AP Stats Ch 17 - Required Practice **Name:** Solutions **Per:** ____

20. Frogs. A wildlife biologist examines frogs for a genetic trait he suspects may be linked to sensitivity to industrial toxins in the environment. Previous research had established that this trait is usually found in 1 of every 8 frogs. He collects and examines a dozen frogs. If the frequency of the trait has not changed, what's the probability he finds the trait in

a) none of the 12 frogs?
b) at least 2 frogs?
c) 3 or 4 frogs?
d) no more than 4 frogs?

a) X | 0 1 2 3 4 5 6 7 8
P |
binompdf(12, $\frac{1}{8}$, 0) $\approx$ $\boxed{.2014}$

b) X | 0 1 2 3 4 5 6 7 8
1 − binomcdf(12, $\frac{1}{8}$, 1) $\approx$ $\boxed{.4533}$

c) X | 0 1 2 3 4 5 6 7 8
binompdf(12, $\frac{1}{8}$, 3) + binompdf(12, $\frac{1}{8}$, 4) $\approx$ $\boxed{.1707}$

d) X | 0 1 2 3 4 5 6 7 8
binomcdf(12, $\frac{1}{8}$, 4) $\approx$ $\boxed{.9887}$

8. Chips. Suppose a computer chip manufacturer rejects 2% of the chips produced because they fail presale testing.

a) What's the probability that the fifth chip you test is the first bad one you find?
b) What's the probability you find a bad one within the first 10 you examine?

a) X | 1st 2 3 4 5 6 7 . . .
geometpdf(.02, 5) $\approx$ $\boxed{.0184}$ (or NNNNY $(.98)^4(.02)$)

b) X | 1st 2 3 4 5 6 7 8 9 10, 11
geometcdf(.02, 10) = $\boxed{.1829}$

10. Chips ahoy. For the computer chips described in Exercise 8, how many do you expect to test before finding a bad one?

Expected value = mean, for geometric, $\mu = \frac{1}{p} = \frac{1}{.02} = \boxed{50 \text{ chips}}$

23. Frogs, part II. Based on concerns raised by his preliminary research, the biologist in Exercise 20 decides to collect and examine 150 frogs.

a) Assuming the frequency of the trait is still 1 in 8, determine the mean and standard deviation of the number of frogs with the trait he should expect to find in his sample.

b) Verify that he can use a Normal model to approximate the distribution of the number of frogs with the trait.

c) He found the trait in 22 of his frogs. Do you think this proves that the trait has become more common? Explain.

a) $EV = \mu = np$ for Binomial

$= (150)(\frac{1}{8}) = \boxed{18.75 \text{ frogs}}$

$\sigma = \sqrt{npq} = \sqrt{150(\frac{1}{8})(\frac{7}{8})} = \boxed{4.05 \text{ frogs}}$

b) $np = 18.75 \geq 10$ so Normal approx is appropriate

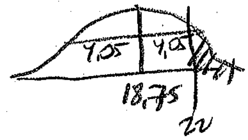
c) w/ Binomial

0 1 2 ⋯ 21 | 22 23 ⋯ 150

1 − binomcdf(150, 1/8, 21)

$= \boxed{.2433}$

w/ Normal approx



normalcdf(22, 999, 18.75, 4.05)

$\boxed{.2111}$

$P(\geq 22 \text{ frogs}) \approx .21 - .24$

which is not that unlikely. This data does not support the conclusion that the trait is more common.

$\frac{1}{8} = .125$, $\frac{22}{150} = .146$ is higher but not higher enough to be significant.

<u>Why do we need Normal approximations</u>? One reason: invNorm-type problems.

Let's reconsider the Frogs problem assuming we take a sample of 150 frogs with p=0.125 (1 out of 8)

a) Use a binomial model to find the probability of a sample of 150 frogs having <u>more than 22</u> frogs with the trait.

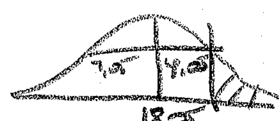
0 1 2 ⋯ 21 22 | 23 ⋯ 150

$P(>22 \text{ frogs}) = 1 - \text{binomcdf}(150, \frac{1}{8}, 22) = \boxed{.1759}$

b) Use a Normal approximation to the binomial model to find the probability of a sample of 150 frogs having more than 22 frogs with the trait.

$\mu = np = (150)(\frac{1}{8}) = 18.75$

$\sigma = \sqrt{npq} = \sqrt{150(\frac{1}{8})(\frac{7}{8})} = 4.05$



$P(>22 \text{ frogs}) = $ normalcdf(22, 999, 18.75, 4.05)

$= \boxed{.2111}$

not very unlikely

c) Now what if we asked this: If we consider a probability of 5% or less to be 'unlikely', <u>how many frogs would have to have the trait in a sample of 150 for this result to be 'unlikely'</u>?

for binomial, we would have to 'guess and check'

0 1 2 ⋯ 22 | 23 | 24 | ⋯ 150

1 − binomcdf(150, 1/8, 23) = .1222

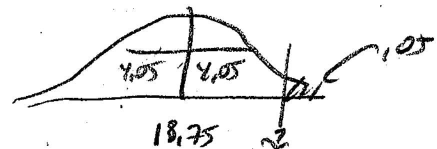
1 − binomcdf(150, 1/8, 26) = .0325

1 − binomcdf(150, 1/8, 25) = .0525

25 - 26 frogs

p crosses into < .05 area

w/ Normal approx, we can use invNorm



x = invNorm(.95, 18.75, 4.05)

$= \boxed{25.4 \text{ frogs}}$

$\boxed{26 \text{ frogs}}$

2. Bernoulli 2. Can we use probability models based on Bernoulli trials to investigate the following situations? Explain.

a) You are rolling 5 dice and need to get at least two 6's to win the game.

b) We record the eye colors found in a group of 500 people.

c) A manufacturer recalls a doll because about 3% have buttons that are not properly attached. Customers return 37 of these dolls to the local toy store. Is the manufacturer likely to find any dangerous buttons?

d) A city council of 11 Republicans and 8 Democrats picks a committee of 4 at random. What's the probability they choose all Democrats?

e) A 2002 Rutgers University study found that 74% of high-school students have cheated on a test at least once. Your local high-school principal conducts a survey in homerooms and gets responses that admit to cheating from 322 of the 481 students.

a) yes
b) no (more than 2 outcomes)
c) yes
d) no (small sample size means 1st selection affects probability of next — not independent)
e) yes

14. Arrows. An Olympic archer is able to hit the bull's-eye 80% of the time. Assume each shot is independent of the others. If she shoots 6 arrows, what's the probability of each result described below.

a) Her first bull's-eye comes on the third arrow.

b) She misses the bull's-eye at least once.

c) Her first bull's-eye comes on the fourth or fifth arrow.

d) She gets exactly 4 bull's-eyes.

e) She gets at least 4 bull's-eyes.

f) She gets at most 4 bull's-eyes.

a) geometpdf (.8,3) = $\boxed{.032}$

b) P(miss at least 1) = 1 − P(make all)
= 1 − binompdf (6, .8, 6)
= $\boxed{.7379}$

c) geometpdf(.8,4) + geometpdf(.8,5)
= $\boxed{.00768}$

d) binompdf (6, .8, 4) = $\boxed{.2458}$

e) P(at least 4) = 1 − P(0 to 3) = 1 − binomcdf (6, .8), 3) = $\boxed{.901}$

f) binomcdf (6, .8, 4) = $\boxed{.3446}$

16. More arrows. Consider our archer from Exercise 14.

a) How many bull's-eyes do you expect her to get?

b) With what standard deviation?

c) If she keeps shooting arrows until she hits the bull's-eye, how long do you expect it will take?

a) $EV = \mu = np$ for Binomial
$= (6)(.8) = \boxed{4.8 \text{ bullseyes}}$

b) $\sigma = \sqrt{npq} = \sqrt{6(.8)(.2)} = \boxed{0.9798 \text{ bulls-eyes}}$

c) $EV = \mu = \frac{1}{p}$ for geometric
$= \frac{1}{.8} = \boxed{1.25 \text{ shots}}$

Chapter 17 Practice Quiz

AP Statistics Quiz C – Chapter 17 ***Name***______________________

The owner of a small convenience store is trying to decide whether to discontinue selling magazines. He suspects that only 5% of the customers buy a magazine and thinks that he might be able to use the display space to sell something more profitable. Before making a final decision he decides that for one day he'll keep track of the number of customers and whether or not they buy a magazine.

1. Assuming the owner is correct in thinking that 5% of the customers purchase magazines, how many customers should he expect before someone buys a magazine?

expected value geometric

$\to = \mu = \frac{1}{p} = \frac{1}{.05} = 20$

$\boxed{20 \text{ customers}}$

2. What is the probability that he does not sell a magazine until the 8th customer? Show work.

X | 1st 2 3 4 5 6 7 8 9 ...

geometpdf(.05, 8) $= (.95)^7(.05) = .0349$

$\boxed{.0349}$

3. What is the probability that exactly 2 of the first 10 customers buy magazines? Show work.

binomial

X | 0 1 2 3 4 5 6 7 8 9 10

binompdf(10, .05, 2) $= {}_{10}C_2(.05)^2(.95)^8 = .0746$

$\boxed{.0746}$

4. What is the probability that at least 5 of his first 50 customers buy magazines?

binomial

X | 0 1 2 3 4 5 6 7 8 ... 50

1 − binomcdf(50, .05, 4) = .1036

$\boxed{.1036}$

5. He had 280 customers that day. Assuming this day was typical for his store, what would be the mean and standard deviation of the number of customers who buy magazines each day?

n = 280 binomial

$\mu = np = 280(.05) = 14$

$\sigma = \sqrt{npq} = \sqrt{280(.05)(.95)} = 3.647$

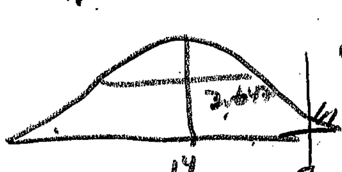
$\boxed{\mu = 14 \text{ customers} \quad \sigma = 3.647 \text{ customers}}$

6. Surprised by a high number of customers who purchased magazines that day, the owner decided that his 5% estimate must have been too low. How many magazine sales would it have taken to convince you? Justify your answer.

$np = (280)(.05) = 14 \geq 10$

$nq = (280)(.95) = 266$

So can use a Normal approximation



"unusual" maybe 5% chance of happening randomly

invNorm(.95, 14, 3.647) = 20

If we had 20 or more, this would be unusual.

—or—

$z = 2$ std devs $= \frac{x - \mu}{\sigma}$

$2 = \frac{x - 14}{3.647}$

$x = 21.29$

If we had 22, that would be unusual if really 5%.