AP® STATISTICS
2011 SCORING GUIDELINES (Form B)

Question 3

Intent of Question

The primary goals of this question were to assess students' ability to (1) describe a situation as a series of Bernoulli trials; (2) calculate probabilities of events involving Bernoulli trials; (3) recognize whether or not an event is likely to occur.

Solution

Part (a):

Let Y denote the number of flights Sam must make until he receives his first upgrade. The random variable Y follows a geometric distribution with $p = 0.1$. The probability that Sam's upgrade will occur after his third flight is calculated below.

$$\begin{aligned} P(Y \geq 4) &= 1 - P(Y \leq 3) \\ &= 1 - \left[P(Y = 1) + P(Y = 2) + P(Y = 3) \right] \\ &= 1 - \left[0.1 + 0.9 \times 0.1 + (0.9)^2 \times 0.1 \right] \\ &= 1 - \left[0.1 + 0.09 + 0.081 \right] \\ &= 0.729 \end{aligned}$$

Part (b):

Let p denote the probability that Sam will be upgraded to first class on a particular flight. Let X denote the number of upgrades Sam will receive in 20 flights. The random variable X follows a binomial distribution with $n = 20$ independent trials and $p = 0.1$. The probability that Sam will be upgraded exactly 2 times in his next 20 flights is calculated as follows.

$$\begin{aligned} P(X = 2) &= \binom{20}{2} \left(0.1^2\right) \left(0.9^{18}\right) \\ &\approx 0.2852 \end{aligned}$$

Part (c):

Let X denote the number of upgrades Sam will receive in 104 flights. The random variable X follows a binomial distribution with $n = 104$ independent trials and $p = 0.1$. Thus,

$$\begin{aligned} P(X > 20) &= 1 - P(X \leq 20) \\ &\approx 1 - 0.9986 \\ &\approx 0.0014. \end{aligned}$$

Because this probability is so small, it is very unlikely that Sam would receive more than 20 upgrades in 104 flights if the airline's claim is correct. This would be expected to happen less than 1 percent of the time, indicating that one should be surprised if Sam receives more than 20 upgrades during the next year.

AP® STATISTICS
2011 SCORING GUIDELINES (Form B)

Question 3 (continued)

Scoring

Parts (a), (b), and (c) are scored as essentially correct (E), partially correct (P), or incorrect (I).

Part (a) is scored as follows:

Essentially correct (E) if the response includes a correct probability and shows supporting work.

Note: An alternative solution to part (a) is:

$$P(\text{first upgrade after third flight}) =$$
$$P(\text{no upgrade on (flight 1 or flight 2 or flight 3)}) =$$
$$P(\text{no upgrade on flight 1 and no upgrade on flight 2 and no upgrade on flight 3}) =$$
$$0.9 \times 0.9 \times 0.9 = 0.729$$

Partially correct (P) if the response includes a correct calculation of a related probability such as $P(Y = 3) = (0.9)(0.9)(0.1)$ or $P(Y = 4) = (0.9)(0.9)(0.9)(0.1)$, with supporting work, *OR* the response includes a correct probability with insufficient supporting work.

Note: The calculator command, 1 – geometcdf(0.1,3) with no other justification should be scored no higher than P, unless the value 0.1 is labeled as *p* somewhere in the response, in which case a score of E can be given.

Incorrect (I) if an answer is provided with no supporting work, *OR* an unreasonable probability (greater than 1 or less than 0) is provided.

Part (b) is scored as follows:

Essentially correct (E) if the response includes a correct probability and shows supporting work.

Partially correct (P) if the response includes supporting work but makes a calculation mistake that results in a reasonable incorrect probability. For example, $0.1^2 \times 0.9^{18}$ should be scored as partially correct, *OR* if the response includes a correct probability with insufficient supporting work.

Notes

- The calculator command, binomialpdf(20,0.1,2), with no other justification should be scored no higher than P, unless somewhere in the response, the value 0.1 is labeled as *p* and the value of 20 is labeled as *n*. In such a case, a score of E can be given for this part.
- If the response in part (a) was downgraded to P for failure to identify the geometric parameter, do not downgrade part (b) for this same oversight.

Incorrect (I) if an answer is provided with no supporting work, *OR* an unreasonable probability (greater than 1 or less than 0) is provided.

AP® STATISTICS
2011 SCORING GUIDELINES (Form B)

Question 3 (continued)

Part (c) is scored as follows:

Essentially correct (E) if the response includes an answer to the question that is linked to a correct probability.

Partially correct (P) for any of the following:

- The response includes an answer to the question *AND* provides a justification based on expected value.
- The response includes an answer to the question that is linked to an incorrect but reasonable probability.
- The response gives a correct probability *AND* includes a correct answer to the question but fails to link the two.
- The response gives a correct probability *BUT* fails to answer the question.

Incorrect (I) if the response fails to meet the criteria for E or P.

4 Complete Response

All three parts essentially correct

3 Substantial Response

Two parts essentially correct and one part partially correct

2 Developing Response

Two parts essentially correct and one part incorrect

OR

One part essentially correct and one or two parts partially correct

OR

Three parts partially correct

1 Minimal Response

One part essentially correct and two parts incorrect

OR

Two parts partially correct and one part incorrect