AP Statistics Ch 6 - Required Practice **Name:** Solutions **Per:** ____

ACT versus SAT, II *There are two major tests of readiness for college, the ACT and the SAT. ACT scores are reported on a scale from 1 to 36. The distribution of ACT scores in recent years has been roughly Normal with mean $\mu = 20.9$ and standard deviation $\sigma = 4.8$. SAT scores (prior to 2005) were reported on a scale from 400 to 1600. SAT scores have been roughly Normal with mean $\mu = 1026$ and standard deviation $\sigma = 209$. The following exercises are based on this information.*

#1. Maria scores 28 on the ACT. Assuming that both tests measure the same thing, what score on the SAT is equivalent to Maria's ACT score? Explain.

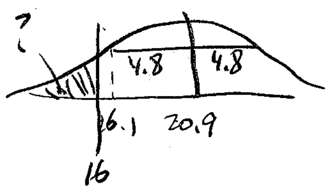
$$z_{ACT} = \frac{28-20.9}{4.8} = 1.47916\ldots = z_{SAT} = \frac{X-1026}{209}$$

$$\frac{X-1026}{209} = 1.47916\ldots$$

$$X - 1026 = 309.14583\ldots$$

$$X = \boxed{1335.15}$$ (Same # std devs above mean as 28 on ACT)

#2. Reports on a student's ACT or SAT usually give the percentile as well as the actual score. Jacob scores 16 on the ACT. What is his percentile? Show your method.



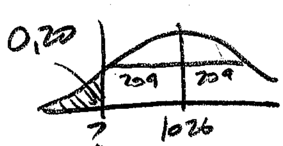
lower: upper: mean SD

normalcdf(-999, 16, 20.9, 4.8)

= .1537

= $\boxed{15.4\%}$

#3. The quintiles of any distribution are the values with cumulative proportions 0.20, 0.40, 0.60, and 0.80. What are the quintiles of the distribution of SAT scores? Show your method.



invNorm(.20, 1026, 209) = $\boxed{850}$

invNorm(.40, 1026, 209) = $\boxed{973}$

invNorm(.60, 1026, 209) = $\boxed{1079}$

invNorm(.80, 1026, 209) = $\boxed{1202}$

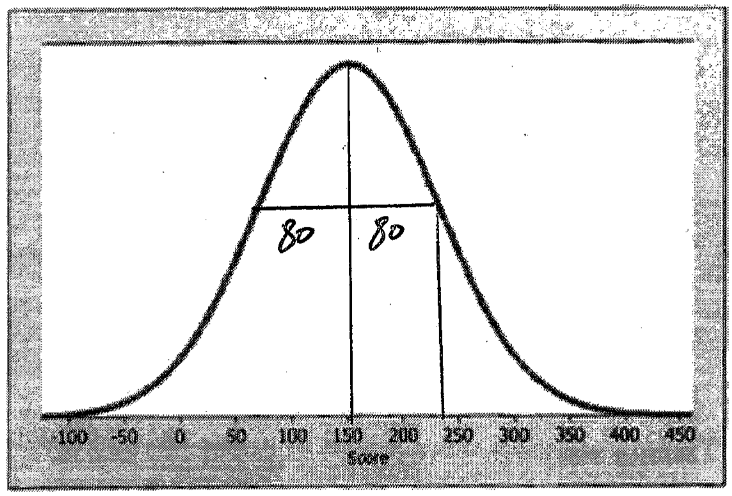
area to left   mean   SD

#4. **Approximate the median, mean, and standard deviation of the Normal distribution graphed below.**

Median: 150

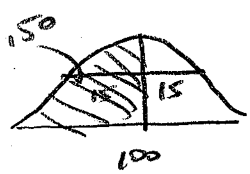
Mean: 150

Standard Deviation: 80



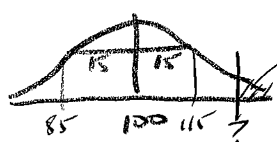
#5. The scores of a reference population on the Wechsler Intelligence Scale for Children (WISC) are approximately Normally distributed with a mean of 100 and standard deviation of 15.

(a) What score would represent the 50th percentile?



$\boxed{100}$ (half the scores are below the mean in a normal distribution)

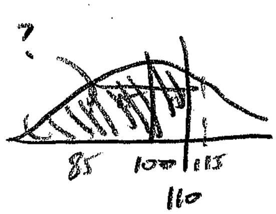
(b) A score in what range would represent the top 1% of the scores?



.01

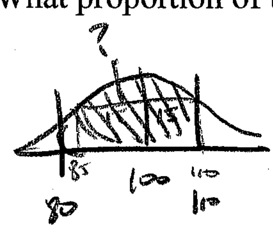
invNorm(.99, 100, 15) = $\boxed{134.9}$ (area to left, mean, SD)

(c) What proportion of the reference population has WISC scores below 110?



normalcdf(-999, 110, 100, 15) = $\boxed{.7475}$ (lower, upper, mean, SD)

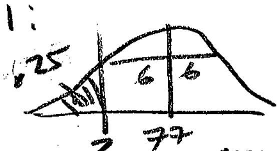
(d) What proportion of the reference population has WISC scores between 80 and 110?



normalcdf(80, 110, 100, 15) = $\boxed{.6563}$ (lower, upper, mean, SD)

#6. The weights of healthy adult male Labrador Retrievers are approximately Normally distributed with a mean of 77 pounds and a standard deviation of 6 pounds. What is the interquartile range of male Labrador Retriever weights?

Q1:



mean SD

invNorm(.25, 77, 6) = 72.953

area to left

Q3:

area to left mean SD

invNorm(.75, 77, 6) = 81.0469

IQR = Q3 − Q1

= 81.0469 − 72.953

= 8.0939

= $\boxed{8.1 \text{ lbs}}$

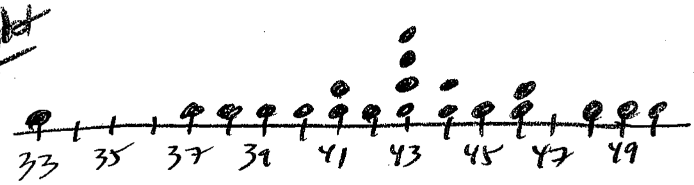
#7. Twenty students were asked to guess the age of a man in a photograph. Here are their guesses:

44	43	48	37	44	40	33	42	43	41
50	49	43	46	46	45	43	38	39	41

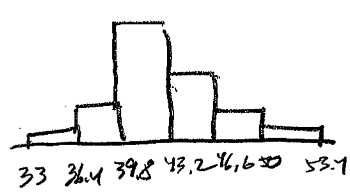
Are these guesses approximately Normally distributed? Provide evidence to support your answer.

$\boxed{\text{Yes}}$ possible ways to justify:

dot plot

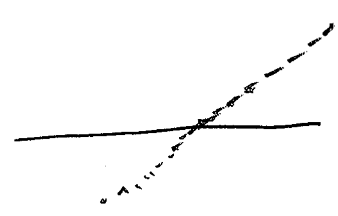


histogram (calculator)



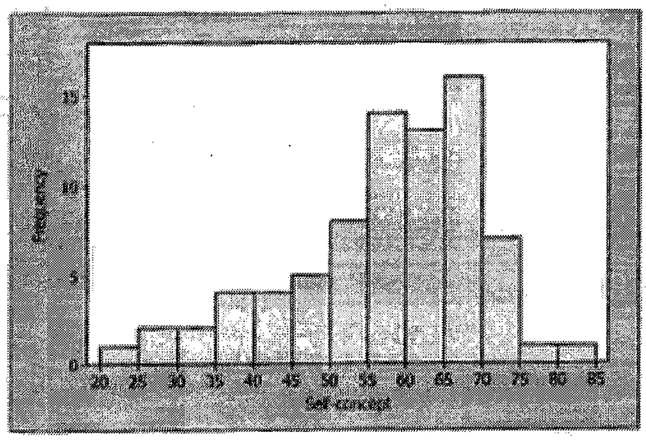
(unimodal and mound-shaped)

Normal probability plot (NPP) (calculator)



(close to linear)

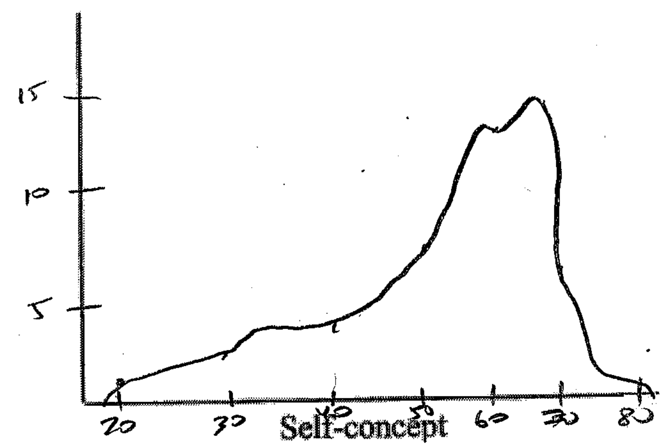
A group of 78 third-grade students in a Midwestern elementary school took a "self-concept" test that measured how well they felt about themselves. Higher scores indicate more positive self-concepts. A histogram and some summary statistics from Minitab for these students' self-concept scores are given below.



Descriptive Statistics: SelfConc

Variable	N	Mean	SE Mean	StDev	Minimum	Q1	Median	Q3	Maximum
SelfConc	78	56.85	1.40	12.35	20.00	50.00	59.00	65.00	80.00

#8. On the graph at right, make a rough sketch of a density curve for these data, based on the histogram above. How would you describe the shape of this density curve?



Unimodal, skewed left, with median of 59.00 and IQR of (65-50=15), no obvious outliers.
(not approximately normal)

#9. Thirty-three students had self-concept scores higher than 62. One student in the group had a self-concept score of 62. Calculate and interpret this student's percentile and z-score.

Not normal, so can't use normalcdf or invNorm.

$78-33-1 = 44$ students

percentile = % of distribution to the left

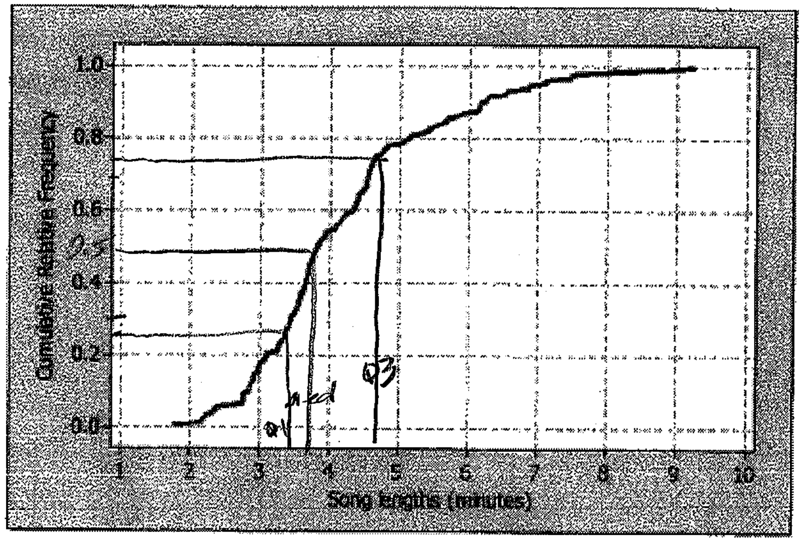
so $\frac{44}{78} = .564$ $\boxed{56.4\%}$

$z = \frac{x-\mu}{\sigma} = \frac{62-56.85}{12.35} =$ $\boxed{0.417 \text{ std devs above mean}}$

#10. What is a "typical" self-concept score for a third-grader in this group? Justify your answer.

$\boxed{\text{Typical score of 59}}$ (use median due to skew)

Below is a cumulative relative frequency graph for the lengths, in minutes, of 200 songs recorded by the Rolling Stones.



#11. What are the median and interquartile range of lengths? Draw lines on the graph to show how you arrived at your answers. $\boxed{\text{median} \approx 3.75 \text{ min}}$

Q1 ≈ 3.3 min

Q3 ≈ 4.6 min

$\boxed{\text{IQR} = 4.6 - 3.3 = 1.3 \text{ min}}$

#12. According to these data, the mean song length was 4.23 minutes, and the standard deviation was 1.38 minutes. A music lover who wants to create a mix of songs wants to have 5 seconds of silence between songs, so he needs to add five seconds to the length of each song. He also wants to express the times in seconds, rather than minutes. Find the mean and standard deviation of the transformed data.

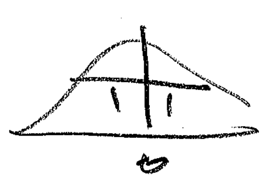
song length min → transform $\boxed{\text{multiply by 60, add 5}}$ → song length secs + silence

mean = 4.23 (60) + 5 = 258.8 seconds

std dev = 1.38 (60) = 82.8 seconds

(+5 shift doesn't affect measures of spread)

#13. What are the mean and standard deviation of the z-scores of song lengths?



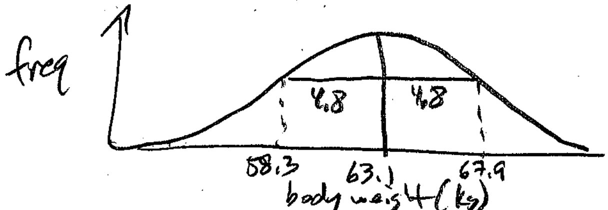
Mean = 0

std dev = 1

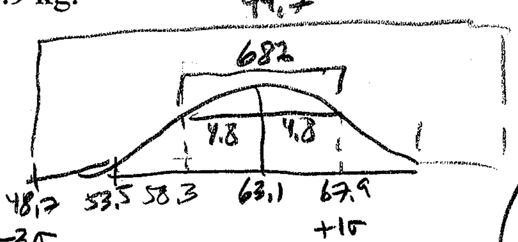
(always, for z-scores)

#14. A study of elite distance runners found a mean body weight of 63.1 kilograms (kg), with a standard deviation of 4.8 kg.

(a) Assuming that the distribution of weights is approximately normal, make an accurate sketch of the weight distribution with the horizontal axis marked in kilograms.



(b) Use the 68-95-99.7 rule to find the percentage of runners whose body weight is between 48.7 and 67.9 kg.



99.7% − 68% = 31.7% between 1 & 3σ

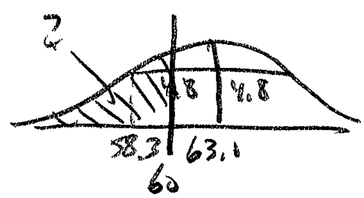
÷2 = 15.85% on each side

so 68 + 15.85 = $\boxed{83.85\%}$

— or —

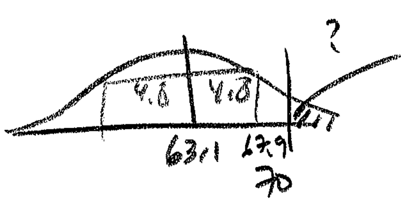
just use normalcdf(−3, 1, 0, 1) = .83999
w/ z-scores
lower upper mean SD

(c) What percentage of runners have body weights below 60 kg?



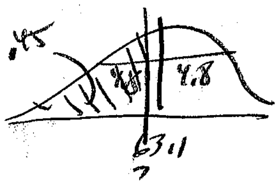
normalcdf(−999, 60, 63.1, 4.8) = $\boxed{.2592 \ (25.92\%)}$
lower upper mean SD

(d) What proportion of runners have body weights above 70 kg?



lower upper mean SD
normalcdf(70, 999, 63.1, 4.8) = $\boxed{.0753}$

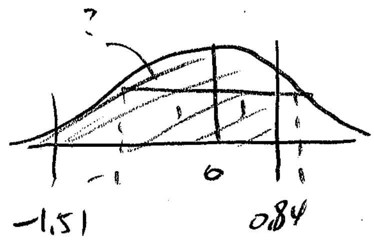
(e) Calculate and interpret the 45th percentile of the runners' body weight distribution.



area to left mean SD
invNorm(.45, 63.1, 4.8) = $\boxed{62.5 \text{ kg}}$

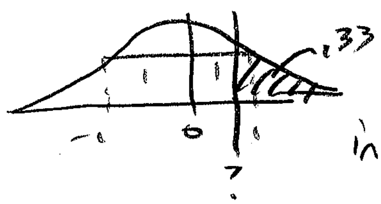
$\boxed{\text{45\% of runners weigh less than 62.5 kg}}$

#15. (a) Find the proportion of observations from a standard normal distribution that satisfies -1.51 < z < 0.84. Sketch the normal curve and shad the area under the curve that is the answer to the question.



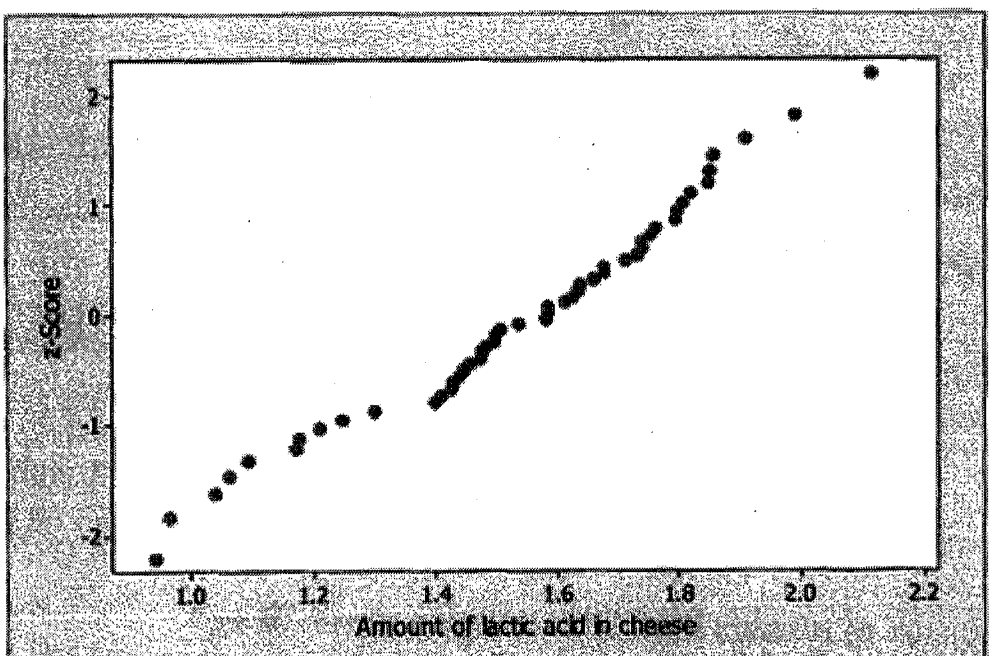
normalcdf(-1.51 [lower], 0.84 [upper], 0 [mean], 1 [SD]) = $\boxed{.7340}$

(b) What z-score in a normal distribution has 33% of all scores above it?



invNorm(.67 [greatest left], 0 [mean], 1 [SD]) = $\boxed{.4399}$

#16. A normal probability plot (NPP) for the amount of lactic acid in a sample of 30 pieces of cheese is shown below. Is the lactic acid distribution approximately normal? Justify your answer.



$\boxed{\text{Yes}}$ the plot is approximately linear.

Chapter 6 Practice Quiz

$\bar{x}$	16.65
s	2.96
min	5
Q1	15
median	16
Q3	19
max	28

1. Students taking an intro stats class reported the number of credit hours that they were taking that quarter. Summary statistics are shown in the table.
   a. Suppose that the college charges \$73 per credit hour plus a flat student fee of \$35 per quarter. For example, a student taking 12 credit hours would pay \$35 + \$73(12) = \$911 for that quarter.

transformation credit hrs $\xrightarrow[+35]{x(73)}$ \$

i. What is the mean fee paid?

$35 + 73(16.65)$
$= \boxed{\$1250.45}$

ii. What is the standard deviation for the fees paid?

$73(2.96) = \boxed{\$216.08}$
(no add for spread)

iii. What is the median fee paid?

$35 + 73(16) = \boxed{\$1203.00}$

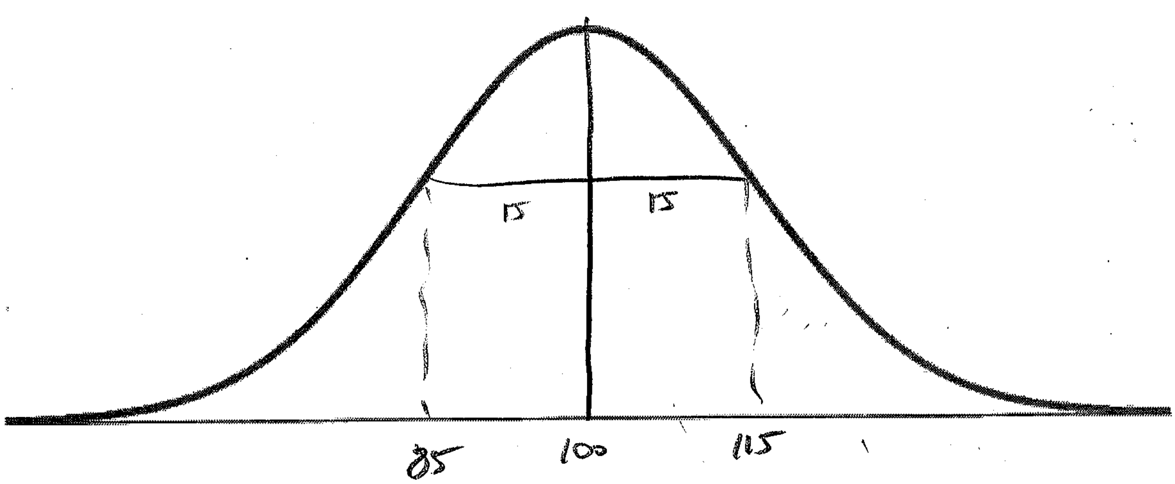
iv. What is the IQR for the fees paid?

$73(19-15) = \boxed{\$292.00}$
(spread)

b. Twenty-eight credit hours seems like a lot. Would you consider 28 credit hours to be unusually high? Explain.

IQR $= 19 - 15 = 4$
upper fence $= 19 + 1.5(4) = 25$ hrs
$\boxed{\text{yes}}$, 28 is an outlier.

2. The Wechsler Adult Intelligence Scale – Revised (WAIS-R) follow a Normal model with mean 100 and standard deviation 15. Draw and clearly label this model.



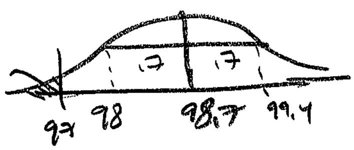
#3. Adult female Dalmatians weigh an average of 50 pounds with a standard deviation of 3.3 pounds. Adult female Boxers weight an average of 57.5 pounds with a standard deviation of 1.7 pounds. One statistics teacher owns an underweight Dalmatian and an underweight Boxer. The Dalmatian weighs 45 pounds, and the Boxer weighs 52 pounds. Which dog is more underweight? Explain.

$z_{dalmatian} = \frac{45-50}{3.3} = -1.515$ $\qquad z_{boxer} = \frac{52-57.5}{1.7} = -3.23$

The boxer is more underweight at 3.23 standard deviations below the mean compared to the dalmatian at only 1.5 std devs below the mean.

#4. Human body temperatures taken through the ear are typically 05°F higher than body temperatures taken orally. Making this adjustment and using the 1992 *Journal of the American Medical Association* article that reports average oral body temperature as 98.2°F, we will assume that a Normal model with an average of 98.7°F and a standard deviation of 0.7 °F is appropriate for body temperatures taken through the ear.

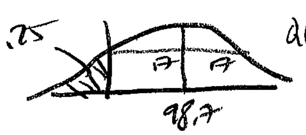
(a) An ear temperature of 97 °F may indicate hypothermia (low body temperature). What percent of people have ear temperatures that may indicate hypothermia?



normalcdf(−999, 97, 98.7, .7) = .007579 (lower, upper, mean, SD)

$\boxed{0.758\%}$

(b) Find the interquartile range for ear temperatures.

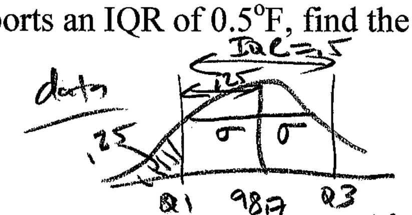


Q1: invNorm(.25, 98.7, .7) = 98.228 (area to left, mean, SD)

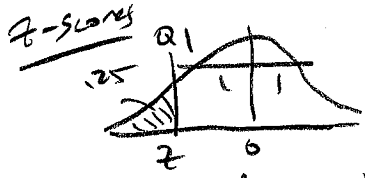
Q3: invNorm(.75, 98.7, .7) = 99.172 (area to left, mean, SD)

IQR = Q3 − Q1 = $\boxed{0.944°F}$

(c) A new thermometer for the ear reports that it is more accurate than the ear thermometers currently on the market. If the average ear temperature reading remains the same and the company reports an IQR of 0.5°F, find the standard deviation for this new ear thermometer.



x = 98.7 − .25 = 98.45°F



invNorm(.25, 0, 1) = −.6745 (area to left, mean, SD)

z = −.6745 linked to x = 98.45°F

$z = \frac{x-\mu}{\sigma}$

$-.6745 = \frac{98.45-98.7}{\sigma}$

$-.6745 = \frac{-.25}{\sigma}$

$-.6745\sigma = -.25$

$\sigma = \frac{-.25}{-.6745} = \boxed{0.37°F}$