Calc III - Ch 12 - Extra Practice

#1b. On a single set of coordinate axes, sketch the points (1,2,3), (5,0,0), (0,3,0), and (2,3,-4).

#2b. Describe and sketch the surface in $\mathbb{R}^3$ represented by the equation $2x+4y=4$.

#2c. Describe and sketch the surface in $\mathbb{R}^3$ represented by the equation $-2x+3y=6$.

#2d. Describe and sketch the surface in $\mathbb{R}^3$ represented by the equation $2x^2+2y=8$.

#2e. Describe and sketch the surface in $\mathbb{R}^3$ represented by the equation $y^2+4z^2-2y-16z+13=0$.

#3b. Find the lengths of the sides of the triangle PQR. Is it a right triangle? Is it an isosceles triangle? $P(2,-1,0)$, $Q(4,1,1)$, $R(4,-5,4)$.

#4b. Find an equation of the sphere with center $(2,-6,4)$ and radius 5. What is the intersection of this sphere with the yz-plane?

#5b. Show that the equation represents a sphere, and find its center and radius: $4x^2+4y^2+4z^2-8x+16y=1$

#6b. Find an equation of a sphere if one of its diameters has endpoints $(5,2,0)$ and $(3,8,8)$.

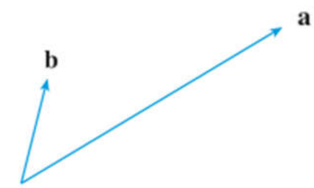
#7b. Describe in words the region of $\mathbb{R}^3$ represented by the equation or inequality: $x>3$.

#7c. Describe in words the region of $\mathbb{R}^3$ represented by the equation or inequality: $y = -4$.

#7d. Describe in words the region of $\mathbb{R}^3$ represented by the equation or inequality: $x^2 + y^2 + z^2 > 2z$.

12.2

#1b. For the given vectors $\vec{a}$ and $\vec{b}$:



sketch each of the following:

$\underline{\vec{a} + \vec{b}}$ $\underline{\vec{a} - \vec{b}}$ $\underline{\vec{b} - 2\vec{a}}$

#2b. Find the vector $\vec{AB}$ if $A(-2,-2)$, $B(5,3)$.

Then sketch $\vec{AB}$ starting at the origin.

#2c. Find the vector $\vec{AB}$ if $A(0,3,1)$, $B(2,3,-1)$.

Then sketch $\vec{AB}$ starting at the origin.

#3b. Find a unit vector that has the same direction as the given vector $-3\vec{i}+7\vec{j}$.

#3c. Find a unit vector that has the same direction as the given vector $\langle -4,2,4\rangle$.

#4b. Suppose that a wind is blowing from the direction N45°W at a speed of 50 km/hr (this mean that the direction the wind pushes the plane is 45° west of the northerly direction). A pilot is steering a plane in the direction N60°E at an airspeed (speed in still air) of 250 km/hr. The true course of the plane is the direction of the resultant of the velocity vectors of the plane and the wind. The ground speed of the plan is the magnitude of the resultant. Find the true course and the ground speed of the plane.

#5b. (For an example, please look at the example worked in the lesson notes.)

#6b. (This problem is setup and solved to the point where there is a system to solve in the lesson notes. From this point, look at the 'star' example to see how to solve the system.

#7. If *A, B,* and *C* are the vertices of a triangle, find $\vec{AB}+\vec{BC}+\vec{CA}$.

12.3

#1b. Which of the following expressions are meaningful and which are meaningless?

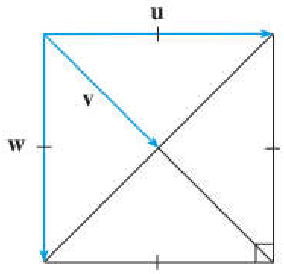
$$\vec{a}\bullet\left(\vec{b}+\vec{c}\right) \qquad \vec{a}\bullet\vec{b}+\vec{c} \qquad \left|\vec{a}\right|\bullet\left(\vec{b}\bullet\vec{c}\right)$$

#2b. Find $\vec{a}\bullet\vec{b}$ *if* $\vec{a}=\langle -2,3\rangle,\ \vec{b}=\langle 3,5\rangle$

#3b. Find $\vec{a}\bullet\vec{b}$ *if* $\vec{a}=\langle s,2t,3\rangle,\ \vec{b}=\langle t^2,5,s^3\rangle$

#4b.Find $\vec{a}\bullet\vec{b}$ *if* $\vec{a}=4\vec{j}-3\vec{k},\ \vec{b}=2\vec{i}+4\vec{j}+6\vec{k}$

#5b. If $\vec{u}$ is a unit vector, find $\vec{u} \bullet \vec{v}$ and $\vec{u} \bullet \vec{w}$.



#6b. Find the angle between the vectors (exact and decimal): $\vec{a} = \langle \sqrt{3}, 1 \rangle$, $\vec{b} = \langle 0, 5 \rangle$

#7b. Find the angle between the vectors (exact and decimal): $\vec{a} = \vec{i} + 2\vec{j} - 2\vec{k}$, $\vec{b} = 4\vec{i} - 3\vec{k}$

#8b. Determine whether the given vectors are orthogonal, parallel, or neither:

$\vec{a} = \langle -3, 9, 6 \rangle$, $\vec{b} = \langle 4, -12, -8 \rangle$

$\vec{a} = \vec{i} - \vec{j} + 2\vec{k}$, $\vec{b} = 2\vec{i} - \vec{j} + \vec{k}$

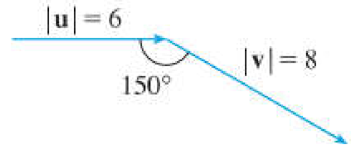
$\vec{a} = \langle x, y, z \rangle$, $\vec{b} = \langle -y, x, 0 \rangle$

#9b. Hint: vectors are orthogonal when their dot product is zero.

#10b. Hint: Two ways to solve:
1) graph in 3D and just figure it out visually.
2) postulate a general vector $\langle a, b, c \rangle$ and use dot product equals zero with the given vectors to find a,b,c.

#11b. Find the work done by a force $\vec{F} = 8\vec{i} - 6\vec{j} + 0\vec{k}$ that moves an object from the point $(0,10,8)$ to the point $(6,12,20)$ along a straight line. The distance is measured in meters and the force in Newtons.

#3b. Find $\left|\vec{u} \times \vec{v}\right|$ and determine whether $\vec{u} \times \vec{v}$ is directed into the page or out of the page.



12.4

#1b. Find the cross product $\vec{a} \times \vec{b}$ and verify that it is orthogonal to both $\vec{a}$ and $\vec{b}$.

$\vec{a} = \langle 6,0,-2\rangle,\ \vec{b} = \langle 0,8,0\rangle$

#2b. Which of the following expressions are meaningful and which are meaningless? If meaningful, state whether the result is a vector or a scalar.

$\left(\vec{a} \bullet \vec{b}\right) x\, \vec{c}$

$\left(\vec{a} \bullet \vec{b}\right) x \left(\vec{c} \bullet \vec{d}\right)$

$\left(\vec{a}\ x\ \vec{b}\right) \bullet \left(\vec{c}\ x\ \vec{d}\right)$

#4b. If $\vec{a} = \langle 2,3,1\rangle$ and $\vec{b} = \langle 1,-2,4\rangle$, find $\vec{a}\ x\ \vec{b}$ and $\vec{b}\ x\ \vec{a}$.

#5b. Show that $\left(\vec{a}\ x\ \vec{b}\right) \bullet \vec{b} = 0$ for all vectors $\vec{a}$ and $\vec{b}$ in $\mathbb{R}^3$.

#6b. Find the area of the parallelogram with vertices: $A(2,2)$, $B(6,3)$, $C(7,6)$, and $D(3,5)$.

#7b. (i) Find a nonzero vector orthogonal to the plane through the given points: $P(2,1,5), Q(-1,3,4), R(3,0,6)$.

(ii) Find the area of triangle *PQR*.

#8b. Find the volume of the parallelepiped with adjacent edges, *PQ, PR,* and *PS*.

$P(3,0,1),\ Q(-1,2,5),\ R(5,1,-1),\ S(0,4,2)$.

#9b. Use the scalar triple product to verify that the vectors

$\vec{u} = \langle 2,10,-4\rangle,\ \vec{v} = \left\langle \frac{5}{2}, \frac{9}{2}, -2 \right\rangle,\ \vec{w} = \langle 6,-2,0\rangle$ are coplanar.

12.5

#1b. Find a vector equation and parametric equations for the line though the point $(2,-5,2)$ and parallel to the vector $\left\langle 1,3,-\frac{2}{3}\right\rangle$.

#2b. Find parametric equations and symmetric equations for the line though the points $(5,1,-3)$ and $(2,4,5)$.

#3b. Find parametric equations and symmetric equations for the line though $(3,1,4)$ and perpendicular to both $\langle 2,1,0\rangle$ and $\langle 4,2,1\rangle$.

#4b. Is the line through $(4,1,-1)$ and $(2,5,3)$ perpendicular to the line through $(-3,2,0)$ and $(5,1,4)$?

#5b. Find a vector equation for the line segment from $(10,3,1)$ to $(5,6,-3)$.

#6b. Find an equation of the plane through the point $(4,0,-3)$ and with normal vector $\langle 0,1,2\rangle$.

#7b. Find an equation of the plane through the points $(3,-1,2)$, $(8,2,4)$, and $(-1,-2,-3)$.

#8b. Find an equation of the plane that passes through the line of intersection of the planes $y+z=2$ and $x+2y=4$ and is perpendicular to the plane $3x-2y+z=3$.

#9b. Find an equation of the plane that passes through the point $(2,3,4)$ and contains the line of intersection of the planes
$x-y-z=4$ and $2x+y+3z=6$

#10b. Where does the line through $(1,2,3)$ and $(6,3,4)$ intersect the plane $x+2y+z=4$?

#11b. Determine whether the plane are parallel, perpendicular, or neither. If neither, find the angle between them. $x+y+z=1, \quad x-y+z=1$

#12b. Find an equation for the plane consisting of all points that are equidistant from the points $(2,5,5)$ and $(-6,3,1)$.

12.6

#1b. Describe and sketch the surface: $x^2 - y^2 = 1$.

#2b. Describe and sketch the surface: $z = 4 - x^2$.

#3b. Describe and sketch the surface: $yz = 4$.

#4b. Draw at least three traces in each of the coordinate planes and identify the surface for $-x^2 + 4y^2 - z^2 = 4$.

#5b. Put the equation in standard form, then name and sketch the surface:
$4y^2 + z^2 - x - 16y - 4z + 20 = 0$.

#7b. Find an equation for the surface obtained by rotating the line $x = 3y$ about the x-axis.

#6b. Sketch the region bounded by the paraboloids $z = x^2 + y^2$ and $z = 2 - x^2 - y^2$.