AP Statistics – Unit 5 Additional Review

#1. A researcher conducted a medical study to investigate whether taking a low-dose aspirin reduces the chance of developing colon cancer. As part of the study, 1,000 adult volunteers were randomly assigned to one of two groups. Half of the volunteers were assigned to the experimental group that took a low-dose aspirin each day, and the other half were assigned to the control group that took a placebo each day. At the end of six years, 15 of the people who took the low-dose aspirin had developed colon cancer and 26 of the people who took the placebo had developed colon cancer. At the significance level $\alpha = 0.05$, do the data provide convincing statistical evidence that taking a low-dose aspirin each day would reduce the chance of developing colon cancer among all people similar to the volunteers?

a) $H_0: p_A = p_P$ percentage who had cancer was the same for aspirin and placebo

$H_a: p_A < p_P$ percentage who had cancer was lower for aspirin than placebo.

conditions

✓ – $n_A p_A = 15$

$n_A q_A = 500 - 15 = 485$ ≥ 10

$n_P p_P = 26$

$n_P q_P = 500 - 26 = 474$

✓ – SRSs (assume volunteers are representative)

✓ – groups indep (randomly assigned)

✓ – 1000 < 10% of all adults

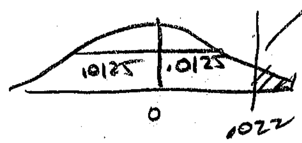
by hand

$\hat{p}_A = \frac{15}{500} = .03 \quad \hat{p}_P = \frac{26}{500} = .052$

$\hat{p}_P - \hat{p}_A = .052 - .03 = .022$

$SE_{\hat{p}_1 - \hat{p}_A} = \sqrt{\frac{(.052)(.948)}{500} + \frac{(.03)(.97)}{500}}$

$= .0125$



this side because $\hat{p}_P - \hat{p}_A$ is positive if $p_A < p_P$

p-value = normalcdf(.022, 999, 0, .0125)

$= \boxed{.0392}$

by calculator

Perform a 2PropZTest using $x_1 = 15$

$n_1 = 500$

$x_2 = 26$

$n_2 = 500$

$p_1 < p_2$

$z = -1.7542$

p-value $= \boxed{.0397}$

with $\alpha = .05$, p-value $= .0392$ is low so we reject H_0.
We do have sufficient statistical evidence to conclude that taking a low-dose aspirin each day reduces the chance of developing colon cancer.

b) Interpret the p-value

If aspirin actually makes no difference to colon cancer, there is a 3.92% probability of seeing a difference in percentage of patients with colon cancer between aspirin and placebo as large as this study found (2.2%) or larger, just due to random chance.

#2. Each person in a random sample of 1,026 adults in the United States was asked the following question.

"Based on what you know about the Social Security system today, what would you like Congress and the President to do during this next year?"

The response choices and the percentages selecting them are shown below.

Completely overhaul the system	19%
Make some major changes	39%
~~Make some minor adjustments~~	30%
Leave the system the way it is now	11%
No opinion	1%

(a) Find a 95% confidence interval for the proportion of all United States adults who would respond "Make some major changes" to the question. Give an interpretation of the confidence interval and give an interpretation of the confidence level.

$\hat{p} = .39$ w/ $n = 1026$

Conditions

✓ $- np = (1026)(.39) = 408.14$ ≥ 10
$nq = (1026)(.61) = 625.86$

✓ – SRS "random sample"

✓ – 1026 < 10% of all US adults

Perform a 1PropZInt in TI-84
using: $x = 408$
$n = 1026$
C-level $= .95$

$\boxed{(.36771, .42761)}$

we are 95% confident that between 36.83 and 42.82 of all people would say, "make major changes".

confidence level:
If we conducted this study many times and found CIs for each, 95% of the CIs would contain the true % of all people who would answer this way

(b) An advocate for leaving the system as it is now commented, "Based on this poll, only 39% of adults in the sample responded that they want some major changes made to the system, while 41% responded that they want only minor changes or no changes at all. Therefore, we should not change the system." Explain why this statement, while technically correct, is misleading.

The 39% value is just for the sample and for the population we believe this value can be anywhere from 36.72 to 42.73 & there would be a similar range of values for the "minor changes" group. Also 39% to 41% is a very small difference – even if it were statistically significant it isn't large enough, practically, to support this statement.

c) If we were happy with a margin of error of .04, how many adults should we sample?

$CI = \hat{p} \pm \boxed{z SE_{\hat{p}}}$ margin of error $= .04$

for 2 sample (and using $\hat{p} = .5$): $z^* \sqrt{\frac{(.5)(.5)}{n} + \frac{(.5)(.5)}{n}} = .04$

$(1.96)\sqrt{\frac{2(.5)(.5)}{n}} = .04$

$\sqrt{\frac{2(.5)(.5)}{n}} = \frac{.04}{1.96}$

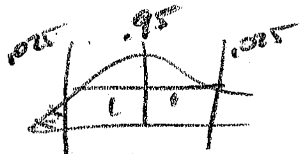
$\frac{2(.5)(.5)}{n} = \left(\frac{.04}{1.96}\right)$

$n = \frac{2(.5)(.5)}{\left(\frac{.04}{1.96}\right)^2}$

$= 1200.5 \uparrow$

$\boxed{1201 \text{ adults}}$

z^* for 95% confidence:



$z^* = \text{invNorm}(.025, 0, 1)$
$= \pm 1.96$

#3. To increase business, the owner of a restaurant is running a promotion in which a customer's bill can be randomly selected to receive a discount. When a customer's bill is printed, a program in the cash register randomly determines whether the customer will receive a discount on the bill. The program was written to generate a discount with a probability of 0.2, that is, giving 20 percent of the bills a discount in the long run. However, the owner is concerned that the program has a mistake that results in the program not generating the intended long-run proportion of 0.2.

The owner selected a random sample of bills and found that only 15 percent of them received discounts. A confidence interval for p, the proportion of bills that will receive a discount in the long run, is 0.15 ± 0.06. All conditions for inference were met.

(a) Consider the confidence interval 0.15 ± 0.06. (.09, .21)

(i) Does the confidence interval provide convincing statistical evidence that the program is not working as intended? Justify your answer.

(ii) Does the confidence interval provide convincing statistical evidence that the program generates the discount with a probability of 0.2 ? Justify your answer.

A second random sample of bills was taken that was four times the size of the original sample. In the second sample 15 percent of the bills received the discount.

(b) Determine the value of the margin of error based on the second sample of bills that would be used to compute an interval for p with the same confidence level as that of the original interval.

(c) Based on the margin of error in part (b) that was obtained from the second sample, what do you conclude about whether the program is working as intended? Justify your answer.

(a-i) NO. The confidence interval contains 0.20 so it is a possible value, So we can't conclude the program is not working.

(a-ii) NO. The confidence interval is (.09, .21) So any value in this interval is a plausible value, including 0.20.

(b) original margin of error = .06 and margin of error $\propto \frac{1}{\sqrt{n}}$

so for 4x n, new margin of error will be $(.06)\frac{1}{\sqrt{4}} = \boxed{.03}$

(c) The new margin of error, .03, changes the confidence interval to $.15 \pm .03 = (.12, .18)$. This new confidence interval does not contain 0.2 which suggests that the program is not working as intended.

#4. Psychologists interested in the relationship between meditation and health conducted a study with a random sample of 28 men who live in a large retirement community. Of the men in the sample, 11 reported that they participate in daily meditation and 17 reported that they do not participate in daily meditation.

The researchers wanted to perform a hypothesis test of

$$H_0 : p_m - p_c = 0$$
$$H_a : p_m - p_c < 0,$$

where p_m is the proportion of men with high blood pressure among all the men in the retirement community who participate in daily meditation and p_c is the proportion of men with high blood pressure among all the men in the retirement community who do not participate in daily meditation.

(a) If the study were to provide significant evidence against H_0 in favor of H_a, would it be reasonable for the psychologists to conclude that daily meditation causes a reduction in blood pressure for men in the retirement community? Explain why or why not.

(a) NO, because this was a study, not an experiment (no random assignment to groups, treatment is not imposed by researchers) therefore no cause-effect conclusions can be made.

The psychologists found that of the 11 men in the study who participate in daily meditation, 0 had high blood pressure. Of the 17 men who do not participate in daily meditation, 8 had high blood pressure.

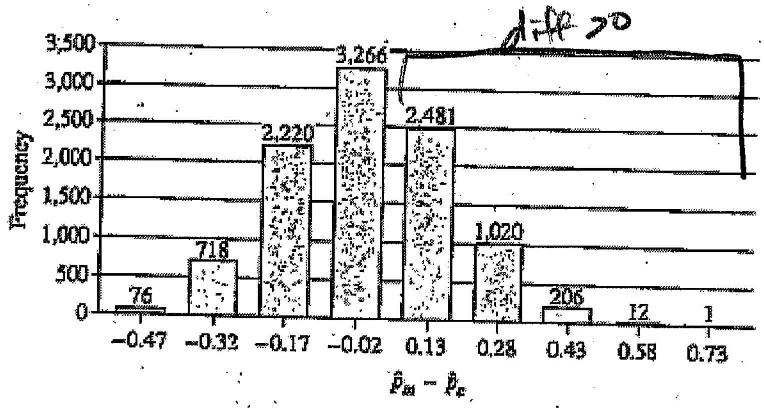
(b) Let $\hat{p}_m$ represent the proportion of men with high blood pressure among those in a random sample of 11 who meditate daily, and let $\hat{p}_c$ represent the proportion of men with high blood pressure among those in a random sample of 17 who do not meditate daily. Why is it not reasonable to use a normal approximation for the sampling distribution of $\hat{p}_m - \hat{p}_c$?

(b) $\hat{p}_M = \frac{0}{11} = 0$ $\hat{p}_c = \frac{8}{17} = .4706$

$n_M p_m = 0$ which is not ≥ 10

we don't meet the minimum sample size required in order to use normal-distribution based inference methods.

Although a normal approximation cannot be used, it is possible to simulate the distribution of $\hat{p}_m - \hat{p}_c$. Under the assumption that the null hypothesis is true, 10,000 values of $\hat{p}_m - \hat{p}_c$ were simulated. The histogram below shows the results of the simulation.



(c) Based on the results of the simulation, what can be concluded about the relationship between blood pressure and meditation among men in the retirement community?

H_0: $p_m = p_c$ no difference in high blood pressure percentages

H_a: $p_m \neq p_c$ there is a difference between high blood pressure 2 meditators v. non-meditators

Conditions

~~x 2 tails np test~~ (but we are not using normal, so OK)

✓ – SRS states "representative"

✓ – n < 10% pop "large retirement community"

X – groups indep (can't really assume this because meditators self-select)

we need a p-value for $P(\hat{p}_m - \hat{p}_c > 0) = \frac{3720}{10000} = .3720$

adding up counts for all the positive values in simulation

this should really be 2-sided, so p-value = 2(.3720) = .7440

with $\alpha = .05$ p-value = .7440 is high so we fail to reject H_0. We do not have sufficient statistical evidence to conclude there is a relationship between blood pressure and meditation in this community.

#5. As lab partners, Sally and Betty collected data for a significance test. Both calculated the same Z statistics, but Sally found the results were significant at the 0.05 level while Betty found that the results were not. When checking their results, the women found that the only difference in their work was that Sally had used a two-sided test, while Betty used a one-sided test.

Which of the following could have been their test statistic?

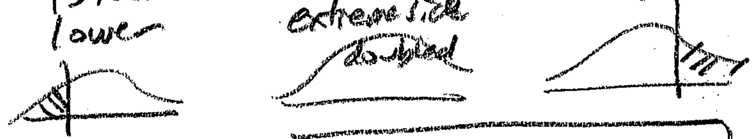
a) -1.980 normalcdf(-999,-1.98,0,1) = .0239 normalcdf(-1.98, 999, 0, 1) = .9761

b) -1.690 double whichever is < 0.5

c) 1.340

d) 1.690

e) 1.780



Z-Score	1 sided lower	2 sided extreme side doubled	1 sided upper
-1.980	.0239	.0473	.9761
-1.690	.0455	.0917	.9545
1.340	.9099	.1802	.0901
1.690	.9545	.0910	.0455
1.780	.9625	.0751	.0375

Sally significant ($p < .05$) 2 sided

Betty not significant ($p > .05$) 1-sided

So we need a z-score where the 2-sided p-value is < .05 and one of the 1-sided p-values is ≥ .05

$\boxed{z = -1.980}$ is the only one of these values it could do

$\boxed{a}$