AP Stats Ch 26 - Required Practice **Name:** Solutions **Per:** ____

12. NYPD and gender. The table below shows the rank attained by male and female officers in the New York City Police Department. Do these data indicate that men and women are equitably represented at all levels of the department?

Rank	Male	Female	
Officer	21,900	4,281	26181
Detective	4,058	806	4864
Sergeant	3,898	415	4313
Lieutenant	1,333	89	1422
Captain	359	12	371
Higher ranks	218	10	228
	31766	5613	37379

a) What's the probability that a person selected at random from the NYPD is a female?
b) What's the probability that a person selected at random from the NYPD is a detective?
c) Assuming no bias in promotions, how many female detectives would you expect the NYPD to have?
d) To see if there is evidence of differences in ranks attained by males and females, will you test goodness-of-fit, homogeneity, or independence?
e) State the hypotheses.
f) Test the conditions.
g) How many degrees of freedom are there?
h) Find χ^2 and the P-value.
i) State your conclusion.
j) If you concluded that the distributions are not the same, analyze the differences using the standardized residuals of your calculations.

a) $\hat{p}_F = \frac{5613}{37379} = \boxed{.15}$

b) $\hat{p}_{det} = \frac{4864}{37379} = \boxed{.13}$

c) $(.13)(5613) = 729.69$ $\boxed{730}$

d) independence (or homogeneity)
← most correct
(seems to be one sample)

(no symbolic version for most χ^2)

e) H_0: Rank is independent of gender
H_A: Rank depends on gender

f) conditions

✓ — counts?
✓ — SRS/indep (this year representative)
✓ — expected counts ≥ 5? (check [B] after)

g) $df = (\text{rows}-1)(\text{cols}-1) = (6-1)(2-1) = \boxed{5}$

h) Perform χ^2-test in TI-84 using data in [A]: $\chi^2 = 290.13$
$p = 1.3 \times 10^{-60} \approx 0$

(i) With $\alpha = .05$, $p = 1.3 \times 10^{-60} \approx 0$ is low, so we reject H_0.
We do have sufficient statistical evidence to conclude that rank in the NYPD depends on gender.

(j) This is usually not asked, because it is a lot of work: std res $= \frac{obs - exp}{\sqrt{exp}}$
hand calculated std residuals:

exp[B]

$$\begin{bmatrix} 22250 & 3931.5 \\ 4133.6 & 730.4 \\ 3665.3 & 647.66 \\ 1208.5 & 213.53 \\ 315.29 & 55.711 \\ 193.76 & 34.238 \end{bmatrix} \qquad \begin{bmatrix} -2.3 & 5.6 \\ -1.2 & 2.8 \\ 3.8 & -9.1 \\ 3.6 & -8.5 \\ 2.5 & -5.9 \\ 1.7 & -4.1 \end{bmatrix}$$

(everything over 2 or under -2 is statistically significant)

There are substantially more female officers and detectives (lower ranks) but fewer females in all the ranks higher than detective.

2. Which test again? For each of the following situations, state whether you'd use a chi-square goodness-of-fit test, a chi-square test of homogeneity, a chi-square test of independence, or some other statistical test.

a) Is the quality of a car affected by what day it was built? A car manufacturer examines a random sample of the warranty claims filed over the past two years to test whether defects are randomly distributed across days of the work week.

b) A medical researcher wants to know if blood cholesterol level is related to heart disease. She examines a database of 10,000 patients, testing whether the cholesterol level (in milligrams) is related to whether a person has heart disease or not.

c) A student wants to find out whether political leaning (liberal, moderate, or conservative) is related to choice of major. He surveys 500 randomly chosen students and performs a test.

a) Goodness-of-fit (uniform = expected percentages)

b) not counts, so can't use a Chi-squared test (χ^2)

c) One sample, two variables χ^2-test of independence

3. Dice. After getting trounced by your little brother in a children's game, you suspect the die he gave you to roll may be unfair. To check, you roll it 60 times, recording the number of times each face appears. Do these results cast doubt on the die's fairness?

a) If the die is fair, how many times would you expect each face to show?

b) To see if these results are unusual, will you test goodness-of-fit, homogeneity, or independence?

c) State your hypotheses.

d) Check the conditions.

Face	Count
1	11
2	7
3	9
4	15
5	12
6	6

e) How many degrees of freedom are there?

f) Find χ^2 and the P-value.

g) State your conclusion.

a) $\frac{1}{6}$ of 60 = $\boxed{\text{10 times}}$

b) goodness-of-fit

c) H_0: Die is fair (same probability for all faces)
H_A: Die is unfair (diff probs)

d) counts? yes
✓ SRS/indep? yes (always for dice)
✓ exp. counts > 5? (yes, all are 10)

e) df = cats − 1 = 6 − 1 = $\boxed{5}$

f) χ^2 GOF-test in calculator

L1 obs	L2 exp
11	10
7	10
9	10
15	10
12	10
6	10

$\boxed{\chi^2 = 5.6 \quad p = .3471}$

g) with $\alpha = .05$, p = .3471 is high so we fail to reject H_0.
We do not have sufficient statistical evidence to conclude the die is unfair.

(these deviations from all 10s are not big enough to be 'significant')

15. Cranberry juice. It's common folk wisdom that drinking cranberry juice can help prevent urinary tract infections in women. In 2001 the *British Medical Journal* reported the results of a Finnish study in which three groups of 50 women were monitored for these infections over 6 months. One group drank cranberry juice daily, another group drank a lactobacillus drink, and the third drank neither of those beverages, serving as a control group. In the control group, 18 women developed at least one infection compared with 20 of those who consumed the lactobacillus drink and only 8 of those who drank cranberry juice. Does this study provide supporting evidence for the value of cranberry juice in warding off urinary tract infections?

a) Is this a survey, a retrospective study, a prospective study, or an experiment? Explain.
b) Will you test goodness-of-fit, homogeneity, or independence?
c) State the hypotheses.
d) Test the conditions.
e) How many degrees of freedom are there?
f) Find χ^2 and the P-value.
g) State your conclusion.
h) If you concluded that the groups are not the same, analyze the differences using the standardized residuals of your calculations.

a) The drink is a treatment (factor) so this is an experiment.

b) homogeneity (multiple populations)

c) H_0: There is no difference in infection rates for the drink groups.

H_A: There is a difference in infection rates between the drink groups.

d)

	control	lactobacillus	cranberry
UTI	18	20	8
no UTI	32	30	42
	50	50	50

→ [A]

counts? ✗ SRS/ind. representative ✓ expected ≥5? (check [B] after) ✓

e) $df = (2-1)\times(3-1) = 2$

f) data → [A] χ^2-Test $\chi^2 = 7.78$ $p = .020$ $df = 2$

g) For $\alpha = .05$, p-value of 0.20 is low so we reject H_0. The evidence is consistent with there being a difference in infection rates between the drink groups.

(B)
expected values

$$\begin{bmatrix} 15.3 & 15.3 & 15.3 \\ 34.7 & 34.7 & 34.7 \end{bmatrix}$$

(all > 5)

$$\text{residuals} = \frac{\text{obs} - \text{exp}}{\sqrt{\text{exp}}}$$

$$\begin{bmatrix} .69 & 1.2 & -1.9 \\ -.46 & -.80 & 1.2 \end{bmatrix}$$

The evidence women who drink cranberry juice are less likely to develop a UTI, and women who drink lactobacillus are (slightly) <u>more</u> likely to develop a UTI

Chapter 26 Practice Quiz

Practice Quiz - Chapter 26 **Name**________________________

1. A biology professor reports that historically grades in her introductory biology course have been distributed as follows: 15% A's, 30% B's, 40% C's, 10% D's, and 5% F's. Grades in her most recent course were distributed as follows:

Grade	A	B	C	D	F	
Frequency	89	121	78	25	12	= 325
expected:	48.75	97.5	130	32.5	16.25	

a. Test an appropriate hypothesis to decide if the professor's most recent grade distribution matches the historical distribution. Give statistical evidence to support your conclusion.

H_0: Grade distribution proportions match the historical performance

H_a: Grade distribution proportions do not match historical performance.

conditions
- ✓ – counts?
- ✓ – SRS? no but representative
- ✓ – expected ≥ 5? yes, shown above

Perform a χ^2-GOF test in TI-84 w/ obs & exp counts above and $df = 5-1 = 4$

$\chi^2 = 62.5385$

$p = 8.4866 \times 10^{-13} \approx 0$

With $\alpha = .05$, p-value ≈ 0 is low, so we reject H_0. We do have sufficient statistical evidence to conclude that grade distribution proportions do not match historical performance.

b. Which grade impacted your decision the most? Explain what this means in the context of the problem.

	A	B	C	D	F
χ^2 contributions	33.23	5.66	20.8	1.73	1.11

The largest contribution is 33.23 for the As. The professor now gives significantly more As than she did before.

2. As part of a survey, students in a large statistics class were asked whether or not they ate breakfast that morning. The data appears in the following table:

one proportion, 2 variables
independence

		Breakfast		
		Yes	No	Total
Sex	Male	66	66	132
	Female	125	74	199
	Total	191	140	331

Is there evidence that eating breakfast is independent of the student's sex? Test an appropriate hypothesis. Give statistical evidence to support your conclusion.

H_0: Eating breakfast is independent of gender.
H_A: Eating breakfast is dependent upon gender
CONDITIONS: counts? ✓ SRS? no but representative ✓ exp. counts > 5 ✓ (check(B) after)

$\chi^2 = 5.34$ p-value = .02
with $\alpha = .05$, p-value of .02 is low so we reject H_0.
There is sufficient statistical evidence to suggest that eating breakfast is not independent of gender.

3. A manufacturing plant for recreational vehicles receives shipments from three different parts vendors. There has been a defect issue with some of the electrical wiring in the recreational vehicles manufactured at the plant. The plant manager wonders if all of the vendors might be contributing equally to the defect issue. The plant manager reviews a sample of quality assurance inspections from the last six months. The data are shown in the table below.

	Purrfect Parts Co.	Made-4-U Co.	25 Hour Parts Co.
Rejected	53	48	70
Perfect	93	71	88
Not perfect but acceptable	22	31	22

Test an appropriate hypothesis to decide if the plant manager is correct. Give statistical evidence to support your conclusion.

H_0: proportions of quality parts distributions are the same for all vendors.
H_A: proportion of quality parts distributions are different between vendors.
CONDITIONS: counts? ✓ SRS? stated ✓ expected counts > 5? ✓
χ^2 test of homogeneity $\chi^2 = 7.4$ p-value = .116
For $\alpha = .05$, p-value = .116 is high so we fail to reject the H_0.
There is no significant statistical evidence that the proportions of quality parts vary by vendor.