AP® STATISTICS
2013 SCORING GUIDELINES

Question 3

Intent of Question

The primary goals of this question were to assess a student's ability to (1) calculate a probability from a normal distribution and (2) apply properties of means and variances of functions of random variables.

Solution

Part (a):

Let W denote the weight of a randomly selected full carton of eggs. W has a normal distribution with mean 840 grams and standard deviation 7.9 grams.

The z-score for a weight of 850 grams is $z = \frac{850 - 840}{7.9} \approx 1.27$.

The standard normal probability table reveals that
$P(W > 850) = P(Z > 1.27) \approx 1 - 0.8980 = 0.1020$.

Part (b):

(i) Let W represent the weight of a randomly selected full carton of eggs, P the weight of the packaging, and X_i the weight of the *ith* egg, for $i = 1, 2, \ldots, 12$.

Note that $W = P + X_1 + X_2 + \ldots + X_{12}$.
Properties of expected values establish that $\mathrm{E}(W) = \mathrm{E}(P) + \mathrm{E}(X_1) + \ldots + \mathrm{E}(X_{12})$.
Because all 12 eggs have the same mean weight, this becomes $\mathrm{E}(W) = \mathrm{E}(P) + 12 \times \mathrm{E}(X_i)$.
We were told that $\mathrm{E}(W) = 840$ and $\mathrm{E}(P) = 20$, so we can solve
$840 = 20 + 12 \times \mathrm{E}(X_i)$ to find $\mathrm{E}(X_i) = \frac{840 - 20}{12} \approx 68.33$ grams.

(ii) Because of independence, properties of variance establish that
$\mathrm{Var}(W) = \mathrm{Var}(P) + \mathrm{Var}(X_1) + \mathrm{Var}(X_2) + \ldots + \mathrm{Var}(X_{12})$.

Because all 12 eggs have the same variance of their weights, this becomes
$\mathrm{Var}(W) = \mathrm{Var}(P) + 12 \times \mathrm{Var}(X_i)$.

We were told that $\mathrm{SD}(W) = 7.9$ and $\mathrm{SD}(P) = 1.7$. Therefore, $\mathrm{Var}(W) = (7.9)^2 = 62.41$ and $\mathrm{Var}(P) = (1.7)^2 = 2.89$.
We can solve $62.41 = 2.89 + 12 \times \mathrm{Var}(X_i)$ to find $\mathrm{Var}(X_i) = \frac{62.41 - 2.89}{12} = 4.96$. Thus,
$\mathrm{SD}(X_i) = \sqrt{(4.96)} \approx 2.23$ grams.

AP® STATISTICS
2013 SCORING GUIDELINES

Question 3 (continued)

Scoring

Parts (a), (b-i), and (b-ii) were scored as essentially correct (E), partially correct (P), or incorrect (I). (Minor arithmetic errors in any part were not penalized).

Part (a) is scored as follows:

Essentially correct (E) if the response correctly includes the following three components:
1. Indicates use of a normal distribution and clearly identifies the correct parameter values (using a *z*-score is sufficient);
2. Uses the correct boundary value;
3. Reports the correct normal probability consistent with components 1 and 2.

Partially correct (P) if the response correctly includes two of the three components listed above.

Incorrect (I) if the response does not satisfy the criteria for an E or a P.

Notes:
1. An error in statistical notation in the response lowers the score one level (that is, from E to P or from P to I).
2. Responses that calculate a probability for a sample mean with n not equal to 1 should be scored an I. For example, using $z = \frac{x - \mu}{\sigma/\sqrt{n}}$, even if the parameters were correctly identified.
3. In component 1, a sketch of a normal curve with the mean labeled is sufficient for indicating use of a normal distribution and identifying the mean.
4. The following were examples of clearly identified parameters for component 1:
   - Writes "$\mu = 840, \sigma = 7.9$."
   - Explicitly labels the mean and standard deviation in a normalcdf calculator statement.
   - Sketches a normal curve, labels 840 as the mean, and labels two additional consecutive values separated by 7.9.
5. For component 3, acceptable correct values were all in the interval from 0.1020 to 0.1038.

Part (b-i) is scored as follows:

Essentially correct (E) if the response correctly uses properties of expected values to set up the correct equation to be solved *AND* correctly solves the equation for the desired expected value
OR
If the response follows a correct numerical procedure to find the correct expected value for one egg.

Partially correct (P) if the response indicates a correct procedure but makes an error in applying properties of expected values.
OR
If the response provides poor communication of the procedure.

Question 3 (continued)

Incorrect (I) if the response does not satisfy the criteria for an E or P.

Note: $\frac{820}{12} = 68.33$ is an example of poor communication, because the two arithmetic steps of subtraction $(840 - 20)$ and division $\left(\frac{820}{12}\right)$ were not documented.

Part (b-ii) is scored as follows:

Essentially correct (E) if the response combines variances and correctly includes the following three components:

1. Subtracts variances
2. Correctly uses the "12" in the calculations
3. Reports the correct standard deviation, consistent with components (1) and (2)

Partially correct (P) if the response combines variances and correctly includes two of the three components listed above.

Incorrect (I) if the response does not satisfy the criteria for an E or P.

Notes:

1. Examples of incorrect calculations with variances that should be scored P (one component incorrect):

$$\sqrt{\frac{7.9^2 + 1.7^2}{12}} = 2.33\text{g}$$

$$\sqrt{7.9^2 - 1.7^2} = 7.71\text{g}$$

$$\frac{\sqrt{7.9^2 - 1.7^2}}{12} = 0.643\text{g}$$

$$\frac{7.9^2 - 1.7^2}{12} = 4.96\text{g}$$

Examples of incorrect calculations with variances that should be scored I (more than one component incorrect):

$$\sqrt{7.9^2 + 1.7^2} = 8.08\text{g}$$

$$\frac{\sqrt{7.9^2 + 1.7^2}}{12} = 0.673\text{g}$$

2. Example of a response that does not combine variances and should be scored I:

$$\sqrt{\frac{7.9^2}{12}} = 2.28$$

Question 3 (continued)

4 Complete Response

All three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

OR

One part essentially correct and two parts partially correct

2 Developing Response

Two parts essentially correct and one part incorrect

OR

One part essentially correct, one part partially correct, and one part incorrect

OR

Three parts partially correct

1 Minimal Response

One part essentially correct and two parts incorrect

OR

One or two parts partially correct