AP Stats Ch 23 - Required Practice **Name:** SOLUTIONS **Per:** ____

#1. **Speed Limit** - The speeds of cars on a small road with a speed limit of 30 mph were monitored. The following speeds were observed:

29	**29**	**24**
34	**32**	**36**
28	**31**	**31**
27		

(a) Use a hypothesis test to determine whether or not the average speed of all vehicles on this road exceeds the speed limit.

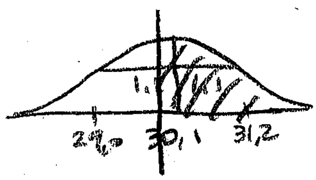
$H_0: \mu = 30$ mph avg. speed does not exceed limit

$H_A: \mu > 30$ mph avg. speed exceeds the limit

conditions ??

$\bar{x} = 30.1 \quad s = 3.4785$ (from 1-varstats)

$SE_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{3.4785}{\sqrt{10}} = 1.1$



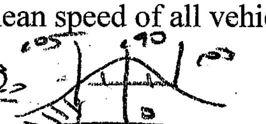
$P(\bar{x} > 30 \mid \mu = 30) =$ normalcdf(30, 999, 30.1, 1.1) $= .5362$
lower upper mean SD

With $\alpha = .05$, p-value $= .5362$ is high so we fail to reject H_0. We do not have sufficient statistical evidence to conclude that average speed exceeds the speed limit.

(b) Use this sample to build a 90% confidence interval for the mean speed of all vehicles traveling on the road and interpret your interval.

$CI = \bar{x} \pm z^* \frac{s}{\sqrt{n}}$

for 90%



$z^* =$ invNorm(.05, 0, 1) $= -1.645$

$CI = 30.1 \pm (1.645) \frac{(3.4785)}{\sqrt{10}}$

$= 30.1 \pm 1.809$

$= (28.29, 31.91)$

We are 90% confident that the true average speed of all cars on this road is between 28.29 and 31.91 mph.

#1 again (this time using t instead of z).

Speed Limit - The speeds of cars on a small road with a speed limit of 30 mph were monitored. The following speeds were observed:

29	**29**	**24**
34	**32**	**36**
28	**31**	**31**
27		

(a) Use a hypothesis test to determine whether or not the average speed of all vehicles on this road exceeds the speed limit.

$H_0: \mu = 30$ Avg speed does not exceed the limit

$H_A: \mu > 30$ Avg speed exceeds the limit.

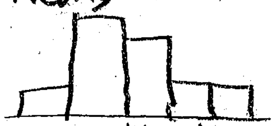
1 var stats:
$\bar{x} = 30.1$
$s = 3.4785$

conditions

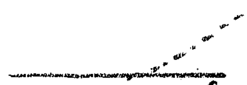
✓ – SRS (no, but assume these cars are representative)

✓ – 10 < 10% of all cars on this road

– Nearly Normal: .. Normal Probability Plot



possible skew .. but



(nearly) .. shows sample is normal

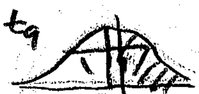
$SE_{\bar{x}} = \frac{s}{\sqrt{n}} = \frac{3.4785}{\sqrt{10}} = 1.1$

$df = n - 1 = 10 - 1 = 9$

by hand:

$t = \frac{\bar{x} - \mu_0}{SE_{\bar{x}}} = \frac{30.1 - 30}{1.1} = .0909$

t_9



0 .0909

$P(t > .0909) = tcdf(\underbrace{.0909}_{lower}, \underbrace{999}_{upper}, \underbrace{9}_{df}) = .4648$

by calculator

I perform a T-test in TI-84 using: data in L1
$\mu_0 = 30$
$\mu > \mu_0$
$t = .0909$
p-value $= .4648$
($df = 10 - 1 = 9$)

With $\alpha = .05$, p-value $= .4648$ is high so we fail to reject H_0. we do not have sufficient statistical evidence to conclude that the average speed of all cars on this road exceeds the speed limit.

(b) Use this sample to build a 90% confidence interval for the mean speed of all vehicles traveling on the road and interpret your interval.

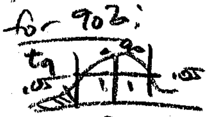
(conditions checked above)

by hand

$CI = \bar{x} \pm t^* \frac{s}{\sqrt{n}}$

for 90%:

t_9



.05 .90 .05
0

$t^* = invT(.05, 9) = 1.833$

by calculator

I perform a T-Interval in TI-84 using: data in L1
C-level = .90

(28.084, 32.116)

$CI = (30.1) \pm (1.833) \frac{3.4785}{\sqrt{10}}$

$= 30.1 \pm 2.0163 = (28.08, 32.12)$

We are 90% confident that the average speed of all cars on this road is between 28.08 and 32.12 mph.

26. Fuel economy. A company with a large fleet of cars hopes to keep gasoline costs down, and sets a goal of attaining a fleet average of at least 26 miles per gallon. To see if the goal is being met, they check the gasoline usage for 50 company trips chosen at random, finding a mean of 25.02 mpg and a standard deviation of 4.83 mpg. Is this strong evidence that they have failed to attain their fuel economy goal?

a) Write appropriate hypotheses.
b) Are the necessary assumptions to perform inference satisfied?
c) Describe the sampling distribution model of mean fuel economy for samples like this.
d) Find the P-value.
e) Explain what the P-value means in this context.
f) State an appropriate conclusion.

a) $H_0: \mu = 26$ The mean fuel economy is 26 mpg. (~~failed to attain their goal~~)
$H_A: \mu < 26$ The mean fuel economy is less than 26 mpg (~~attained their goal~~)

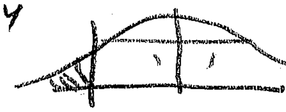
b) ✓ SRS "chosen at random"
✓ 50 < 10% of all trips
✓ Nearly Normal, no data but normal because $n \geq 40$

c) with $\bar{x} = 25.02$ mpg, $s = 4.83$ mpg, $n = 50$
$df = 50 - 1 = 49$
$\mu_{\bar{x}} = 25.02$ mpg
$SE_{\bar{x}} = \frac{4.83}{\sqrt{50}} = .6831$

a t-distribution of 49 degrees with mean of 25.02 mpg & SE of .6831 mpg

or $\boxed{t_{49}(25.02, .6831)}$

d) $t = \frac{25.02 - 26}{.6831} = -1.434$


-1.434   0

p-value = tcdf(-999, -1.434, 49)
lower upper df
$= \boxed{.0789}$

e) The the mean fuel economy is actually 26 mpg, there is a 7.89% chance of having a sample of 50 trips with an mean of 25.02 mpg (or less) just randomly, due to chance.

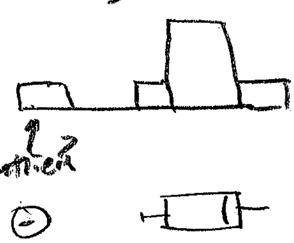
f) With $\alpha = .05$, p-value = .0789 is high so we ~~fail to reject H_0~~.
We do not have sufficient statistical evidence to conclude that the fuel economy is less than 26 mpg.

36. Braking. A tire manufacturer is considering a newly designed tread pattern for its all-weather tires. Tests have indicated that these tires will provide better gas mileage and longer treadlife. The last remaining test is for braking effectiveness. The company hopes the tire will allow a car traveling at 60 mph to come to a complete stop within an average of 125 feet after the brakes are applied. They will adopt the new tread pattern unless there is strong evidence that the tires do not meet this objective. The distances (in feet) for 10 stops on a test track were 129, 128, 130, 132, 135, 123, 102, 125, 128, and 130. Should the company adopt the new tread pattern? Test an appropriate hypothesis and state your conclusion. Explain how you dealt with the outlier, and why you made the recommendation you did.

$H_0: \mu = 125$ ft Mean braking distance is 125 ft.
$H_A: \mu > 125$ ft mean braking distance is greater than 125 ft.

Conditions

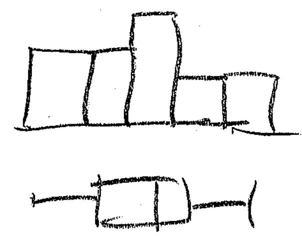
✓ – SRS (assume these are representative)
✓ – 10 stops < 10% of all stops
outlier – Nearly Normal



outlier?

yes

Could just remove this point



~~w/o removing outlier:~~
~~Perform TTest in TI-84~~
~~using data:~~
~~$\mu_0 = 125$~~
~~$\mu > \mu_0$~~

~~$t = .4152$~~
~~p-value = .3438~~

w/ outlier removed:
perform TTest in TI-84
using Modified data:
$\mu_0 = 125$
$\mu > \mu_0$
$t = 3.285$
p-value = $\boxed{.0056}$

with $\alpha = .05$, p-value = .0056 is low, so we reject H_0.
we do have sufficient statistical evidence to conclude the new tread results in longer braking distances.

(Interesting how much this one outlier affects the results!)
— outliers have a very large effect on inference tests for means, because even a single point can drastically change the mean.

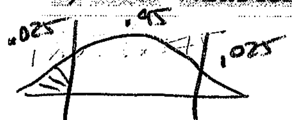
2. *t*-models, part II. Using the *t* tables, software, or a calculator, estimate

a) the critical value of t for a 95% confidence interval with df = 7.

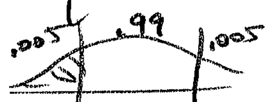
b) the critical value of t for a 99% confidence interval with df = 102.

c) the P-value for $t \le 2.19$ with 41 degrees of freedom.

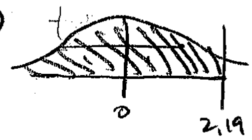
d) the P-value for $|t| > 2.33$ with 12 degrees of freedom.

a) 

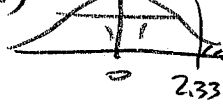
$t^* = \text{invT}(\underbrace{.025}_{\text{left area}}, \underbrace{7}_{\text{df}}) = \boxed{2.365}$

b) 

$t^* = \text{invT}(\underbrace{.005}_{\text{left area}}, \underbrace{102}_{\text{df}}) = \boxed{2.625}$

c) 

$\text{tcdf}(\underbrace{-999}_{\text{lower}}, \underbrace{2.19}_{\text{upper}}, \underbrace{41}_{\text{df}}) = \boxed{.983}$

d) 

$\text{tcdf}(\overbrace{2.33}^{\text{lower}}, \overbrace{999}^{\text{upper}}, \overbrace{12}^{\text{df}}) = \boxed{0.038}$

8. Rain. Based on meteorological data for the past century, a local TV weatherman estimates that the region's average winter snowfall is 23", with a margin of error of ±2 inches. Assuming he used a 95% confidence interval, how should viewers interpret this news? Comment on each of these statements.

a) During 95 of the last 100 winters, the region got between 21" and 25" of snow. — confident intervals are about means of multiple samples, ~~not the values~~

b) There's a 95% chance the region will get between 21" and 25" of snow this winter. — Not a prediction of what will happen in a given year,

c) There will be between 21" and 25" of snow on the ground for 95% of the winter days. — not based on a sample of days

d) Residents can be 95% sure that the area's average snowfall is between 21" and 25".

e) Residents can be 95% confident that the average snowfall during the last century was between 21" and 25" per winter. — we know exactly what the mean was for the sample (23"per yr) there is no uncertainty so the margin of error would be zero.

Chapter 23 Practice Quiz

AP Statistics Quiz C – Chapter 23 ***Name*** ______________________

Textbook authors must be careful that the reading level of their book is appropriate for the target audience. Some methods of assessing reading level require estimating the average word length. We've randomly chosen 20 words from a randomly selected page in *Stats: Modeling the World* and counted the number of letters in each word:

5, 5, 2, 11, 1, 5, 3, 8, 5, 4, 7, 2, 9, 4, 8, 10, 4, 5, 6, 6

1. Suppose that our editor was hoping that the book would have a mean word length of 6.5 letters. Does this sample indicate that the authors failed to meet this goal? Test an appropriate hypothesis and state your conclusion.

$H_0: \mu = 6.5$ avg word length is 6.5 letters

$H_A: \mu \neq 6.5$ avg word length is not 6.5 letters

Conditions

✓ – SRS "randomly selected"

✓ – 20 < 10% of all words on page

✓ – Nearly Normal

Perform a T-test in TI-84

using: data

$\mu_0 = 6.5$

$\mu \neq \mu_0$

$t = -1.6654$

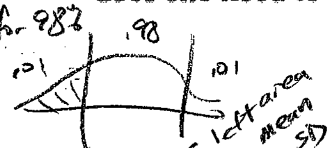
p-value = .1122

with $\alpha = .05$, p-value = .1122 is high so we fail to reject H_0. We do not have sufficient statistical evidence to conclude this book has mean word length which is different from 6.5 (the authors seem to have met their goal.)

2. For a more definitive evaluation of reading level the editor wants to estimate the text's mean word length to within 0.5 letters with 98% confidence. How many randomly selected words does she need to use?

1var stats: $s = 2.6852$

z^* ~~t^*~~ for 98%



$t^* \approx z^* = \text{invNorm}(.01, 0, 1)$

$= \times 2.3263$

$ME = .5$

$t^* \frac{s}{\sqrt{n}} = .5$

$(2.3263)\frac{2.6852}{\sqrt{n}} = .5$

$\frac{2.6852}{\sqrt{n}} = \frac{.5}{2.3263}$

$.5\sqrt{n} = (2.6852)(2.3263)$

$\sqrt{n} = \frac{(2.6852)(2.3263)}{.5}$

$n = \left(\frac{(2.6852)(2.3263)}{.5}\right)^2 = 156.079$ $\boxed{157 \text{ words}}$