AP Stats Ch 19 - Required Practice **Name:** Solutions **Per:** ____

#1. **DV Seniors.** Suppose that we take an SRS of n=40 seniors from DV and ask them if they are staying in state for college. 34 of these seniors say that they are staying in state. Find the 95% confidence interval which contains the proportion of all seniors at DV who will stay in state for college.

a) What is $\hat{p}$?

$\hat{p} = \frac{34}{40} = \boxed{.85}$

b) Check the assumptions:

✓ 1) SRS? assume representative

✓ 2) Individuals are independent? assumed

✓ 3) n < 10% of population? 40 < 10% of all seniors

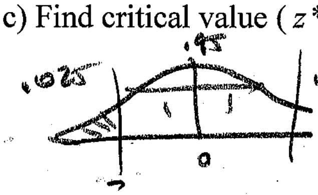
4) success/failure ($np \geq 10,\ nq \geq 10$)?

$np = (40)(.85) = 34 \geq 10$ ✓

$nq = (40)(.15) = 6 \geq 10$ X no

(we will proceed but note this in our conclusion)

c) Find critical value (z^*) for the confidence level.



$z^* = \text{invNorm}(.025, 0, 1) = -1.96$ (ignore) $\boxed{1.96}$

d) Find the standard error for this statistic (use the AP Stats Exam formula sheet 'recipe' – SE = standard dev).

(for sample proportions, $SE_{\hat{p}} = \sqrt{\frac{\hat{p}\hat{q}}{n}}$)

$SE_{\hat{p}} = \sqrt{\frac{(.85)(.15)}{40}} = \boxed{.0565}$

e) Find the margin of error ($ME = (z^*)(SE_{\hat{p}})$)

$ME = (1.96)(.0565) = \boxed{.11074}$

f) Find the confidence interval ($CI = \hat{p} \pm ME$)

$CI = .85 \pm .11074$

$\boxed{(.7393, .9607)}$

Now interpret the confidence interval in the context of the problem:

We are 95% confident that the true percentage of all DV seniors who are staying in state for college is between 73.93% and 96.07%. (But, we may not be able to trust this result because conditions for inference are not met, $nq \not\geq 10$).

g) The Margin of Error here is 11%. If we increase the sample size, n, the standard error will decrease and the margin of error will decrease. Currently, n = 40. How high would n need to be if we wanted a margin of error of only 5% (without changing the confidence level)?

$CI = \hat{p} \pm \underbrace{\boxed{z^* SE_{\hat{p}}}}_{\text{Margin of error}}$

$z^* SE_{\hat{p}} = MofE$

$(1.96)\sqrt{\frac{(.85)(.15)}{n}} = .05$

Solve for n algebraically:

divide by 1.96: $\sqrt{\frac{(.85)(.15)}{n}} = \frac{.05}{1.96}$

square: $\frac{(.85)(.15)}{n} = \left(\frac{.05}{1.96}\right)^2$

multiply by n: $(.85)(.15) = \left(\frac{.05}{1.96}\right)^2 n$

divide: $\frac{(.85)(.15)}{\left(\frac{.05}{1.96}\right)^2} = n = 195.99$ always round up

$\boxed{196 \text{ students}}$

h) The required n calculated in part g uses the sample proportion from this specific sample. If we took a difference sample with this same sample size, the $\hat{p}$ value would likely vary due to natural sampling variation, which slightly affects standard deviation and therefore margin of error. How can we select an n that would _guarantee_ that we have a margin of error of only 5% regardless of the sample's $\hat{p}$ value?

to guarantee, use $\hat{p} = 0.5$:

$z^* SE_{\hat{p}} = MofE$

$(1.96)\sqrt{\frac{(.5)(.5)}{n}} = .05$

$\sqrt{\frac{(.5)(.5)}{n}} = \frac{.05}{1.96}$

$\frac{(.5)(.5)}{n} = \left(\frac{.05}{1.96}\right)^2$

$n = \frac{(.5)(.5)}{\left(\frac{.05}{1.96}\right)^2} = 384.16 \uparrow = \boxed{385}$

#2. **Gambling.** A city ballot includes a local initiative that would legalize gambling. The issue is hotly contested, and two groups decide to conduct polls to predict the outcome. The local newspaper finds that 53% of 1200 randomly selected voters plan to vote "yes", while a college Statistics class finds 54% of 450 randomly selected voters in support. Both groups will create 95% confidence intervals.

a) Without finding the confidence intervals, explain which one will have the larger margin of error.

The statistics class will have the larger margin of error due to the smaller sample size.

b) Use your calculator to find both confidence intervals (assume all conditions are met).

class ~~1-prop Zint~~ 1-prop Zint
.54 of 450
x: 243
n: 450
C-level: .95
$\boxed{(.49395, .58605)}$

paper 1-prop Zint
.53 of 1200
x: 636
n: 1200
C-level: .95
$\boxed{(.50176, .55824)}$

c) Which group concludes that the outcome is too close to call? Why?

The statistics class must conclude the election is too close to call because 0.5 is within their confidence interval.
(The newspaper would conclude the measure will pass because all their likely values are higher than 0.5)

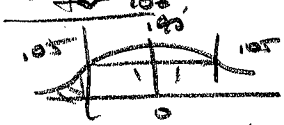
#3. **Junk Mail.** Direct mail advertisers send solicitations (a.k.a. "junk mail") to thousands of potential customers in the hope that some will buy the company's product. The response rate is usually quite low. Suppose a company wants to test the response to a new flyer, and sends it to 1000 people randomly selected from their mailing list of over 200,000 people. They get orders from 123 of the recipients.

a) Create a 90% confidence interval for the percentage of people the company contacts who may buy something.

$\hat{p} = \frac{123}{1000} = .123$

conditions
✓ $n\hat{p} = 123$
$n\hat{q} = 1000 - 123 = 877$
✓ SRS "randomly selected"
✓ 1000 < 10% of 200000

z^* for 90%:



$z^* = \text{invNorm}(.05, 0, 1)$
$= \pm 1.64$

$CI = \hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$
$= .123 \pm (1.64) \sqrt{\frac{(.123)(.877)}{1000}}$
$= .123 \pm .0171$
$= \boxed{(.1059, .1401)}$

b) Interpret this confidence interval in the context of the problem.

We are 90% confident that between 10.6% and 14.0% of all customers contacted will order from the new flyer.

c) Explain what "90% confidence" means in the context of the problem.

If we took many samples of 1000 and computed confidence intervals for each, 90% of these confidence intervals would contain the true proportion of all customers who would order from the flyer.

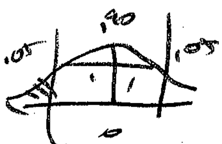
d) The company must decide whether to now do a mass mailing. The mailing won't be cost-effective unless it produces at least a 5% response rate. What does your confidence interval suggest? Explain.

The company should do the mass mailing because all the likely values for percentage ordering (the confidence interval) are far above the required 5%.

#4. **DV Seniors (again).** Suppose our senior class has 743 students. It is believed that 42% of our seniors attend ASU after graduation. We want to calculate a confidence interval for the proportion of our seniors who are planning to go to ASU next year.

a) How many randomly selected seniors must we ask to create an interval in which we are 90% confident within a margin of error of 0.12?

z^*



$z^* = \text{invNorm}(.05, 0, 1) = 1.64$

$ME = .12$

$z^*\sqrt{\frac{\hat{p}\hat{q}}{n}} = .12$

$(1.64)\sqrt{\frac{(.42)(.58)}{n}} = .12$

$\sqrt{\frac{(.42)(.58)}{n}} = \frac{.12}{1.64}$

$\frac{(.42)(.58)}{n} = \left(\frac{.12}{1.64}\right)^2$

$n = \frac{(.42)(.58)}{\left(\frac{.12}{1.64}\right)^2} = 45.499 \uparrow$

$\boxed{46 \text{ seniors}}$

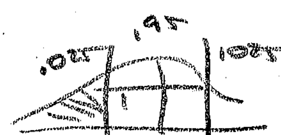
b) How about in which we are 90% confident, with a margin of error of 0.10? Before you start your calculations, do you think (circle one) n will *increase* or *decrease*?

$n \uparrow$ SE $\downarrow$ ME $\downarrow$

$(1.64)\sqrt{\frac{(.42)(.58)}{n}} = .10$

$n = \frac{(.42)(.58)}{\left(\frac{.10}{1.64}\right)^2} = 65.5 \uparrow$ $\boxed{66 \text{ seniors}}$

c) Repeat parts a and b but with a confidence level of 95%.



$z^* = \text{invNorm}(.025, 0, 1) = 1.96$

$ME = .12$

$(1.96)\sqrt{\frac{(.42)(.58)}{n}} = .12$

$n = \frac{(.42)(.58)}{\left(\frac{.12}{1.96}\right)^2} = 64.987 \uparrow$

$\boxed{65 \text{ seniors}}$

$ME = .10$

$(1.96)\sqrt{\frac{(.42)(.58)}{n}} = .10$

$n = \frac{(.42)(.58)}{\left(\frac{.10}{1.96}\right)^2} = 93.58 \uparrow$

$\boxed{94 \text{ seniors}}$

d) Should we be concerned about these new sample sizes? Explain why, or why not.

The 10% pop conditions says we should use n higher than $(.10)(743) = 74.3$, so the 94 seniors sample technically doesn't meet conditions for inference.

#5. **n and Margin of Error.** Calculate the 95% confidence interval if a sample n=50 has $\hat{p}$ = 0.4:

$$z^*\sqrt{\frac{\hat{p}\hat{q}}{n}} = (1.96)\sqrt{\frac{(.4)(.6)}{50}} = .1358$$

Now compute the 95% confidence intervals for $\hat{p}$ = 0.4 if n is increased to be…

4 times as large (n=200):

$$(1.96)\sqrt{\frac{(.4)(.6)}{200}} = .0679$$

$$\frac{.0679}{.1358} \approx \frac{1}{2}$$

9 times as large (n=450):

$$(1.96)\sqrt{\frac{(.4)(.6)}{450}} = .0453$$

$$\frac{.0453}{.1358} \approx \frac{1}{3}$$

100 times as large (n=5000):

$$(1.96)\sqrt{\frac{(.4)(.6)}{5000}} = .0136$$

$$\frac{.0136}{.1358} \approx \frac{1}{10}$$

Do you see a pattern here? How, specifically, does sample size change the margin of error?

Margin of error varies as $\frac{1}{\sqrt{n}}$ 4x larger, ME $= \frac{1}{\sqrt{4}} = \frac{1}{2}$ as big

100x larger, ME $= \frac{1}{\sqrt{100}} = \frac{1}{10}$ as big

3) **TV Viewership.** The PBS T.V. station is interested in knowing the percentage of their viewers who watch NOVA on Wednesday evenings. How large should their sample size be if they want to be within 3% with 95% confidence?

$$z^*\sqrt{\frac{\hat{p}\hat{q}}{n}} = .03$$

$$(1.96)\sqrt{\frac{(.5)(.5)}{n}} = .03$$

$$\sqrt{\frac{(.5)(.5)}{n}} = \frac{.03}{1.96}$$

$$\frac{(.5)(.5)}{n} = \left(\frac{.03}{1.96}\right)^2$$

$$n = \frac{(.5)(.5)}{\left(\frac{.03}{1.96}\right)^2} = 1067.11$$

$\boxed{\text{1068 viewers}}$

4) How large should n be if an original sample size is n = 200 and we want the confidence interval to be twice as wide?

ME varies as $\frac{1}{\sqrt{n}}$, so if $\frac{1}{\sqrt{n}} = 2$

$$\frac{1}{\sqrt{n}} = \frac{2}{1}$$

$$2\sqrt{n} = 1$$

$$\sqrt{n} = \frac{1}{2}$$

$n = \frac{1}{4}$ original n

new n $= \frac{200}{4}$

$\boxed{= \text{50 viewers}}$

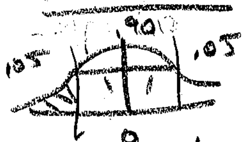
Chapter 19 Practice Quiz

AP Statistics Quiz C – Chapter 19 ***Name*** ____________________

A state's Department of Education reports that 12% of the high school students in that state attend private high schools. The State University wonders if the percentage is the same in their applicant pool. Admissions officers plan to check a random sample of the over 10,000 applications on file to estimate the percentage of students applying for admission who attend private schools.

1. The admissions officers want to estimate the true percentage of private school applicants to within ±4%, with 90% confidence. How many applications should they sample? $\hat{p} = .12$

z^* for 90%



$z^* = \text{invNorm}(.05, 0, 1) = \pm 1.64$

$$z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = .04$$

$$(1.64)\sqrt{\frac{(.12)(.88)}{n}} = .04$$

$$n = \frac{(.12)(.88)}{\left(\frac{.04}{1.64}\right)^2} = 178.59$$

$\boxed{\text{179 applications}}$

2. They actually select a random sample of 450 applications, and find that 46 of those students attend private schools. Create the confidence interval. (using 90% z^*)

$\hat{p} = \frac{46}{450} = .1022$

conditions
- ✓ $np = 450(.1022) = 46$, $nq = 450(.8978) = 404$ ≥ 10
- ✓ SRS "random sample"
- ✓ $450 < 10\%$ of 10,000

$$CI = \hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}}$$

$$= .1022 \pm (1.64)\sqrt{\frac{(.1022)(.8978)}{450}}$$

$$= .1022 \pm .0235$$

$$= \boxed{(.0787, .1257)}$$

3. Interpret the confidence interval in this context.

We are 90% confident ~~that~~ between 7.87% and 12.57% of all State University applicants attended private high schools.

4. Explain what 90% confidence means in this context.

If we took multiple samples of 450 applications and computed confidence intervals for each, 90% of the confidence intervals would contain the true percentage of all applicants who attended private high schools.

5. Should the admissions officers conclude that the percentage of private school students in their applicant pool is lower than the statewide enrollment rate of 12%? Explain.

$\boxed{\text{No}}$, 12% is within the 90% confidence interval.