Calculus 3 Ch16 Part 1 Review Name ________________________________

NOTE: Read each problem carefully…most require setup of integral only, but some require evaluation. Use the formulas provided in the problem.

#1a) Find $div\,\vec{F}$ and $curl\,\vec{F}$ if

$\vec{F}(x,y,z) = \langle x^2yz,\quad xy^2z,\quad xyz^2 \rangle$

$$\vec{F} = \langle P, Q, R \rangle$$

$$div\,\vec{F} = \nabla \bullet \vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

$$curl\,\vec{F} = \nabla x\,\vec{F} = \begin{vmatrix} + & - & + \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ P & Q & R \end{vmatrix}$$

#1b) Find $div\,\vec{F}$ and $curl\,\vec{F}$ if $\vec{F}(x,y,z) = \langle 1,\quad x+yz,\quad xy-\sqrt{z} \rangle$

#2a) Determine whether or not $\vec{F}(x,y) = \langle e^x \cos y, e^x \sin y \rangle$ is conservative. If it is, find the potential function f such that $\nabla f = \vec{F}$.

$$conservative\ if\ \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

$$\int_C \vec{F} \bullet dr = f(end) - f(start)$$

#2b) Determine whether or not $\vec{F}(x,y) = \langle e^x \sin y, e^x \cos y \rangle$ is conservative. If it is, find the potential function f such that $\nabla f = \vec{F}$.

#2c) Determine whether or not $\vec{F}(x,y) = \langle 3x^2 + 2y^2,\quad 4xy + 3 \rangle$ is conservative. If it is, find the potential function f such that $\nabla f = \vec{F}$.

$$\iint_S f(x,y,z)\,dS = \iint_D f(x,y,g(x,y))\sqrt{1+\left(\frac{\partial g}{\partial x}\right)^2+\left(\frac{\partial g}{\partial y}\right)^2}\,dA$$
$$\iint_S f(x,y,z)\,dS = \iint_D f\left(\vec{r}(u,v)\right)\left|\vec{r}_u \times \vec{r}_v\right|\,dA$$

#3a) Set up the integral to find the surface area of the part of the plane $3x+2y+z=6$ that lies in the first octant. *(Do not evaluate the integral)*

#3b) Set up the integral to find the surface area of the part of the plane with vector equation $\vec{r}(u,v)=\langle 1+v,\ u-2v,\ 3-5u+v\rangle$ that is given by $0\le u\le 1,\quad 0\le v\le 1$.
(Do not evaluate the integral)

#3c) Set up the integral to find the surface area of the part of the paraboloid $z=x^2+y^2$ that lies inside the cylinder $x^2+y^2=4$.
(Do not evaluate the integral)

#3d) Set up the integral to find the surface area of the part of the plane $z=4-y$ that lies inside the cylinder $x^2+y^2=4$.
(Do not evaluate the integral)

Scalar Integrand: $\int_C f(x,y,z)\bullet ds = \int_a^b f\left(\vec{r}(t)\right)\left|\vec{r}'(t)\right|\,dt$

#4a) Set up the integral to evaluate $\int_C y^3\,ds$ if C is defined by:

$x=t^3,\ y=t\ \ for\ \ 0\le t\le 2$. *(Do not evaluate the integral)*

#4b) Set up the integral to evaluate $\int_C xy\,ds$ if C is defined by:

$x=t^2,\ y=2t\ \ for\ \ 0\le t\le 1$. *(Do not evaluate the integral)*

$$\text{conservative if } \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$$

$$\int_C \vec{F} \bullet dr = f(end) - f(start)$$

#5a) Set up **and evaluate** $\int_C \vec{F} \bullet \vec{dr}$ for $\vec{F} = \langle x^2,\ y^2 \rangle$ along curve C defined by the arc of the parabola $y = 2x^2$ from $(-1,2)$ to $(2,8)$.
(hint: check to see if this field is conservative)
(NOTE: You must **also evaluate** this integral)

#5b) Set up **and evaluate** $\int_C \vec{F} \bullet \vec{dr}$ for $\vec{F} = \langle xy^2,\ x^2 y \rangle$ along curve C defined by

$$\vec{r}(t) = \left\langle t + \sin\left(\frac{1}{2}\pi t\right),\quad t + \cos\left(\frac{1}{2}\pi t\right) \right\rangle,\quad 0 \le t \le 1$$

(hint: check to see if this field is conservative)
(NOTE: You must **also evaluate** this integral)

$$\text{Green's Theorem}: \oint_C \vec{F} \bullet \vec{dr} = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) dA$$

#6a) Set up the line integral $\int_C (xy)\,dx + (x^2 y^3)\,dy$ along C which traces the path around a triangle from $(0,0)$ to $(1,0)$ to $(1,2)$ and back to $(0,0)$.
(Do not evaluate the integral)

#6b) Set up the line integral $\int_C (xy^2)\,dx + (2x^2 y)\,dy$ along C which traces the path around a triangle from $(0,0)$ to $(2,2)$ to $(2,4)$ and back to $(0,0)$.
(Do not evaluate the integral)

$$\text{tangent plane } \vec{n} = \vec{r}_u \times \vec{r}_v$$

$$ax + by + cz = \vec{n} \bullet \vec{r}_0$$

#7a) Find the equation of the tangent plane to the parametric surface defined by $x = u + v,\ y = 3u^2,\ z = u - v$ at $(2,3,0)$.

#7b) Find the equation of the tangent plane to the parametric surface defined by $x = u^2,\ y = v^2,\ z = uv$ at $u = 1,\ v = 1$.

$$\iint_S f(x,y,z)\, dS = \iint_D f(x,y,g(x,y))\sqrt{1 + \left(\frac{\partial g}{\partial x}\right)^2 + \left(\frac{\partial g}{\partial y}\right)^2}\, dA$$

#8a) Set up the surface integral $\int_S (z)\, dS$ where *S* is the part of the paraboloid $z = x^2 + y^2$ that lies inside the cylinder $x^2 + y^2 = 4$.
(Do not evaluate the integral)

#8b) Set up the surface integral $\int_S (xz)\, dS$ where *S* is the part of the paraboloid $z = 16 - x^2 - y^2$ that lies above the plane $z = 7$.
(Do not evaluate the integral)

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Ch16 Part 2

NOTE: On all problems, set up the integrals including limits of integration but <u>do not evaluate</u>.

$$\iint_S \vec{F} \bullet d\vec{S} = \iint_D \vec{F} \bullet \left(\vec{r}_u \ x \ \vec{r}_v \right) dA$$

#1a) Set up the integral to evaluate the flux of $\vec{F}$ over S if $\vec{F} = \langle xy, yz, zx \rangle$ and S is the part of the paraboloid $z = 4 - x^2 - y^2$ that lies above the square $0 \le x \le 1,\ 0 \le y \le 1$ and has upward orientation.

#1b) Set up the integral to evaluate the flux of $\vec{F}$ over S if $\vec{F} = \langle x^2, xy, z \rangle$ and S is the part of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 1$ with upward orientation.

Divergence Theorem: $\iint_S \vec{F} \bullet d\vec{S} = \iiint_E div\ \vec{F}\ dV$

$$div\ \vec{F} = \nabla \bullet \vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

#2a) Using the Divergence Theorem, write the **<u>triple-integral</u>** which is equivalent to the surface integral $\iint_S \vec{F} \bullet d\vec{S}$ which calculates the flux of $\vec{F}$ across S if $\vec{F} = \langle x^2, xy, z \rangle$ and S is the part of the paraboloid $z = x^2 + y^2$ that lies below the plane $z = 1$. *(You must set up the integral for the **<u>triple-integral side</u>**)*

#2b) Using the Divergence Theorem, write the **<u>triple-integral</u>** which is equivalent to the surface integral $\iint_S \vec{F} \bullet d\vec{S}$ which calculates the flux of $\vec{F}$ across S if $\vec{F} = \langle x^2, -x^2yz, 2xy^2 \rangle$ and S is the cylinder $x^2 + y^2 = 4$ between the planes $z = x + 4$ and $z = 0$. *(You must set up the integral for the **<u>triple-integral side</u>**)*

$$\begin{array}{c}\textit{Stokes'}\\ \textit{Theorem}\end{array} : \iint_S \left(curl\ \vec{F} \right) \bullet \vec{n}\ dA = \int_a^b \vec{F}\left(\vec{r}(t) \right) \bullet \vec{r}'(t)\, dt$$

#3a) Using Stokes' Theorem, write the **single-integral** which is equivalent to the surface integral which calculates $\iint_S \left(curl\ \vec{F} \right) \bullet \vec{dS}$ where $\vec{F} = \langle x^2z^2,\ \ y^2z^2,\ \ xyz \rangle$ and S is the part of the paraboloid $z = x^2 + y^2$ that inside the cylinder $x^2 + y^2 = 4$ and is oriented upward. *(You must set up the integral for the* ***single-integral side****)*

#3b) Using Stokes' Theorem, write the **single-integral** which is equivalent to the surface integral which calculates $\iint_S \left(curl\ \vec{F} \right) \bullet \vec{dS}$ where $\vec{F} = \langle 2x^2,\ \ -2y^2,\ \ x^2z \rangle$ and S is the part of the sphere $x^2 + y^2 + z^2 = 13$ that lies above the plane $z = 3$ and is oriented upward. *(You must set up the integral for the* ***single-integral side****)*

$$\begin{array}{c}\textit{Divergence}\\ \textit{Theorem}\end{array} : \iint_S \vec{F} \bullet \vec{dS} = \iiint_E div\ \vec{F}\, dV$$

$$div\ \vec{F} = \nabla \bullet \vec{F} = \frac{\partial P}{\partial x} + \frac{\partial Q}{\partial y} + \frac{\partial R}{\partial z}$$

#4) Using the Divergence Theorem, write the **triple-integral** which is equivalent to the surface integral $\iint_S \vec{F} \bullet \vec{dS}$ which calculates the flux of $\vec{F}$ across S if

$\vec{F} = \langle 3x^2,\ \ xyz,\ \ z^3 \rangle$ and S is the surface of a sphere of radius 9 centered at the origin. *(You must set up the integral for the* ***triple-integral side****)*

$$\begin{array}{l}Stokes' \\ Theorem\end{array} : \int_C \vec{F} \bullet dr = \iint_S \left(curl\ \vec{F} \right) \bullet \vec{n}\ dA$$

$$curl\ \vec{F} = \nabla x \vec{F} = \begin{vmatrix} + & - & + \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ P & Q & R \end{vmatrix}$$

#5a) Using Stokes' Theorem, write the **double-integral** which is equivalent to the line (path) integral $\int_C \vec{F} \bullet \vec{dr}$ which sums the contributions of the field $\vec{F} = \left\langle x^2z^2,\ \ x^2,\ \ xyz \right\rangle$ along the path which is the intersection of the paraboloid $z = x^2 + y^2$ and the cylinder $x^2 + y^2 = 4$. (Assume the path is oriented such that the normal vector to the enclosed surface is in the 'upward' direction).
*(You must set up the integral for the **double-integral side***, but remember that you can choose any surface that is bounded by the curve of the given surface.*)*

#5b) Using Stokes' Theorem, write the **double-integral** which is equivalent to the line (path) integral $\int_C \vec{F} \bullet \vec{dr}$ which sums the contributions of the field $\vec{F} = \left\langle x + y^2,\ \ y + z^2,\ \ z + x^2 \right\rangle$ along the path which is the intersection of the sphere $x^2 + y^2 + z^2 = 13$ and the plane $z = 3$. (Assume the path is oriented such that the normal vector is in the 'upward' direction).
*(You must set up the integral for the **double-integral side***, but remember that you can choose any surface that is bounded by the curve of the given surface.*)*