Calc III Formulas for Ch16 Test

polar	*cylindrical*	*spherical*
$x = r\cos\theta$	$x = r\cos\theta$	$x = \rho\sin\phi\cos\theta$
$y = r\sin\theta$	$y = r\sin\theta$	$y = \rho\sin\phi\sin\theta$
	$z = z$	$z = \rho\cos\phi$
$dA = r\,dr\,d\theta$	$dV = r\,dz\,dr\,d\theta$	$dV = \rho^2\sin\phi\,d\rho\,d\phi\,d\theta$
$x^2+y^2=r^2$	$x^2+y^2=r^2$	$r = \rho\sin\phi$
		$x^2+y^2+z^2=\rho^2$

$$\vec{F} = \langle P, Q, R \rangle$$

$$div\ \vec{F} = \nabla\bullet\vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$curl\ \vec{F} = \nabla x\,\vec{F} = \begin{vmatrix} + & - & + \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ P & Q & R \end{vmatrix}$$

Line (path) Integrals…

$$\begin{matrix}Scalar\\Integrand\end{matrix}: \int_C f(x,y,z)\bullet ds = \int_a^b f\left(\vec{r}(t)\right)\left|\vec{r}'(t)\right| dt$$

$$\begin{matrix}Vector\\Field\end{matrix}: \int_C \vec{F}\bullet d\vec{r} = \int_a^b \vec{F}\left(\vec{r}(t)\right)\bullet\vec{r}'(t)\,dt$$

$$\begin{matrix}Green's\\Theorem\end{matrix}: \oint_C \vec{F}\bullet d\vec{r} = \oiint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y}\right) dA$$

$$\begin{matrix}Stokes'\\Theorem\end{matrix}: \int_C \vec{F}\bullet d\vec{r} = \iint_S \left(curl\ \vec{F}\right)\bullet d\vec{S} = \iint_S \left(curl\ \vec{F}\right)\bullet\vec{n}\ dA$$

Surface Integrals…

$$tangent\ plane\ \vec{n} = \vec{r_u}\ x\ \vec{r_v}$$

$$\begin{matrix}Scalar\\Integral\end{matrix}: \iint_S f(x,y,z)\,dS = \iint_D f\left(\vec{r}(u,v)\right)\left|\vec{r_u}\ x\ \vec{r_v}\right| dA\ \ or = \iint_D f\left(x,y,g(x,y)\right)\sqrt{1+\left(\frac{\partial g}{\partial x}\right)^2+\left(\frac{\partial g}{\partial y}\right)^2}\ dA$$

$$\begin{matrix}Flux\\Integral\end{matrix}: \iint_S \vec{F}\bullet d\vec{S} = \iint_S \vec{F}\bullet\vec{n}\ dS = \iint_D \vec{F}\bullet\left(\vec{r_u}\ x\ \vec{r_v}\right) dA\ \ or = \iint_D \left(-P\frac{\partial g}{\partial x} - Q\frac{\partial g}{\partial y} + R\right) dA$$

$$\begin{matrix}Divergence\\Theorem\end{matrix}: \iint_S \vec{F}\bullet d\vec{S} = \iiint_E div\ \vec{F}\ dV$$

Line Integrals

Is integrand a scalar or vector function?

scalar function $f(x,y)$

$$\int_C f(x,y)\,ds = \int_a^b f(\vec{r})\,|\vec{r}'|\,dt$$

(WORK / CIRCULATION)

vector function $\vec{F}(x,y) = \langle P, Q \rangle$

Is the field conservative?

$\frac{\partial P}{\partial y} \neq \frac{\partial Q}{\partial x}$ → ***no***

Is C a closed path?

no

$$\int_C \vec{F} \cdot \vec{dr} = \int_a^b \vec{F}(r) \cdot \vec{r}'\,\vec{dt}$$

yes

Green's Theorem

$$\oint_C P\,dx + Q\,dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

generalizes to Stokes' Theorem

$$\int_C \vec{F} \cdot \vec{dr} = \iint_S \left(curl\ \vec{F} \right) \cdot \vec{n}\ dA$$

$\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ → ***yes***

$$\int_C \vec{F} \cdot dr = f(end) - f(start)$$

$$where\ \nabla f = \vec{F}$$

Surface Integrals

Is integrand a scalar or vector function?

scalar function $f(x,y)$

Can z be written in terms of x and y?

$S : z = g(x,y)$

yes

$\vec{r}(x,y) = \langle x, y, g(x,y) \rangle$

$$\iint f\,dS$$

$$= \iint_D f(r)\sqrt{z_x^{\ 2} + z_y^{\ 2} + 1}\,dA$$

no

$\vec{r}(u,v)$

$$\iint f\,dS$$

$$= \iint_D f(r)\,|\vec{r_u}\ x\ \vec{r_v}|\,dA$$

(FLUX)

vector function $\vec{F} = \langle P, Q, R \rangle$

Is S closed?

yes

$$\iint_S \vec{F} \cdot \vec{dS} = \iiint_E div\ \vec{F}\ dV$$

Divergence Theorem

no

Can z be written in terms of x and y?

yes

$S : z = g(x,y)$

$$\iint \vec{F} \cdot \vec{dS}$$

$$= \iint_D \left(-Pz_x - Qz_y + R \right) dA$$

no

$$\iint \vec{F} \cdot \vec{dS}$$

$$= \iint_D \vec{F}(\vec{r}) \cdot (\vec{r_u}\ x\ \vec{r_v})\,dA$$