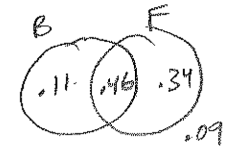
Unit 4 Practice Test ***Probability – Part IV*** **Name** Solutions

Use the following information for questions 1-2:
In an AP Stats class, 57% of students eat breakfast in the morning and 80% of students floss their teeth. Forty-six percent of students eat breakfast and also floss their teeth.



B 1. What is the probability that a student from this class eats breakfast but does not floss their teeth?
A) 9% B) 11% C) 34% D) 57% E) 91%

E 2. What is the probability that a student from this class eats breakfast or flosses their teeth?
A) 9% B) 11% C) 34% D) 57% E) 91%

B 3. Five juniors and four seniors have applied for two open student council positions. School administrators have decided to pick the two new members randomly. What is the probability they are both juniors or both seniors?
A) 0.395 B) 0.444 C) 0.506 D) 0.569 E) 0.722

JJ or SS

$\frac{5}{9}\cdot\frac{4}{8} + \frac{4}{9}\cdot\frac{3}{8} = .444$

B 4. A ~~fair coin~~ has come up "heads" 10 times in a row. The probability that the coin will come up heads on the next flip is
A) less than 50%, since "tails" is due to come up.
B) 50%.
C) greater than 50%, since it appears that we are in a streak of "heads."
D) It cannot be determined.

E 5. According to the National Telecommunication and Information Administration, 56.5% of U.S. households owned a computer in 2001. What is the probability that of three randomly selected U.S. households at least one owned a computer in 2001?
A) 18.0% B) 43.5% C) 56.5% D) 82.0% E) 91.8%

0 1 2 3

1 − binompdf(3, .565, 0) = .9176

A 6. According to the National Telecommunication and Information Administration, 50.5% of U.S. households had Internet access in 2001. What is the probability that four randomly selected U.S. households all had Internet access in 2001?
A) 6.5% B) 12.6% C) 49.5% D) 50.5% E) 93.5%

binompdf(4, .505, 4) = .065

C 7. Which of these has a Binomial model?
(½ E) × A) the number of people we survey until we find someone who has taken Statistics
× B) the number of people we survey until we find two people who have taken Statistics
✓ C) the number of people in a class of 25 who have taken Statistics
× D) the number of aces in a five-card Poker hand not independent
E) the number of sodas students drink per day p not independent?

A 8. Which of these has a Geometric model?
✓ A) the number of people we survey until we find someone who has taken Statistics
× B) the number of people we survey until we find two people who have taken Statistics
C) the number of people in a class of 25 who have taken Statistics
D) the number of aces in a five-card Poker hand
E) the number of sodas students drink per day

C 9. BatCo, a company that sells batteries, claims that 99.5% of their batteries are in working order. How many batteries would you expect to buy, on average, to find one that does not work?
A) 5 B) 100 C) 200 D) 995 E) 2000

$p = 1 - .995 = .005$

geometric $EV = \mu = \frac{1}{p} = \frac{1}{.005} = 200$

B 10. Some marathons allow two runners to "split" the marathon by each running a half marathon. Alice and Sharon plan to split a marathon. Alice's half-marathon times average 92 minutes with a standard deviation of 4 minutes, and Sharon's half-marathon times average 96 minutes with a standard deviation of 2 minutes. Assume that the women's half-marathon times are independent. The expected time for Alice and Sharon to complete a full marathon is $92 + 96 = 188$ minutes. What is the standard deviation of their total time?

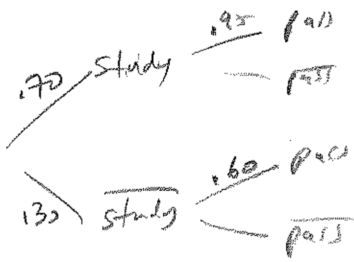
A) 2 minutes B) 4.5 minutes C) 6 minutes
D) 20 minutes E) It cannot be determined

$\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2$

$\sigma_{X+Y}^2 = (4)^2 + (2)^2$

$\sigma = \sqrt{20} = 4.47$

11. **Passing the test** Assume that 70% of teenagers who go to take the written drivers license test have studied for the test. Of those who study for the test, 95% pass; of those who do not study for the test, 60% pass. What is the probability that a teenager who passes the written drivers license test did not study for the test?

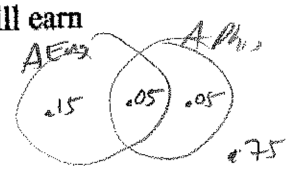


$$P(\overline{\text{study}} \mid \text{pass}) = \frac{(.30)(.60)}{(.70)(.95) + (.30)(.60)}$$

$$= \boxed{.2130}$$

12. **Grades** You believe that there is a 20% chance that you will earn an A in your English class, a 10% chance that you will earn an A in your Physics class, and a 5% chance that you will earn an A in both classes.

a. Find the probability that you do not get an A in either English or Physics.



$P(\bar{E} \cap \bar{P}) = 1 - P(E \cup P)$

$= 1 - .25 = \boxed{.75}$

b. Are "earning an A in English" and "earning an A in Physics" disjoint events? Explain.

No, not disjoint because $P(A_{Eng} \cap A_{Phy}) \neq 0$

c. Are "earning an A in English" and "earning an A in Physics" independent events? Explain.

$P(E) = .20$

$P(E \mid P) = \frac{.05}{.10} = \frac{5}{10} = .50$

$P(E) \neq P(E \mid P)$ so $\boxed{\text{not independent}}$

13. **Heights of Adults** According to the National Health Survey, heights of adults may have a Normal model with mean heights of 69.1" for men and 64.0" for women. The respective standard deviations are 2.8" and 2.5."

a. Based on this information,

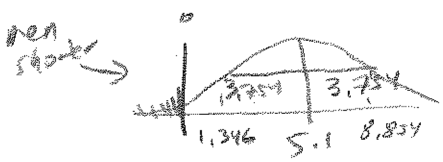
i. how much taller are men than women, on average?

$$\mu_{M-W} = \mu_M - \mu_W$$
$$= 69.1 - 64.0 = \boxed{5.1''}$$

ii. what is the standard deviation for the difference in men's and women's heights?

$$\sigma^2_{M-W} = \sigma_M^2 + \sigma_W^2 = (2.8)^2 + (2.5)^2$$
$$\sigma_{M-W} = \sqrt{14.09} = \boxed{3.754''}$$

b. Assume that women date men without considering the height of the man (i.e., that the heights of the couple are independent). What is the probability that a woman dates a man shorter than she is? men − women = neg if man is shorter



normalcdf$(-999, 0, 5.1, 3.754) = \boxed{.0871}$

14. **Luxury cars** According to *infoplease*, 18.8% of the luxury cars manufactured in 2003 were silver. A large car dealership typically sells 50 luxury cars a month.

a. Explain why you think that the luxury car sales can be considered Bernoulli trials.

- Only 2 outcomes (silver, not silver)
- probabilities independent and constant (large dealership, large 'pool' of cars)
- repeatedly selecting cars (multiple identical trials) and 50 is less than 10% of all cars.

b. What is the probability that the fifth luxury car sold is the first silver one? geometric model

geometpdf$(.188, 5) = \boxed{.08173}$

c. Let X represent the number of silver luxury cars sold in a typical month. What is the probability model for X? Specify the model (name and parameters), and tell the mean and standard deviation. $\boxed{\text{Binomial model}}$ $\boxed{n = 50 \quad p = .188}$

So $\mu = np = 50(.188) = \boxed{9.4}$

$\sigma = \sqrt{npq} = \sqrt{50(.188)(.812)} = \boxed{2.76375}$

15. **Home ownership** According to the Bureau of the Census, 68.0% of Americans owned their own homes in 2003. A local real estate office is curious as to whether a higher percentage of Americans own their own homes in its area. The office selects a random sample of 200 people in the area to estimate the percentage of those people that own their own homes.

a. Verify that a Normal model is a useful approximation for the Binomial in this situation.

$n = 200$
$p = .68$
$q = .32$

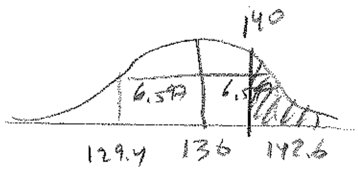
$np = 200(.68) = 136 \geq 10$ ✓

$nq = 200(.32) = 64 \geq 10$ ✓

np & $nq \geq 10$ so, yes, Normal approximation of Binomial is appropriate.

(Also, 200 people is <10% of population in area so removing these does not change the p value)

b. What is the probability that at least 140 people will report owning their own home?



$\mu = np = 200(.68) = 136$

$\sigma = \sqrt{npq} = \sqrt{200(.68)(.32)} = 6.597$

normalcdf (140, 99999, 136, 6.597)

$= \boxed{.2721}$

c. Based on the sample, how many people would it take for you to be convinced that a higher percentage of Americans own their own homes in that area? Explain.

140 isn't high enough because 27% probability of this occurring by chance.

unusual ~ 2σ above mean:

$$z = \frac{x - \mu}{\sigma}$$

$$2 = \frac{x - 136}{6.597}$$

$x = 149.19$

If we saw 150 out of the 200 sample owning, that would be unusual (2.5% chance of occurring due to natural variation)