Honors Calculus III / Differential Equations Course Description

Honors Calculus III / Differential Equations is a year-long Desert Vista High School class which corresponds to the two separate semester courses at Rio Salado Community College MAT241 Calculus with Analytic Geometry III and MAT277 Modern Differential Equations.

1st semester – Calculus III: The first semester is a complete Calculus III course, including all vector calculus topics (Stokes' and Divergence Theorem are included). We use the textbook "Calculus w/Early Transcendentals" by James Stewart (6th edition):

Unit 1 (ch12): Vectors and the Geometry of Space

- 3D-coordinate systems
- Intro to Vectors, Properties, Adding/Subtracting vectors, multiply vector by scalar
- Dot Product, vector projections, direction angles, physics application: work
- Cross Product, determining parallel/perpendicular vectors, triple product, torque
- Equations of Lines and Planes, Parametric Equations
- Cylinders and Quadric Surfaces

Unit 2 (ch13): Vector Functions

- Vector Functions and Space Curves, ellipse parametrizations, limits of vector functions
- Derivatives and Integrals of Vector Functions, Unit Tangent Vector
- Arc Length and Curvature, Osculating circles, Normal/Binormal Vectors, TNB frame
- Motion in Space: Velocity and Acceleration, Projectile Motion

Unit 3 (ch14): Partial Derivatives

- Mulitivariable Functions, Level curves/surfaces/contour maps
- Limits and Continuity of Multivariable Functions
- Partial Derivatives, Clairaut's Theorem
- Tangent Planes, Linear Approximations, Differentials
- The Chain Rule for Multivariable Functions, Implicit Differentiaion
- Directional Derivatives, Gradient Vector
- Maximum and Minimum Values (without constraints, with domain boundary constraints)
- Lagrange Multipliers (optimization with constraints)

Unit 4 (ch15): Multiple Integrals

- Double Integrals over Rectangles
- Iterated Integrals, Fubini's Theorem
- Double Integrals over General Regions in the domain
- Double Integrals in Polar Coordinates
- Applications of Double Integrals: mass/density, probability density functions
- Triple Integrals in Rectangular Coordinates
- Triple Integrals in Cylindrical Coordinates
- Triple Integrals in Spherical Coordinates
- Extra Content: Disk/Shell method of volumes for solids of revolution
- Extra Content: General Coordinate Transforamtions/Jacobian of a Transform

Unit 5 (ch16): Vector Calculus

- Vector Fields
- Line (path) Integrals for scalar and vector functions
- The Fundamental Theorem for Line Integrals, Conservative Fields, Potential Function
- Green's Theorem, Circulation
- Curl and Divergence
- Parametric Surfaces and their Areas
- Surface Integrals for scalar and vector functions, Flux Integrals
- Stokes' Theorem
- The Divergence (Gauss') Theorem

2nd semester – Differential Equations: The second semester is a complete Introduction to Ordinary Differential Equations course, including an introduction to numerical analysis methods using MATLAB/OCTAVE programming. We use the textbook "A First Course in Differential Equations with Modeling Applications" by Dennis G. Zill (9th edition):

Unit 6 (ch1/2): Methods of Solving First-Order Differential Equations

- Intro to differential equations and terminology
- Initial Value Problems
- Slope Fields, solution curves, 1D phase portraits, Isoclines
- Solving first order linear DEs using Separable Variables
- Solving first order linear DEs using an Integrating Factor
- Solving first order Exact Equations
- Solving first order DEs by substitutions: Bernoulli forms, Composition forms
- Euler's Method for approximate solution curves

Unit 7 (ch3): Modeling/Applications of First-Order Differential Equations

- Linear DE models for Growth/Decay
- Linear DE models for Compound Interest
- Linear DE models for Radioactivity/Carbon Dating
- Linear DE models for Newton's Law of Cooling/Warming
- Linear DE models for Series Electrical Circuits
- Non-Linear DE models for Falling masses with air resistance
- Non-Linear DE models for Logistic Growth Model

Unit 8 (ch4): Methods of Solving Higher-Order Differential Equations

- Terminology, using the Wronskian to determine if solutions are linearly independent
- Reduction of Order to find a 2nd solution from a given solution
- Solving homogeneous linear DEs with constant coefficients (auxiliary equation)
- Solving non-homogeneous DEs using the method of Undetermined Coefficients
- Solving non-homogeneous DEs using the method of Variation of Parameters
- Solving Cauchy-Euler form DEs

Unit 9 (ch5): Modeling/Applications of Higher-Order Differential Equations

- Linear DE models for Mass/Spring Systems (oscillation, damping, resonance, beat phenomena)
- Linear DE models for LRC Series Electrical Circuits

Unit 10 (ch7): The Laplace Transform

- Definition of the Laplace Transform
- Inverse Laplace Transform
- Laplace Transform of Derivatives
- Using Laplace Transforms to Solve Differential Equations
- Laplace Transform Properties: Shifting on s-axis
- The Unit Step Function
- Laplace Transform Properties: Shifting on t-axis
- Derivatives of a Transform (derivative on s-axis)
- The Dirac Delta Functions

Unit 11 (ch8): Systems of Differential Equations

- Terminology, Matrix Form of a System, Solution Curves/Phase Plane Trajectories
- Eigenvalues, Eigenvectors, and DE system solutions
- The Repeated Eigenvalues case
- The Complex Eigenvalues case
- Phase Portraits
- Solving non-homogeneous systems with the Method of Undetermined Coefficients
- Solving non-homogeneous systems with the Method of Variation of Parameters
- Solving a non-homogenerous system of DEs with initial conditions
- Writing a single higher-order DE as a system of first-order DEs

Unit 12: MATLAB/OCTAVE Programming

- The concepts of numerical analysis
- OCTAVE Software User Interface
- Two Files for OCTAVE analysis of a differential equations
- 2^{nd}-order DE solutions using OCTAVE
- Predator-Prey Models and solutions using OCTAVE
- Other Practical Examples