Derivation of mean and standard deviation of a Binomial distribution

Start by considering a single trial in a Bernoulli setting. The probability of success is p and the probability of failure is q. We could make a probability model for this single trial. If we define a random variable for the number of successes, the outcomes would be that we could have either 0 successes (if the single trial failed) or 1 success (if the single trial succeeded):

X	0	1
P	q	p

For this distribution, we could find a mean (expected value) and variance:

$$\mu = E(X) = \sum x \bullet P(x) = 0q + 1p$$
$$\mu = E(X) = p$$

$$\sigma^2 = Var(X) = \sum (x-\mu)^2 \bullet P(x)$$
$$\sigma^2 = Var(X) = (0-p)^2 q + (1-p)^2 p$$
$$\sigma^2 = Var(X) = p^2 q + q^2 p$$
$$\sigma^2 = Var(X) = pq(p+q)$$
$$\sigma^2 = Var(X) = pq(1)$$
$$\sigma^2 = Var(X) = pq$$

If we have more than one trial, these vary independently from each other, so for multiple trials, we are combining the distributions of multiple random variables. The means would add algebraically, but the variances add (Pythagorean Theorem of Statistics). To be formal, let's define a new random variable, Y, which combines n trials of random variables X:

$$Y = X_1 + X_2 + X_3 + \ldots . X_n$$

$$\mu_Y = \mu_{X_1} + \mu_{X_2} + \mu_{X_3} + \ldots . + \mu_{X_n}$$
$$\mu_Y = p + p + p + \ldots . + p$$
$$\mu_Y = np$$

$$\sigma^2{}_Y = \sigma^2{}_{X_1} + \sigma^2{}_{X_2} + \sigma^2{}_{X_3} + \ldots . + \sigma^2{}_{X_n}$$
$$\sigma^2{}_Y = pq + pq + pq + \ldots\ldots + pq$$
$$\sigma^2{}_Y = npq$$
$$\sigma_Y = \sqrt{npq}$$

So the mean of a Binomial distribution is np and the standard deviation is $\sqrt{npq}$.