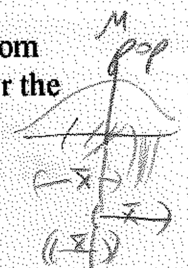
Unit 6 Practice Test – *Inference for Means – Part VI* Name Solutions

D 1. Which statement correctly compares t-distributions to the normal distribution?

✓ I. t distributions are also mound shaped and symmetric.
✗ II. t distributions have less spread than the normal distribution.
✓ III. As degrees of freedom increase, the variance of t distributions becomes smaller.

A) I only B) II only C) I and II only **[D)]** I and III only E) I, II, and III

E 2. A marketing company reviewing the length of television commercials monitored a random sample of commercials over several days. They found that a 95% confidence interval for the mean length (in seconds) of commercials aired daily was (23, 27). Which is true?



✗ A) 95% of the commercials they checked were between 23 and 27 seconds long.
✗ B) 95% of all the commercials aired were between 23 and 27 seconds a day.
✗ C) Commercials average between 23 and 27 seconds long on 95% of the days.
✗ D) 95% of all samples would show mean commercial length between 23 and 27 seconds.
✓ **[E)]** We're 95% sure that the mean commercial length is between 23 and 27 seconds.

B 3. A random sample of 120 classrooms at a large university found that 70% of them had been cleaned properly. What is the standard error of the sample proportion? $SE_{\hat{p}} = \sqrt{\frac{\hat{p}\hat{q}}{n}} = \sqrt{\frac{(.7)(.3)}{120}} = .0418$

A) 0.028 **[B)]** 0.042 C) 0.046 D) 0.082 E) 0.458

C 4. A coffee house owner knows that customers pour different amounts of coffee into their cups. She samples cups from 10 costumers she believes to be representative of the customers and weighs the cups, finding a mean of 12.5 ounces and standard deviation of 0.5 ounces. Assuming these cups of coffee can be considered a random sample of all cups of coffee which of the following formulas gives a 95% confidence interval for the mean weight of all cups of coffee?

A) $12.5 \pm 1.969\left(\frac{0.5}{\sqrt{10}}\right)$ B) $12.5 \pm 2.228\left(\frac{0.5}{\sqrt{10}}\right)$ **[C)]** $12.5 \pm 2.262\left(\frac{0.5}{\sqrt{10}}\right)$

D) $12.5 \pm 2.228\left(\frac{0.5}{\sqrt{9}}\right)$ E) $12.5 \pm 2.262\left(\frac{0.5}{\sqrt{9}}\right)$

CI = statistic ± (critical)(std dev)
$= 12.5 \pm \underbrace{2.262}_{\text{from t-table, 95\%, df}=9}\left(\frac{0.5}{\sqrt{10}}\right)$

E 5. Doctors at a technology research facility randomly assigned equal numbers of people to use computer keyboards in two rooms. In one room a group of people typed a manuscript using standard keyboards, while in the other room people typed the same manuscript using ergonomic keyboards to see if those people could type more words per minute. After collecting data for several days the researchers tested the hypothesis $H_0: \mu_1 - \mu_2 = 0$ against the one-tail alternative and found $P = 0.22$. Which is true?

✗ A) The people using ergonomic keyboards type 22% more words per minute.
✗ B) There's a 22% chance that people using ergonomic keyboards type more words per minute.
✗ C) There's a 22% chance that there's really no difference in typing speed.
✗ D) There's a 22% chance another experiment will give these same results. (or more extreme)
[E)] None of these.

B 6. An elementary school principal wants to know the mean number of children in families whose children attend this school. He checks all the families using the school's registration records, and we use the TI-83 to create a 95% confidence interval based on a t-distribution. This procedure was not appropriate. Why?

A) Since these families are from only one school, the family sizes may be skewed.
B) The entire population of families was gathered so there is no reason to do inference.
C) The recent record-setting family with twelve children is probably an outlier.
D) The population standard deviation is known, so he should have used a z-model.
E) At a given school families are not randomly selected.

E 7. Trainers need to estimate the level of fat in athletes to ensure good health. Initial tests were based on a small sample but now the trainers double the sample size for a follow-up test. The main purpose of the larger sample is to…

A) reduce response bias. B) decrease the variability in the population.
C) reduce non-response bias. D) reduce confounding due to other variables.
E) decrease the standard deviation of the sampling model.

D 8. Based on data from two very large independent samples, two students tested a hypothesis about equality of population means using $\alpha = 0.02$. One student used a one-tail test and rejected the null hypothesis, but the other used a two-tail test and failed to reject the null. Which of these might have been their calculated value of t?

w/ large n, t dist → Normal
← so t is like z and can find one-tail
← pvalue = normalcdf(z, 999, 0, 1)

A) 1.22 B) 1.55 C) 1.88 D) 2.22 E) 2.66

.111/.222 .06/.12 .03/.06 .013/.026 .004/.008

T F

D 9. The two samples whose statistics are given in the table are thought to come from populations with equal variances. What is the pooled estimate of the population standard deviation?

n	Mean	SD
50	22	3
55	25	4

A) 1.87 B) 3.50 C) 3.52 D) 3.56 E) 5.00

use calculator 2SampTTest, and choose "pooled = YES" scroll down $Sx_p = 3.55948071$ is pooled std dev

C 10. A contact lens wearer read that the producer of a new contact lens boasts that their lenses are cheaper than contact lenses from another popular company. She collected some data, then tested the null hypothesis $H_0: \mu_{old} - \mu_{new} = 0$ against the alternative $H_A: \mu_{old} - \mu_{new} > 0$. Which of the following would be a Type II error?

	T	F
R	I	
NR		II

A) Deciding that the new lenses are cheaper, when in fact they really are.
B) Deciding that the new lenses are cheaper, when in fact they are not.
C) Deciding that the new lenses are not really cheaper, when in fact they are.
D) Deciding that the new lenses are not really cheaper, when in fact they are not.
E) Applying these results to all contact lenses, old and new.

H_0 false but don't reject

$\mu_{old} - \mu_{new}$ not $= 0$ but we think it is. really its cheaper but we think not cheaper

11. **Housing costs** A government report on housing costs says that single-family home prices nationwide are skewed to the right, with a mean of $235,700.

a. We collect price data from a random sample of 50 homes in Orange County, California. Why is it okay to use these data for inference even though the population is skewed?

Because sample size is large enough to assume Nearly Normal.

b. The standard deviation of the 50 homes in our sample was $25,500. Specify the sampling model (shape, center, spread) for the mean price of such samples.

$s = 25{,}500 \quad \mu_{\bar{x}} = \mu = \$235{,}700 \quad SE_{\bar{x}} = \frac{25500}{\sqrt{50}} = \$3606.24$ $\boxed{t_{49}(235700, 3606.24)}$

c. This sample of randomly chosen homes produced a 90% confidence interval for the mean price in Orange County of ($233954, $246046). Does this interval provide evidence that single-family home prices are unusually high in this county? Explain briefly.

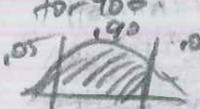
No, the national value $235,700 is in the interval and is a possible value for this county which suggests no difference.

d. Suppose we hope improve our estimate by choosing a new sample. How many home prices must we survey to have 90% confidence of estimating the mean local price to within $2000?

$CI = \bar{x} \pm t^* SE_{\bar{x}}$

$= \bar{x} \pm \boxed{t^* \frac{s}{\sqrt{n}}}$ margin of error

for 90%:



$t^* = \text{invT}(.95, df)$

~Normal so use $z^* = \text{invNorm}(.95, 0, 1) = 1.64$

$t^* \frac{s}{\sqrt{n}} = 2000$

$1.64 \frac{25500}{\sqrt{n}} = 2000$

$\frac{25500}{\sqrt{n}} = \frac{2000}{1.64}$

$2000\sqrt{n} = 1.64(25500)$

$\sqrt{n} = \frac{1.64(25500)}{2000}$

$n = \left(\frac{1.64(25500)}{2000}\right)^2 = 437.221$

$\boxed{\text{at least 438 homes}}$

if you wanted to you could now go back and use $df = 437$ to get t^*

$t^* = \text{invT}(.95, 437) = 1.648$ (z^* was 1.644) so not much difference

12. **Gas mileage** Hoping to improve the gas mileage of their cars, a car company has made an adjustment in the manufacturing process. Random samples of automobiles coming off the assembly line have been measured each week that the plant has been in operation. The data from before and after the manufacturing adjustments were made are in the table. It is believed that measurements of gas mileage are normally distributed. Write a complete conclusion about the manufacturing adjustments based on the statistical software printout shown below.

SET M1 24 21 26 25 23 24 19 22 20 24 20 21 27 22

SET M2 22. 24 28 28 27 24 22 24 27 25 27 23 28

Two Sample T for M1 vs M2

	N	Mean	StDev	SEMean
M1	14	22.71	2.40	0.64
M2	13	25.31	2.29	0.64

95% CI for $\mu_2 - \mu_1$ (0.74, 4.45) difference of means

T-Test $\mu_1 = \mu_2$ (vs. $\mu_1 < \mu_2$): T= 2.88 P = 0.0041 DF = 24.98

With p-value = .0041 there is strong evidence for H_A, that $\mu_1 < \mu_2$ (there is an improvement in gas mileage.)

We are 95% confident that this increase is between 0.74 and 4.45 mpg.

13. **Test identification** Suppose you were asked to analyze each of the situations described below. (*NOTE: Do not do these problems!*) For each, indicate which procedure you would use (pick the appropriate number from the list), the test statistic (z or t), and, if t, the number of degrees of freedom. A procedure *may* be used more than once.

	Type	z/t?	*df*
a.	1	z	—
b.	3	t	23
c.	2	z	—
d.	5	t	14
e.	5	t	9
f.	4	t	calculator or (35) worst case (n−1 w/smallest n) 19

1. proportion - 1 sample
2. difference of proportions, 2 samples
3. mean – 1 sample
4. difference of means – independent samples
5. mean of differences – matched pairs

a. A union organization would like to represent the employees at the local market. A sample of the employees revealed 74 of 120 were in favor of the union. Does the union have the required 3 to 2 majority? proportion 1 sample proportion (z) 1-sample

b. An oral surgeon is interested in estimating how long (quantitative) it takes to extract all four wisdom teeth. The doctor records the times for 24 randomly chosen surgeries. Estimate the time it takes to perform the surgery with a 95% confidence interval. single sample mean (t)

c. A microwave manufacturing company receives large shipments of thermal shields from two suppliers. A sample from each supplier's shipment is selected and tested for the rate of defects. The microwave manufacturing company's contract with each supplier states the shipment with the smallest rate of defect will be accepted. Do the shipments' defect rates vary from each other? proportion difference of proportion different samples not related (not pairs)

d. The owner of a construction company would like to know if his current work teams can build room additions quicker than the time allotted for by the contract. A random sample of 15 room additions completed recently revealed an average completion time of 0.32 days faster than contracted. Is this strong evidence that the teams can complete room additions in less than the contract times? single sample
mean of differences t assuming both team and mean contract times are varying then this is a matched-paired scenario (actual time − contract time)

e. A farmer would like to know if a new fertilizer increases his crop yield. In an effort to decide this, the farmer recorded the yield for 10 different fields prior to adding fertilizer and after adding the fertilizer. The farmer assumes the crop yields are approximately normal. Does the fertilizer work as advertised? measurement before/after same fields = matched pairs

f. In a study to determine whether there is a difference between the average jail time convicted bank robbers and car thieves are sentenced to, the law students randomly selected 20 cases of each type that resulted in jail sentences during the previous year. A 90% confidence interval was created from the results.

different distributions (no matching) difference of means.

14. **Improving productivity** A packing company considers hiring a national training consultant in hopes of improving productivity on the packing line. The national consultant agrees to work with 18 employees for one week as part of a trial before the packing company makes a decision about the training program. The training program will be implemented if the average product packed increases by more than 10 cases per day per employee. The packing company manager will test a hypothesis using $\alpha = 0.05$.

a. Write appropriate hypotheses (in words *and* in symbols).

$H_0: \mu_d = 10$ The difference (post-pre) is 10 cases per day per employee.
$H_A: \mu_d > 10$ The difference is more than 10 cases per day per employee.

	T	F
R	I	
NR		II

b. In this context, which do you consider to be more serious – a Type I or a Type II error? Explain briefly.

For the company, Type I is worse: That the difference is not > 10 but we decide to pay the money to hire the consultant.

c. After this trial produced inconclusive results the manager decided to test the training program again with another group of employees. Describe two changes he could make in the trial to increase the power of the test, and explain the disadvantages of each.

1) He could increase α. ($\alpha\uparrow, \beta\downarrow, 1-\beta\uparrow$) but that increases the likelihood of a Type I error.
2) He could increase n ($n\uparrow$, $\sigma\downarrow$ all widths decrease). This would improve the power and reduce errors, but disadvantage is the cost and time associated with taking and processing more samples.

15. **Flight costs** Every year Educational Services (ETS) selects readers for the Advanced Placement Exams. Recently the AP Statistics exam has been graded in Lincoln, Nebraska. One objective of ETS is to achieve equity in grading by inviting teachers to be readers from all parts of the nation. However budgets are a consideration also. The accountants at ETS wonder if the flights from cities west of Lincoln are the same as flight costs from cities east of Lincoln. A random sample of the expense vouchers from last year was reviewed for the cost of airline tickets. Costs (in dollars) are shown in the table.

East	West
265	257
298	320
340	295
219	288
199	366
398	275
359	430
309	397
105	253
253	366

Indicate what inference procedure you would use to see if there is a significant difference in the costs of airline flights between the west and east coasts to Lincoln, Nebraska, then decide if it is okay to actually perform that inference procedure. (Check the appropriate assumptions and conditions and indicate whether you could or could not proceed. You do not have to do the actual test.) t-test for difference of means

CONDITIONS

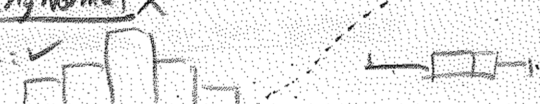
SRS/indep each sample — stated ✓

groups indep ✓ — yes

$n < 10\%$ pop ✓ — Samples < 10% of all flights

Nearly Normal X

East: ✓



West: X

West's histogram is very skewed right. Should *not* proceed with the test.