 & 15 & 12 \end{vmatrix} = \dfrac{-1}{7\sqrt{140}} \langle 24-45, -(12-33), 15-22 \rangle$

$= \dfrac{-1}{7\sqrt{140}} \langle -21, 21, -7 \rangle$

Normal plane contains $\vec{B}$ & $\vec{N}$, $\perp$ to $\vec{T}$, so $\vec{n} = \vec{T} = \langle 1, 2, 3 \rangle$, $\vec{r_0} = \langle 1, 1, 1 \rangle$

$ax + by + cz = \vec{n} \cdot \vec{r_0}$

$1x + 2y + 3z = \langle 1, 2, 3 \rangle \cdot \langle 1, 1, 1 \rangle = 1 + 2 + 3 = 6$, so $\boxed{x + 2y + 3z = 6}$

Osculating plane contains $\vec{T}$ & $\vec{N}$, $\perp$ to $\vec{B}$, so $\vec{n} = \vec{B} = \langle 11, 15, 12 \rangle$, $\vec{r_0} = \langle 1, 1, 1 \rangle$

$ax + by + cz = \vec{n} \cdot \vec{r_0}$

$11x + 15y + 12z = \langle 11, 15, 12 \rangle \cdot \langle 1, 1, 1 \rangle = 38$, so $\boxed{11x + 15y + 12z = 38}$

13.4

#1b. Find the velocity, acceleration, and speed of a particle with the given position function. Sketch the path of the particle and draw the velocity and acceleration vectors for the specified value of t.

$\vec{r}(t)=\langle 3\cos t,\ 2\sin t\rangle,\quad t=\frac{\pi}{3}$

$\vec{v}(t)=\vec{r}\,'(t)=\langle -3\sin t, 2\cos t\rangle$

$\vec{a}(t)=\vec{v}\,'(t)=\langle -3\cos t, -2\sin t\rangle$

Speed $=|\vec{v}|=\sqrt{9\sin^2 t+4\cos^2 t}$

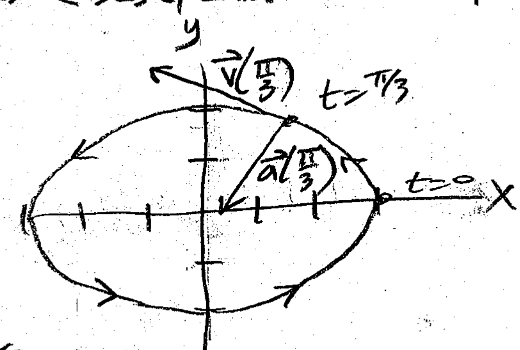
at $t=\pi/3$

$\vec{v}(\pi/3)=\langle -3\sin\pi/3,\ 2\cos\pi/3\rangle=\langle -\frac{3\sqrt{3}}{2},\ 1\rangle$

$\vec{a}(\pi/3)=\langle -3\cos\pi/3,\ -2\sin\pi/3\rangle=\langle -\frac{3}{2},\ -\sqrt{3}\rangle$

Speed $=\sqrt{9(\sin\pi/3)^2+4(\cos\pi/3)^2}=4.6$

$\vec{r}(t)=\langle 3\cos t,\ 2\sin t\rangle$ is an ellipse:



t	$\langle 3\cos t, 2\sin t\rangle$
0	$\langle 3, 0\rangle$
$\pi/3$	$\langle 3(\frac{1}{2}), 2(\frac{\sqrt{3}}{2})\rangle$

#2b. Find the velocity, acceleration, and speed of a particle with the given position function.

$\vec{r}(t)=\langle \sqrt{2}t,\ e^t,\ e^{-t}\rangle$

$\boxed{\vec{v}(t)=\vec{r}\,'(t)=\langle \sqrt{2}, e^t, -e^{-t}\rangle}$

Speed $=|\vec{v}|=\sqrt{(\sqrt{2})^2+(e^t)^2+(e^{-t})^2}$

$=\boxed{\sqrt{2+e^{2t}+e^{-2t}}}$

$\boxed{\vec{a}(t)=\vec{v}\,'(t)=\langle 0, e^t, e^{-t}\rangle}$

#3b. Find the position vector of a particle that has the given acceleration and the specified initial velocity and position:

$\vec{a}(t)=\langle t, e^t, e^{-t}\rangle,\ \vec{v}(0)=\langle 0,0,1\rangle,\ \vec{r}(0)=\langle 0,1,1\rangle$

$\vec{v}(t)=\int \vec{a}(t)\,dt$

$=\langle \int t\,dt, \int e^t dt, \int e^{-t} dt\rangle$

$=\langle \frac{1}{2}t^2+C_1, e^t+C_2, -e^{-t}+C_3\rangle$

use $\vec{v}(0)=\langle 0,0,1\rangle$:

$\langle 0,0,1\rangle=\langle \frac{1}{2}(0)^2+C_1, e^{(0)}+C_2, -e^0+C_3\rangle$

$0+C_1=0$, $1+C_2=0$, $-1+C_3=1$

$C_1=0$, $C_2=-1$, $C_3=2$

so $\vec{v}(t)=\langle \frac{1}{2}t^2, e^t-1, -e^{-t}+2\rangle$

$\vec{r}(t)=\int \vec{v}(t)\,dt$

$=\langle \int \frac{1}{2}t^2 dt, \int (e^t-1)dt, \int(-e^{-t}+2)dt\rangle$

$=\langle \frac{1}{6}t^3+C_4, e^t-t+C_5, e^{-t}+2t+C_6\rangle$

use $\vec{r}(0)=\langle 0,1,1\rangle$

$\langle 0,1,1\rangle=\langle \frac{1}{6}(0)^3+C_4, e^0-(0)+C_5, e^0+2(0)+C_6$

$0+C_4=0$, $1+C_5=1$, $1+C_6=1$

$C_4=0$, $C_5=0$, $C_6=0$

$\boxed{\vec{r}(t)=\langle \frac{1}{6}t^3, e^t-t, e^{-t}+2t\rangle}$

#4b. The position function of a particle is given by $\vec{r}(t)=\langle 3t^2, 4t, t^2-9t\rangle$. When is the speed a minimum?

$\vec{v}(t)=\vec{r}\,'(t)=\langle 6t, 4, 2t-9\rangle$

speed $=|\vec{v}|=\sqrt{36t^2+16+(2t-9)^2}$

$=\sqrt{36t^2+16+4t^2-36t+81}$

$=\sqrt{40t^2-36t+97}$

any function is minimum when it's derivative $=0$ or DNE.

$\frac{d\,\text{speed}}{dt}=\frac{1}{2}(40t^2-36t+97)^{-1/2}(80t-36)$

$=\frac{80t-36}{2\sqrt{40t^2-36t+97}}=\frac{40t-18}{\sqrt{40t^2-36t+97}}$

this $=0$, when $40t-18=0$

$40t=18$

$t=\frac{18}{40}=0.45$

is this ever DNE? (denom $=0$?)

$40t^2-36t+97=0$

$t=\frac{36\pm\sqrt{36^2-4(40)(97)}}{2(40)}$

$\frac{36\pm\sqrt{-14224}}{80}$

(no real solutions)

so only when $t=0.45$

to verify min, test values around it:

$-\infty$ ——— $t=0.45$ ——— ∞

(0)	(1)
$\frac{d(\text{speed})}{dt}$	$\frac{d(\text{speed})}{dt}$
$\frac{(-)}{(+)}$	$\frac{(+)}{(+)}$
↘	↗

$\boxed{\text{relative minimum at } t=0.45}$

#5b. A projectile is fired from a position 200 m above the ground with an initial speed of 500 m/s and angle of elevation of 30°. Find (i) the range of the projectile, (ii) the maximum height reached, and (iii) the speed at impact.

$\vec{a}(t)=\langle 0,-9.81\rangle$ m/s^2

$\vec{v}(t)=\langle 0+C_1, -9.81t+C_2\rangle$

$\vec{v}(0)=\langle 500\cos 30^\circ, 500\sin 30^\circ\rangle$

$=\langle 433.013, 250\rangle$

$\langle 433.013, 250\rangle=\langle 0+C_1, -9.81(0)+C_2\rangle$

$C_1=433.013,\quad C_2=250$

$\vec{v}(t)=\langle 433.013, -9.81t+250\rangle$

$\vec{r}(t)=\langle 433.013t+C_3, -4.9t^2+250t+C_4\rangle$

$\vec{r}(0)=\langle 0, 200\rangle$

$\langle 0,200\rangle=\langle 433.013(0)+C_3, -4.9(0)^2+250(0)+C_4\rangle$

$C_3=0,\quad C_4=200$

$\vec{r}(t)=\langle 433.013t, -4.9t^2+250t+200\rangle$

(i) range $=r_x$ when $r_y=0$

$-4.9t^2+250t+200=0$

$t=\frac{-250\pm\sqrt{250^2+4(4.9)(200)}}{2(-4.9)}=\cancel{-.788},\ 51.8$ sec

range $=r_x=433.013(51.8)=\boxed{22433.6\text{ m}}$

(ii) max height when $v_y=0$

$-9.81t+250=0,\quad 9.81t=250$

$t=\frac{250}{9.81}=25.48$ sec

height $=r_y=-4.9(25.48)^2+250(25.48)+200$

$=\boxed{3388.7\text{ m}}$

(iii) Speed at impact $=|\vec{v}|$ at 51.8 sec

$\vec{v}(51.8)=\langle 433.013, -9.81(51.8)+250\rangle$

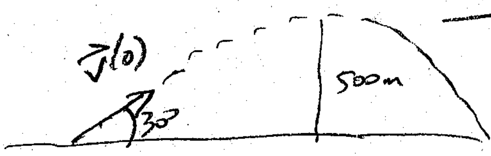
$=\langle 433.013, -258.158\rangle$

Speed $=\sqrt{433.013^2+258.158^2}$

$=\boxed{503.6\text{ m/s}}$

#6b. A gun is fired from the ground with angle of elevation 30°. What is the muzzle ~~shell~~ speed (initial projectile speed) if the maximum height of the shell is 500 m?

muzzle speed = M



$\vec{a}(t) = \langle 0, -9.81 \rangle$

$\vec{v}(t) = \int \vec{a}(t)\,dt = \langle 0 + C_1, -9.81t + C_2 \rangle$

$\vec{v}(0) = \langle M\cos 30°, M\sin 30° \rangle$

$= \langle \frac{\sqrt{3}}{2}M, \frac{1}{2}M \rangle$

$\langle \frac{\sqrt{3}}{2}M, \frac{1}{2}M \rangle = \langle 0 + C_1, -9.81(0) + C_2 \rangle$

$C_1 = \frac{\sqrt{3}}{2}M, \quad C_2 = \frac{1}{2}M$

$\vec{v}(t) = \langle \frac{\sqrt{3}}{2}M, -9.81t + \frac{1}{2}M \rangle$

$\vec{r}(t) = \int \vec{v}(t)\,dt = \langle \frac{\sqrt{3}}{2}Mt + C_3, -4.9t^2 + \frac{1}{2}Mt + C_4 \rangle$

$\vec{r}(0) = \langle 0, 0 \rangle$

$\langle 0, 0 \rangle = \langle \frac{\sqrt{3}}{2}M(0) + C_3, -4.9(0)^2 + \frac{1}{2}M(0) + C_4 \rangle$

$C_3 = 0, \quad C_4 = 0$

$\vec{r}(t) = \langle \frac{\sqrt{3}}{2}Mt, -4.9t^2 + \frac{1}{2}Mt \rangle$

max height when $v_y = 0$

$-9.81t + \frac{1}{2}M = 0, \quad 9.81t = \frac{1}{2}M$

$t = \frac{M}{2(9.81)}$

max height is $r_y = -4.9t^2 + \frac{1}{2}Mt$, at $t = \frac{M}{2(9.81)}$

$500 = -4.9\left(\frac{M}{2(9.81)}\right)^2 + \frac{1}{2}M\left(\frac{M}{2(9.81)}\right)$

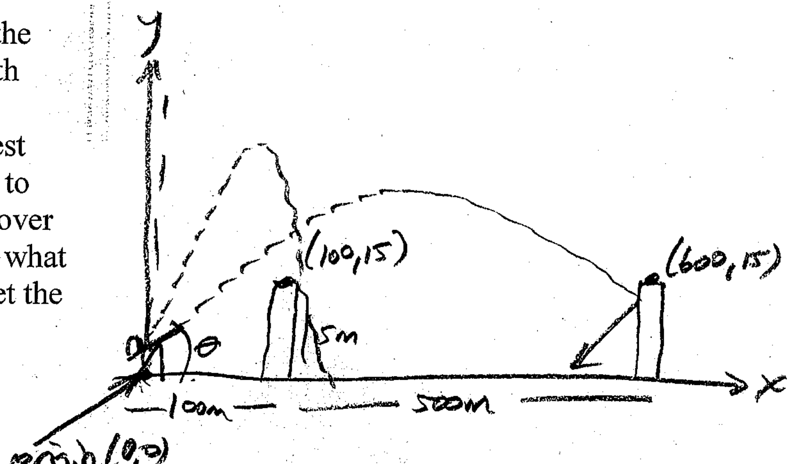
$500 = -0.01272911\,M^2 + 0.02548419\,98\,M^2$

$500 = .012755\,M^2$

$M^2 = 39200$

$M = \sqrt{39200} = 197.99 \approx \boxed{198 \text{ m/s}}$

#7b. (Very challenging) A medieval city has the shape of a square and is protected by walls with length 500 m and height 15 m. You are the commander of an attacking army and the closest you can get to the wall is 100 m. Your plan is to set fire to the city by catapulting heated rocks over the wall (with and initial speed of 80 m/s). At what range of angles should you tell your army to set the catapult? (Assume the path of the rocks is perpendicular to the wall).



$\vec{a}(t) = \langle 0, -9.81 \rangle$

to make it in, rocks must either go through point (100,15) or (600,15)

$\vec{v}(t) = \int \vec{a}(t)\,dt = \langle 0 + C_1, -9.81t + C_2 \rangle$

$\vec{v}(0) = \langle 80\cos\theta, 80\sin\theta \rangle$

$\langle 80\cos\theta, 80\sin\theta \rangle = \langle 0 + C_1, -9.81(0) + C_2 \rangle$

$C_1 = 80\cos\theta, \quad C_2 = 80\sin\theta$

$\vec{v}(t) = \langle 80\cos\theta, -9.81t + 80\sin\theta \rangle$

$\vec{r}(t) = \int \vec{v}(t)\,dt = \langle 80\cos\theta\, t + C_2, -4.9t^2 + 80\sin\theta\, t + C_3 \rangle$

$\vec{r}(0) = \langle 0, 0 \rangle$

$\langle 0, 0 \rangle = \langle 80\cos\theta(0) + C_2, -4.9(0)^2 + 80\sin\theta(0) + C_3 \rangle$

$C_2 = 0, \quad C_3 = 0$

$\vec{r}(t) = \langle 80\cos\theta\, t, -4.9t^2 + 80\sin\theta\, t \rangle$

short throw, must go through $\vec{r} = \langle 100, 15 \rangle = \langle 80\cos\theta\, t, -4.9t^2 + 80\sin\theta\, t \rangle$

$\begin{cases} 80\cos\theta\, t = 100 \rightarrow t = \frac{100}{80\cos\theta} \\ -4.9t^2 + 80\sin\theta\, t = 15 \end{cases}$

$-4.9\left(\frac{100}{80\cos\theta}\right)^2 + 80\sin\theta\left(\frac{100}{80\cos\theta}\right) = 15$, $-7.65625\underbrace{\sec^2\theta}_{\tan^2\theta+1} + 100\tan\theta = 15$

$-7.65625(\tan^2\theta + 1) + 100\tan\theta - 15 = 0$, $-7.65625\tan^2\theta + 100\tan\theta - 22.65625 = 0$

make $\tan\theta = u$: $-7.65625u^2 + 100u - 22.65625 = 0$, solve by calculator, $u = 0.2306, 12.83$

so $\tan\theta = 0.2306 \rightarrow \underline{\theta = 12.98^\circ}$, or $\tan\theta = 12.83 \rightarrow \underline{\theta = 85.54^\circ}$

long throw, must go through $\vec{r} = \langle 600, 15 \rangle = \langle 80\cos\theta\, t, -4.9t^2 + 80\sin\theta\, t \rangle$

$\begin{cases} 80\cos\theta\, t = 600 \rightarrow t = \frac{600}{80\cos t} \\ -4.9t^2 + 80\sin\theta\, t = 15 \end{cases}$

$-4.9\left(\frac{600}{80\cos\theta}\right)^2 + 80\sin\theta\left(\frac{600}{80\cos\theta}\right) = 15$, $-275.625\sec^2\theta + 600\tan\theta = 15$, $-275.625(\tan^2\theta + 1) + 600\tan\theta = 15$

This time, $\tan\theta = 0.7275$ or $1.44936 \rightarrow \theta = \underline{36.04^\circ}$ or $\underline{55.39^\circ}$

short throw $\theta = 12.98^\circ$ —— 85.54°

long throw $\theta = 36.04^\circ$ —— 55.40

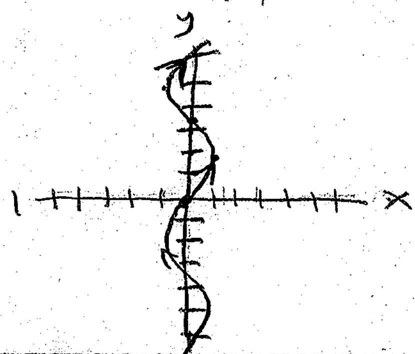
$\boxed{\text{range of angles} = 36^\circ - 55^\circ}$

Ch13 Test Review

#1. Sketch the curve with the given vector equation. Indicate with an arrow the direction in which t increases (always do this for vector function curves, whether it asks for it or not).

$\vec{r}(t) = \langle \sin t,\ t \rangle$.

t	$\langle \sin t, t \rangle$
0	$\langle 0, 0 \rangle$
$\frac{\pi}{2}$	$\langle 1, \frac{\pi}{2} \rangle$
π	$\langle 0, \pi \rangle$
$\frac{3\pi}{2}$	$\langle -1, \frac{3\pi}{2} \rangle$

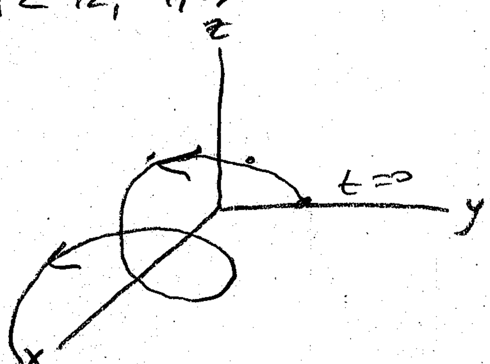


#2. Sketch the curve with the given vector equation. $\vec{r}(t) = \langle t,\ \cos 2t,\ \sin 2t \rangle$.

t	$\langle t, \cos 2t, \sin 2t \rangle$
0	$\langle 0, 1, 0 \rangle$
$\pi/8$	$\langle \pi/8, \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$
$\pi/4$	$\langle \pi/4, 0, 1 \rangle$
$3\pi/8$	$\langle 3\pi/8, -\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$
$\pi/2$	$\langle \pi/2, -1, 0 \rangle$

circle in yz

(a 'helix')



#3. Find a vector equation and parametric equations for the line segment that joins P to Q..
$P(1,-1,2)$, $Q(4,1,7)$.

$\vec{r}(t) = (1-t)\vec{r_0} + t\vec{r_1}$

$\vec{r}(t) = (1-t)\langle 1,-1,2 \rangle + t\langle 4,1,7 \rangle$

$= \langle 1-t, -1+t, 2-2t \rangle + \langle 4t, t, 7t \rangle$

$$\boxed{\begin{array}{l} \vec{r}(t) = \langle 1+3t, -1+2t, 2+5t \rangle \quad 0 \le t \le 1 \\ x = 1+3t,\ y = -1+2t,\ z = 2+5t \end{array}}$$

#4. Find a vector equation and parametric equations for the line segment that joins P to Q..
$P(0,0,0)$, $Q(1,2,3)$.

$\vec{r}(t) = (1-t)\vec{r_0} + t\vec{r_1}$

$= (1-t)\langle 0,0,0 \rangle + t\langle 1,2,3 \rangle$

$= \langle 0,0,0 \rangle + \langle t, 2t, 3t \rangle$

$$\boxed{\begin{array}{l} \vec{r}(t) = \langle t, 2t, 3t \rangle \quad 0 \le t \le 1 \\ x = t,\ y = 2t,\ z = 3t \end{array}}$$

#5. Find a vector equation that represents the curve of intersection of the two surfaces $x^2 + y^2 = 4$ and $z = xy$.

as a system: $\begin{cases} z = xy \to y = \frac{z}{x} \\ x^2 + y^2 = 4 \end{cases}$

$x^2 + \left(\frac{z}{x}\right)^2 = 4$, $\frac{z^2}{x^2} = 4 - x^2$

$z^2 = (4-x^2)x^2$, so $z = \pm\sqrt{(4-x^2)x^2}$

then write z and y in terms of x

but because z has +/- this would

require two equations.... try something

else...

$x^2+y^2=4$ is a circle which has

standard parametrization of $\langle r\cos\theta, r\sin\theta \rangle$

maybe use t as angle parameter θ;

$x^2+y^2=4 \Rightarrow \vec{r}_1(t) = \langle 2\cos t, 2\sin t \rangle$

(where $x = r\cos t = 2\cos t$, $y = 2\sin t$)

so $z = xy = (2\cos t)(2\sin t) = 4\cos t\sin t$

if we make z this, we are staying on both surfaces:

$$\boxed{\begin{array}{c} \vec{r}(t) = \langle 2\cos t, 2\sin t, 4\cos t\sin t \rangle \\ 0 \le t \le 2\pi \\ \text{(once around the circle)} \end{array}}$$

#6. Find a vector equation that represents the curve of intersection of the two surfaces $z = 4x^2 + y^2$ and $y = x^2$.

as a system: $\begin{cases} y = x^2 \\ z = 4x^2 + y^2 \end{cases}$

intersection: $y = x^2$ into $z = 4x^2 + y^2$

$z = 4x^2 + (x^2)^2 = 4x^2 + x^4$

$y = x^2$

both in terms of x, so make x the parameter, $t = x$

$\boxed{\vec{r}(t) = \langle t, t^2, 4t^2 + t^4 \rangle}$

$\boxed{-\infty < t < \infty}$

#8. Find the unit tangent vector $\vec{T}(t)$ at the point with the given value of the parameter t:

$\vec{r}(t) = \langle 4\sqrt{t}, t^2, t \rangle$, $t = 1$. $4t^{1/2}$ $\quad \vec{T}(1) = \frac{\vec{r}'(1)}{|\vec{r}'(1)|}$

$\vec{r}'(t) = \langle 2t^{-1/2}, 2t, 1 \rangle$

$= \langle \frac{2}{\sqrt{t}}, 2t, 1 \rangle$

$\vec{r}'(1) = \langle 2, 2, 1 \rangle$

$|\vec{r}'(1)| = \sqrt{2^2 + 2^2 + 1^2} = 3$

$\vec{T}(1) = \frac{\langle 2, 2, 1 \rangle}{3} = \boxed{\langle \frac{2}{3}, \frac{2}{3}, \frac{1}{3} \rangle}$

#7. Find the derivative of the vector function $\vec{r}(t) = \langle t \sin t, t^2, t \cos 2t \rangle$. (product rule)

$\vec{r}'(t) = \langle t\cos t + \sin t(1), 2t, t(-2\sin 2t) + \cos 2t(1) \rangle$

$= \boxed{\langle t\cos t + \sin t, 2t, -2t\sin 2t + \cos 2t \rangle}$

#9. Find parametric equations for the tangent line to the curve with the given parametric equations at the specified point:

$x = 1 + 2\sqrt{t}$, $y = t^3 - t$, $z = t^3 + t$; $(3, 0, 2)$.

tan line: $\vec{r}(t) = \vec{r}_0 + t\vec{v}$ ← this is curve's $\vec{r}'$

$\vec{r}(t) = \langle 1 + 2t^{1/2}, t^3 - t, t^3 + t \rangle$ (curve)

$\vec{r}'(t) = \langle t^{-1/2}, 3t^2 - 1, 3t^2 + 1 \rangle = \vec{v}$ for the tan line (curve)

t value at point:

$\vec{r}_{curve} = \langle 1 + 2\sqrt{t}, t^3 - t, t^3 + t \rangle \leftrightarrow$ match $(3, 0, 2)$

$1 + 2\sqrt{t} = 3$, $2\sqrt{t} = 2$, $\sqrt{t} = 1$, $\boxed{t = 1}$

so $\vec{r}'_{curve}(1) = \vec{v}_{\text{for tan line}} = \langle 1^{-1/2}, 3(1)^2 - 1, 3(1)^2 + 1 \rangle$

$= \langle 1, 2, 4 \rangle$

and $\vec{r}_0 = \langle 3, 0, 2 \rangle$

tangent line: $\vec{r}(t) = \vec{r}_0 + t\vec{v}$

$= \langle 3, 0, 2 \rangle + t\langle 1, 2, 4 \rangle$

$\boxed{x = 3 + t, \ y = 2t, \ z = 2 + 4t \quad -\infty < t < \infty}$

#10. Find $\vec{r}(t)$ if $\vec{r}'(t)=\langle 2t,\ 3t^2,\ \sqrt{t}\rangle$ and $\vec{r}(1)=\langle 1,1,0\rangle$.

$\vec{r}(t)=\int \vec{r}'(t)dt$

$=\langle \int 2t\,dt, \int 3t^2dt, \int t^{1/2}dt\rangle$

$=\langle t^2+C_1, t^3+C_2, \frac{2}{3}t^{3/2}+C_3\rangle$

$\langle 1,1,0\rangle=\langle 1^2+C_1, 1^3+C_2, \frac{2}{3}(1)^{3/2}+C_3\rangle$

$1+C_1=1$, $1+C_2=1$, $\frac{2}{3}+C_3=0$

$C_1=0$ $C_2=0$ $C=-2/3$

$\boxed{\vec{r}(t)=\langle t^2, t^3, \frac{2}{3}t^{3/2}-2/3\rangle}$

#11. Find $\vec{r}(t)$ if $\vec{r}'(t)=\langle t,\ e^t,\ te^t\rangle$ and $\vec{r}(0)=\vec{i}+\vec{j}+\vec{k}$. $=\langle 1,1,1\rangle$

$\vec{r}(t)=\int \vec{r}'(t)dt$

$=\langle \int t\,dt, \int e^t dt, \int te^t dt\rangle$

$=\langle \frac{1}{2}t^2+C_1, e^t+C_2$ by parts: $u=t$ $dv=e^t$ $\rangle$

$\frac{du}{dt}=1$ $\int dv=\int e^t dt$

$du=dt$ $v=e^t$

$uv-\int v\,du$

$te^t-\int e^t dt$

$te^t-e^t+C_3$

$=\langle \frac{1}{2}t^2+C_1, e^t+C_2, te^t-e^t+C_3\rangle$

$\langle 1,1,1\rangle=\langle \frac{1}{2}(0)^2+C_1, e^0+C_2, (0)e^0-e^0+C_3\rangle$

$0+C_1=1$, $1+C_2=1$, $0-1+C_3=1$

$C_1=1$ $C_2=0$ $C_3=2$

$\boxed{\vec{r}(t)=\langle \frac{1}{2}t^2+1, e^t, te^t-e^t+2\rangle}$

#12. Find the length of the curve $\vec{r}(t)=\langle 2t,\ t^2,\ \frac{1}{3}t^3\rangle$, $0\le t\le 1$. $L=\int_a^b|\vec{r}'(t)|dt$

$\vec{r}'(t)=\langle 2, 2t, t^2\rangle$

$|\vec{r}'(t)|=\sqrt{4+4t^2+t^4}=\sqrt{(t^2+2)^2}=t^2+2$

$L=\int_0^1(t^2+2)dt$

$=[\frac{1}{3}t^3+2t]_0^1$

$=(\frac{1}{3}(1)^3+2(1))-(\frac{1}{3}(0)^3+2(0))$

$=\frac{1}{3}+2=\frac{1}{3}+\frac{6}{3}$

$=\boxed{\frac{7}{3}}$

#13. Find the length of the curve $\vec{r}(t)=\vec{i}+t^2\vec{j}+t^3\vec{k}$, $0\le t\le 1$. $L=\int_a^b|\vec{r}'(t)|dt$

$=\langle 1, t^2, t^3\rangle$

$\vec{r}'(t)=\langle 0, 2t, 3t^2\rangle$

$|\vec{r}'(t)|=\sqrt{4t^2+9t^4}=\sqrt{t^2(4+9t^2)}$

$=\sqrt{t^2}\sqrt{4+9t^2}=t\sqrt{4+9t^2}$

$L=\int_0^1 t\sqrt{4+9t^2}\,dt$ u-sub: $u=4+9t^2$

$\frac{du}{dt}=18t$, $du=18t\,dt$

$t\,dt=\frac{1}{18}du$

$=\frac{1}{18}\int_4^{13}u^{1/2}du$

$=\frac{1}{18}\left[\frac{2}{3}u^{3/2}\right]_4^{13}=\frac{1}{27}\left[u^{3/2}\right]_4^{13}$

$=\frac{1}{27}(13^{3/2}-4^{3/2})$

$\boxed{=\frac{1}{27}((\sqrt{13})^3-8)\approx 1.4397}$

#14. Find the length of the curve (correct to four decimal places):

$\vec{r}(t) = \langle \sin t,\ \cos t,\ \tan t \rangle, \quad 0 \le t \le \frac{\pi}{4}$.

$L = \int_a^b |\vec{r}'(t)|\,dt$

$\vec{r}'(t) = \langle \cos t, -\sin t, \sec^2 t \rangle$

$|\vec{r}'(t)| = \sqrt{\cos^2 t + \sin^2 + \sec^4 t}$

$= \sqrt{1 + \sec^4 t}$

$L = \int_0^{\pi/4} \sqrt{1+\sec^4 t}\, dt$

$= \boxed{1.27798}$

(enter $\left(\frac{1}{\cos t}\right)^4$ for $\sec^4 t$)

#15. Find the curvature κ of $\vec{r}(t) = \langle t,\ t^2,\ t^3 \rangle$ at the point $(1,1,1)$.

since it isn't asking for $\vec{T}, \vec{N}, \ldots$

$K = \frac{|\vec{r}' \times \vec{r}''|}{|r'|^3}$

$\vec{r}'(t) = \langle 1, 2t, 3t^2 \rangle$

$\vec{r}''(t) = \langle 0, 2, 6t \rangle$

$\vec{r}' \times \vec{r}'' = \begin{vmatrix} + & - & + \\ 1 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix}$

$= \langle 12t^2 - 6t^2, -(6t - 0), 2 - 0 \rangle$

$= \langle 6t^2, -6t, 2 \rangle$

$|\vec{r}' \times r''| = \sqrt{36t^4 + 36t^2 + 4}$

at $(1,1,1)$ $t=1$

$\vec{r}'(1) = \langle 1, 2(1), 3(1)^2 \rangle = \langle 1, 2, 3 \rangle$

$|\vec{r}'(1)| = \sqrt{1^2+2^2+3^2} = \sqrt{14}$

$|\vec{r}' \times \vec{r}''|_{t=1} = \sqrt{36+36+4} = \sqrt{76}$

$K = \frac{|\vec{r}' \times \vec{r}''|}{|\vec{r}'|^3} = \boxed{\frac{\sqrt{76}}{(\sqrt{14})^3} = \frac{\sqrt{76}}{14\sqrt{14}}}$

#16. Find the unit tangent vector $\vec{T}(t)$, unit normal vector $\vec{N}(t)$, and curvature κ for

$\vec{r}(t) = \langle 2\sin t,\ 5t,\ 2\cos t \rangle$.

$\vec{r}'(t) = \langle 2\cos t, 5, -2\sin t \rangle$

$|\vec{r}'(t)| = \sqrt{4\cos^2 t + 4\sin^2 t + 25} = \sqrt{4+25} = \sqrt{29}$

$\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} = \boxed{\frac{1}{\sqrt{29}} \langle 2\cos t, 5, -2\sin t \rangle}$

$\vec{T}'(t) = \frac{1}{\sqrt{29}} \langle -2\sin t, 0, -2\cos t \rangle$

$|\vec{T}'(t)| = \frac{1}{\sqrt{29}} \sqrt{4\sin^2 t + 4\cos^2 t} = \frac{1}{\sqrt{29}} \sqrt{4}$

$= \frac{2}{\sqrt{29}}$

$\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|} = \frac{\frac{1}{\sqrt{29}} \langle -2\sin t, 0, -2\cos t \rangle}{\frac{2}{\sqrt{29}}}$

$\vec{N}(t) = \frac{1}{2} \langle -2\sin t, 0, -2\cos t \rangle$

$\boxed{\vec{N}(t) = \langle -\sin t, 0, -\cos t \rangle}$

$K = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|} = \frac{\left(\frac{2}{\sqrt{29}}\right)}{\sqrt{29}} = \frac{\left(\frac{2}{\sqrt{29}}\right)}{\left(\frac{\sqrt{29}}{1}\right)}$

$= \frac{2}{\sqrt{29}} \cdot \frac{1}{\sqrt{29}} = \boxed{\frac{2}{29}}$

#17. Find the unit tangent vector $\vec{T}(t)$, unit normal vector $\vec{N}(t)$, and curvature κ for $\vec{r}(t) = \left\langle t, \frac{1}{2}t^2, t^2 \right\rangle$.

$\vec{r}'(t) = \langle 1, t, 2t \rangle$

$|\vec{r}'(t)| = \sqrt{1+t^2+4t^2} = \sqrt{1+5t^2}$

$$\boxed{\vec{T}(t) = \frac{\vec{r}'(t)}{|\vec{r}'(t)|} = \frac{1}{\sqrt{1+5t^2}}\langle 1, t, 2t \rangle}$$

$$\vec{T}'(t) = \frac{1}{\sqrt{1+5t^2}}\frac{d}{dt}\left[\langle 1,t,2t\rangle\right] + \langle 1,t,2t\rangle \frac{d}{dt}\left[(1+5t^2)^{-1/2}\right]$$

(product rule)

$$= \frac{1}{\sqrt{1+5t^2}}\langle 0,1,2\rangle + \langle 1,t,2t\rangle\left(-\tfrac{1}{2}(1+5t^2)^{-3/2}(10t)\right)$$

$$= \frac{\langle 0,1,2\rangle}{\sqrt{1+5t^2}} + \langle 1,t,2t\rangle\left(\frac{-5t}{\sqrt{1+5t^2}(1+5t^2)}\right)$$

$$= \frac{\langle 0,1,2\rangle(1+5t^2)}{\sqrt{1+5t^2}(1+5t^2)} + \frac{\langle 1,t,2t\rangle(-5t)}{\sqrt{1+5t^2}(1+5t^2)}$$

$$= \frac{\langle 0, 1+5t^2, 2+10t^2\rangle + \langle -5t, -5t^2, -10t^2\rangle}{\sqrt{1+5t^2}(1+5t^2)}$$

$$\vec{T}'(t) = \frac{1}{(1+5t^2)^{3/2}}\langle -5t, 1, 2\rangle$$

$$|\vec{T}'(t)| = \frac{1}{(1+5t^2)^{3/2}}\sqrt{25t^2+1+4}$$

$$= \frac{1}{(1+5t^2)^{3/2}}\sqrt{25t^2+5}$$

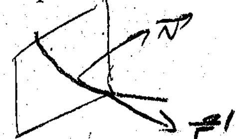
$$\vec{N}(t) = \frac{\vec{T}'(t)}{|\vec{T}'(t)|} = \frac{\left(\frac{1}{(1+5t^2)^{3/2}}\right)\langle -5t,1,2\rangle}{\left(\frac{1}{(1+5t^2)^{3/2}}\right)\sqrt{25t^2+5}}$$

$$\boxed{\vec{N}(t) = \frac{1}{\sqrt{25t^2+5}}\langle -5t, 1, 2\rangle}$$

$$\kappa = \frac{|\vec{T}'(t)|}{|\vec{r}'(t)|} = \frac{\left(\frac{1}{(1+5t^2)^{3/2}}\right)\sqrt{25t^2+5}}{\sqrt{1+5t^2}}$$

$$\boxed{\kappa} = \frac{\sqrt{25t^2+5}}{(1+5t^2)^{3/2}\sqrt{1+5t^2}} = \boxed{\frac{\sqrt{25t^2+5}}{(1+5t^2)^2}}$$

#18. Find the equation of the normal plane of the curve at the given point $x=t,\ y=t^2,\ z=t^3;\ (1,1,1)$.



$\vec{n}$ for normal plane is $\vec{r}'$!

$\vec{r}(t) = \langle t, t^2, t^3 \rangle$

$\vec{r}'(t) = \langle 1, 2t, 3t^2 \rangle$

at $(1,1,1)$ $t=1$ (match xs)

$\vec{r}'(1) = \langle 1,2,3\rangle = \vec{n}$

$\vec{r_0} = \langle 1,1,1\rangle$

$ax+by+cz = \vec{n}\cdot\vec{r_0}$

$1x+2y+3z = \langle 1,2,3\rangle \cdot \langle 1,1,1\rangle$

$= (1)(1)+(2)(1)+(3)(1)$

$= 1+2+3$

$$\boxed{x+2y+3z=6}$$

#19. Find the velocity, acceleration, and speed of a particle with the given position function $\vec{r}(t) = \langle t^2+1,\ t^3,\ t^2-1\rangle$.

$$\boxed{\vec{v} = \vec{r}' = \langle 2t, 3t^2, 2t\rangle}$$

speed $= |\vec{v}| = \sqrt{4t^2+9t^4+4t^2}$

$$\boxed{\text{speed} = \sqrt{8t^2+9t^4}}$$

$$\boxed{\vec{a} = \vec{v}' = \langle 2, 6t, 2\rangle}$$

#20. Find the velocity, acceleration, and speed of a particle with the given position function $\vec{r}(t)=\sqrt{2}t\,\vec{i}+e^{t}\,\vec{j}+e^{-t}\,\vec{k}$.

$\boxed{\vec{v}=\vec{r}\,'=\langle \sqrt{2}, e^{t}, -e^{-t}\rangle}$

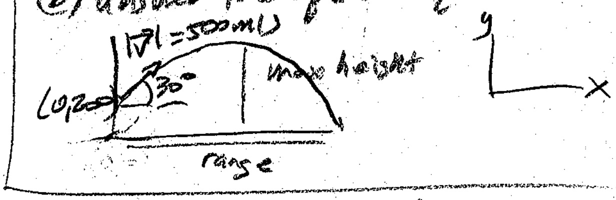
$\text{speed}=|\vec{v}|=\sqrt{(\sqrt{2})^2+(e^t)^2+(-e^{-t})^2}$

$\text{speed}=\boxed{\sqrt{2+e^{2t}+e^{-2t}}}$

$\boxed{\vec{a}=\vec{v}\,'=\langle 0, e^{t}, e^{-t}\rangle}$

#21. A projectile is fired from a position 200 m above the ground with an initial speed of 500 m/s and angle of elevation of 30°. Find (i) the range of the projectile, (ii) the maximum height reached, and (iii) the speed at impact.

① Start with acceleration and integrate twice, using initial conditions, to get $\vec{v}$ & $\vec{r}$.
② answer the specific questions.



① $\vec{a}=\langle 0,-9.81\rangle$ (m/s²)

$\vec{v}(t)=\int \vec{a}(t)dt=\langle \int 0dt, \int -9.81dt\rangle$

$\vec{v}(t)=\langle 0+C_1, -9.81t+C_2\rangle$

$\vec{v}(0)=\langle 500\cos 30°, 500\sin 30°\rangle$
$=\langle 433.013, 250\rangle$

$\langle 433.013, 250\rangle=\langle 0+C_1, -9.81(0)+C_2\rangle$

$0+C_1=433.013$, $0+C_2=250$
$C_1=433.013$, $C_2=250$

so $\boxed{\vec{v}(t)=\langle 433.013, -9.81t+250\rangle}$

now, $\vec{r}(t)=\int \vec{v}dt$

$\vec{r}(t)=\langle \int 433.013dt, \int(-9.81t+250)dt\rangle$

$\vec{r}(t)=\langle 433.013t+C_3, -4.9t^2+250t+C_4\rangle$

$\vec{r}(0)=\langle 0,200\rangle$

$\langle 0,200\rangle=\langle 433.013(0)+C_3, -4.9(0)^2+250(0)+C_4\rangle$

$0+C_3=0$, $0+C_4=200$
$C_3=0$, $C_4=200$

so $\boxed{\vec{r}(t)=\langle 433.013t, -4.9t^2+250t+200\rangle}$

② using $\vec{r}$ & $\vec{v}$ to answer questions...

(i) $\boxed{\text{range}}$ is x comp. of $\vec{r}$, when y of $\vec{r}=0$: (on ground)

r_y: $-4.9t^2+250t+200=0$

$t=\dfrac{-250\pm\sqrt{250^2+4(4.9)(200)}}{2(-4.9)}=-.787, \boxed{51.8}$ sec

$r_x=433.013(51.8)=\boxed{22433.6\text{m}}$

(ii) $\boxed{\text{max height}}$ when y comp. of $\vec{v}=0$:

$v_y=-9.81t+250=0$

$t=\frac{250}{9.81}=25.48$ sec

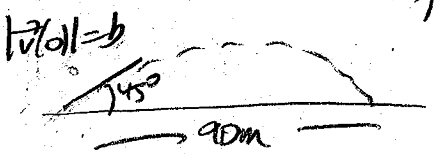
max height $=r_y$ at ↗ $=-4.9(25.48)^2+250(25.48)+200$
$=\boxed{3388.7\text{m}}$

(iii) $\boxed{\text{speed}}$ at impact $(t=51.8)$

$\vec{v}(51.8)=\langle 433.013, -9.81(51.8)+250\rangle$
$=\langle 433.013, -258.158\rangle$

$\text{speed}=|\vec{v}|=\sqrt{433.013^2+258.158^2}$
$=\boxed{503.6\text{ m/s}}$

#22. A ball is thrown at an angle of 45° to the ground. If the ball lands 90 m away, what was the initial speed of the ball? *ball speed = b*



$\vec{a}(t) = \langle 0, -9.81 \rangle$ (m/s^2)

$\vec{v}(t) = \int \vec{a}(t)\,dt = \langle 0 + C_1, -9.81t + C_2 \rangle$

$\vec{v}(0) = \langle b\cos 45°, b\sin 45° \rangle = \langle \frac{\sqrt{2}}{2}b, \frac{\sqrt{2}}{2}b \rangle$

$\langle \frac{\sqrt{2}}{2}b, \frac{\sqrt{2}}{2}b \rangle = \langle 0 + C_1, -9.81(0) + C_2 \rangle$

$0 + C_1 = \frac{\sqrt{2}}{2}b$, $\quad 0 + C_2 = \frac{\sqrt{2}}{2}b$

$\underline{C_1 = \frac{\sqrt{2}}{2}b}$ $\quad \underline{C_2 = \frac{\sqrt{2}}{2}b}$

so $\vec{v}(t) = \langle \frac{\sqrt{2}}{2}b, -9.81t + \frac{\sqrt{2}}{2}b \rangle$

$\vec{r}(t) = \int \vec{v}(t)\,dt = \langle \frac{\sqrt{2}}{2}bt + C_3, -4.9t^2 + \frac{\sqrt{2}}{2}bt + C_4 \rangle$

$\vec{r}(0) = \langle 0, 0 \rangle$

$\langle 0, 0 \rangle = \langle \frac{\sqrt{2}}{2}b(0) + C_3, -4.9(0)^2 + \frac{\sqrt{2}}{2}b(0) + C_4 \rangle$

$0 + C_3 = 0$, $\quad 0 + C_4 = 0$

$\underline{C_3 = 0}$ $\quad \underline{C_4 = 0}$

so $\vec{r}(t) = \langle \frac{\sqrt{2}}{2}bt, -4.9t^2 + \frac{\sqrt{2}}{2}bt \rangle$

---

range = 90m occurs when $r_y = 0$ (on ground)

$r_y = -4.9t^2 + \frac{\sqrt{2}}{2}bt = 0$

$t(-4.9t + \frac{\sqrt{2}}{2}b) = 0$; $\quad t = 0$ or $-4.9t + \frac{\sqrt{2}}{2}b = 0$

$t = \frac{\sqrt{2}b}{2(4.9)} = \frac{\sqrt{2}b}{9.81}$

range is r_x at this time; $\quad r_x = \frac{\sqrt{2}}{2}b\left(\frac{\sqrt{2}b}{9.81}\right) = 90$

$\frac{b^2}{9.81} = 90$, $\quad b^2 = 90(9.81)$

$b = \sqrt{90(9.81)}$

$$\boxed{b = 29.71 \text{ m/s}}$$

#23. Find the curvature of the ellipse $x = 3\cos t$, $y = 4\sin t$ at the points $(3,0)$ and $(0,4)$.

$\kappa = \dfrac{|\vec{r}' \times \vec{r}''|}{|r'|^3}$ (since they are not asking for T)

need 3D to do cross prod.

$\vec{r}(t) = \langle 3\cos t, 4\sin t, 0\rangle$

$\vec{r}'(t) = \langle -3\sin t, 4\cos t, 0\rangle$

$\vec{r}''(t) = \langle -3\cos t, -4\sin t, 0\rangle$

$\vec{r}' \times \vec{r}'' = \begin{vmatrix} + & - & + \\ -3\sin t & 4\cos t & 0 \\ -3\cos t & -4\sin t & 0 \end{vmatrix}$

$= \langle 0-0, -(0-0), 12\sin^2 t + 12\cos^2 t\rangle$

$= \langle 0, 0, 12\rangle$

$|\vec{r}' \times \vec{r}''| = 12$ (at all points)

$(3,0)$	$(0,4)$
$x = 3\cos t = 3$	$y = 4\sin t = 4$
$\cos t = 1$	$\sin t = 1$
$\boxed{t = 0}$	$\boxed{t = \pi/2}$
$\vec{r}'(0) =$	$\vec{r}'(\frac{\pi}{2}) =$
$\langle -3\sin 0, 4\cos 0, 0\rangle$	$\langle -3\sin\frac{\pi}{2}, 4\cos\frac{\pi}{2}, 0\rangle$
$\langle 0, 4, 0\rangle$	$\langle -3, 0, 0\rangle$
$\lvert\vec{r}'\rvert = 4$	$\lvert\vec{r}'\rvert = 3$
$\kappa = \dfrac{\lvert\vec{r}' \times \vec{r}''\rvert}{\lvert r'\rvert^3}$	$\kappa = \dfrac{\lvert\vec{r}' \times \vec{r}''\rvert}{\lvert\vec{r}'\rvert^3}$
$= \dfrac{12}{(4)^3} = 0.1875$	$= \dfrac{12}{(3)^3} \approx 0.444$

#24. An athlete puts a shot (throws an object) at an angle of 45° to the horizontal at an initial speed 43 ft/s. It leaves the athlete's hand 7 ft above the ground. (i) Where is the shot 2 seconds later? (ii) What is the maximum height of the object? (iii) Where does the object land?

$\vec{a}(t) = \langle 0, -32\rangle$ (ft/s²)

$\vec{v}(t) = \int \vec{a}(t)\,dt = \langle 0 + C_1, -32t + C_2\rangle$

$\vec{v}(0) = \langle 43\cos 45°, 43\sin 45°\rangle$

$= \langle 43\frac{\sqrt{2}}{2}, 43\frac{\sqrt{2}}{2}\rangle$

$\langle 43\frac{\sqrt{2}}{2}, 43\frac{\sqrt{2}}{2}\rangle = \langle 0 + C_1, -32(0) + C_2\rangle$

$0 + C_1 = \frac{43\sqrt{2}}{2}$, $0 + C_2 = \frac{43\sqrt{2}}{2}$

$C_1 = \frac{43\sqrt{2}}{2}$ $\quad C_2 = \frac{43\sqrt{2}}{2}$

so $\vec{v}(t) = \langle \frac{43\sqrt{2}}{2}, -32t + \frac{43\sqrt{2}}{2}\rangle$

$\vec{r}(t) = \int \vec{v}(t)\,dt = \langle \frac{43\sqrt{2}}{2}t + C_3, -16t^2 + \frac{43\sqrt{2}}{2}t + C_4\rangle$

$\vec{r}(0) = \langle 0, 7\rangle$

$\langle 0, 7\rangle = \langle \frac{43\sqrt{2}}{2}(0) + C_3, -16(0)^2 + \frac{43\sqrt{2}}{2}(0) + C_4\rangle$

$0 + C_3 = 0$, $0 + C_4 = 7$

$C_3 = 0$ $\quad C_4 = 7$

So $\vec{r}(t) = \langle \frac{43\sqrt{2}}{2}t, -16t^2 + \frac{43\sqrt{2}}{2}t + 7\rangle$

(i) where is shot at $t = 2$?

$\vec{r}(2) = \langle \frac{43\sqrt{2}}{2}(2), -16(2)^2 + \frac{43\sqrt{2}}{2}(2) + 7\rangle$

$= \boxed{\langle 60.8, 3.81\rangle}$ 60.8 ft downrange, 3.81 ft above ground

(ii) max ht when $v_y = 0$; $-32t + \frac{43\sqrt{2}}{2} = 0$, $t = \frac{43\sqrt{2}}{2(32)} = 0.95$ sec

$ht = r_y = -16(0.95)^2 + \frac{43\sqrt{2}}{2}(0.95) + 7$

$= \boxed{21.44 \text{ ft}}$

(iii) where does the object land ($r_y = 0$ on ground)

when? $r_y = -16t^2 + \frac{43\sqrt{2}}{2}t + 7 = 0$

(quadratic formula or calculator graph)

at $t = -0.208$, $t = 2.108$ sec

$r_x = \frac{43\sqrt{2}}{2}(2.108) = \boxed{64.09 \text{ ft downrange}}$