AP Stats Ch 16 - Required Practice **Name:** solutions **Per:** ____

#1) Imagine an experiment in which you flip a unfair coin (P(H) = 3/4) three times.

a) Write out the sample space for the experiment outcomes:

S = {	TTT,	HTT,	THT,	TTH,	THH,	HTH,	HHT,	HHH }
# heads, X	0	1	1	1	2	2	2	3

b) Let *X* be a random variable representing the number of heads in the experiment. Below each entry in the sample space listing above, write the value of *X* for that experiment outcome.

c) Now list the possible values of the random variable *X* (the possible number of heads). Below each value, list all the outcomes that produce this random variable value, and compute the probability of that value occurring.

# heads, X	0	1	2	3
P	TTT	HTT or THT or TTH	THH or HTH or HHT	HHH
	$\frac{1}{4}\frac{1}{4}\frac{1}{4}$	$\frac{3}{4}\frac{1}{4}\frac{1}{4}+\frac{1}{4}\frac{3}{4}\frac{1}{4}+\frac{1}{4}\frac{1}{4}\frac{3}{4}$	$\frac{1}{4}\frac{3}{4}\frac{3}{4}+\frac{3}{4}\frac{1}{4}\frac{3}{4}+\frac{3}{4}\frac{3}{4}\frac{1}{4}$	$\frac{3}{4}\frac{3}{4}\frac{3}{4}$
	$\frac{1}{64}$	$\frac{3}{64}+\frac{3}{64}+\frac{3}{64}$	$\frac{9}{64}+\frac{9}{64}+\frac{9}{64}$	$\frac{27}{64}$
		$\frac{9}{64}$	$\frac{27}{64}$	

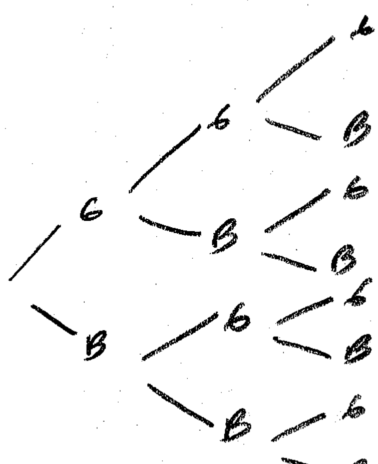
d) Now create a table showing each random variable value, along with its probability. This is called the **probability model**:

# heads, X	0	1	2	3
P	$\frac{1}{64}$	$\frac{9}{64}$	$\frac{27}{64}$	$\frac{27}{64}$

$$E(x) = (0)\left(\frac{1}{64}\right) + (1)\left(\frac{9}{64}\right) + (2)\left(\frac{27}{64}\right) + (3)\left(\frac{27}{64}\right) = 2.25 \text{ heads}$$

#2) Let X be the number of boys in a 3 child family. Find:
a) The expected value of X.
b) The variance of X.
c) The standard deviation of X.

# Boys, X	0	1	2	3
P	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$



S	P	X
GGG	1/8	0
GGB	1/8	1
GBG	1/8	1
GBB	1/8	2
BGG	1/8	1
BGB	1/8	2
BBG	1/8	2
BBB	1/8	3

(a) $E(X) = (0)(\frac{1}{8}) + (1)(\frac{3}{8}) + (2)(\frac{3}{8}) = 1.5$ boys

(b) $\sigma^2 = (0-1.5)^2(\frac{1}{8}) + (1-1.5)^2(\frac{3}{8}) + (2-1.5)^2(\frac{3}{8}) + (3-1.5)^2(\frac{1}{8})$

$= 0.75$ boys^2

(c) $\sigma = \sqrt{0.75} = 0.866$ boys

#3. Given independent random variables X and Y with means and standard deviation as shown, find the mean and standard deviation of each of the following new variables:
a) 3X
b) Y + 6

	Mean	SD
X	10	2
Y	20	5

a) multiply affects everything:

$\mu_{3X} = 3\mu_X = 3(10) = \boxed{30}$

$\sigma_{3X} = 3\sigma_X = 3(2) = 6$

b) adding doesn't affect spread:

$\mu_{Y+6} = \mu_Y + 6 = (20) + 6 = \boxed{26}$

$\sigma_{Y+6} = \sigma_Y = \boxed{5}$

* SEED: any integer, STO, rand *

#4a) **Speed Dating (use a simulation)**: To save time and money, many single people have decided to try speed dating. At a speed dating event, women sit in a circle and men spend about five minutes getting to know a woman before moving on to the next one. Suppose that the height *M* of male speed daters follows a Normal distribution with a mean of 68 inches and a standard deviation of 4 inches and the height *F* of female speed daters follows a Normal distribution with a mean of 64.5 inches and a standard deviation of 3 inches. What is the probability that a randomly selected female speed dater is taller than the randomly selected male speed dater she is paired with?

'STO'

1) Load random male heights into L1: randNorm(68, 4, 100) → L1
2) Load random female heights into L2: randNorm(64.5, 3, 100) → L2
3) height differences in L3 (men − women): L3 = L1 − L2
4) to make it easier to see negatives (where female is taller), sort:
   SortA(L3, L1, L2) (sorts on L3 in ascending order, bringing L1, L2 along)
5) look at L3: count the number of negatives <u>25</u> (for my trial)
   For this simulation, 25 of 100 pairs had taller female
   so experimental P(female taller) = $\boxed{0.25}$

— repeat this, and the value will vary, but stay near 0.24 —

Now look at μ, σ for L1, L2, L3 distributions (separate 1varstats on each)

<u>for my trial:</u>

male height (L1)	female height (L2)	difference (L3)
$\bar{x}_{L1} = 67.7357$	$\bar{x}_{L2} = 64.4331$	$\bar{x}_{L3} = 3.3026$
$s_{L1} = 3.7683$	$s_{L2} = 2.9844$	$s_{L3} = 5.0998$

notice: $\mu_{diffs} \approx \mu_M - \mu_F$

but $s_{diff} \neq s_M - s_F$

actually, $s_{diff} >$ than either s_M or s_F

what turns out to be true is <u>variances</u> add:

$$s^2_{Diff} \approx s^2_M + s^2_F$$

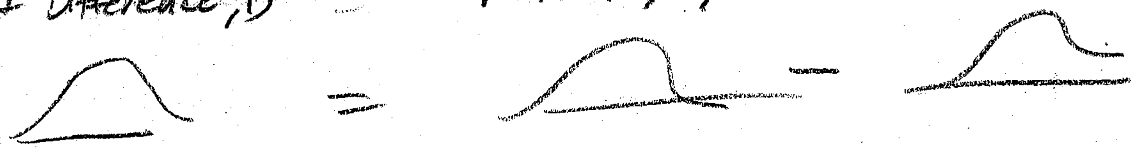
$$(5.0998)^2 \approx (3.7683)^2 + (2.9844)^2$$

$$26.01 \approx 23.11$$

close!

#4b) **Speed Dating (use rules for combining means and variances):** To save time and money, many single people have decided to try speed dating. At a speed dating event, women sit in a circle and men spend about five minutes getting to know a woman before moving on to the next one. Suppose that the height *M* of male speed daters follows a Normal distribution with a mean of 68 inches and a standard deviation of 4 inches and the height *F* of female speed daters follows a Normal distribution with a mean of 64.5 inches and a standard deviation of 3 inches. What is the probability that a randomly selected female speed dater is taller than the randomly selected male speed dater she is paired with?

height Difference, D = Male height, M − Female height, F



algebra: $D = M - F$

means: $\mu_D = \mu_M - \mu_F$

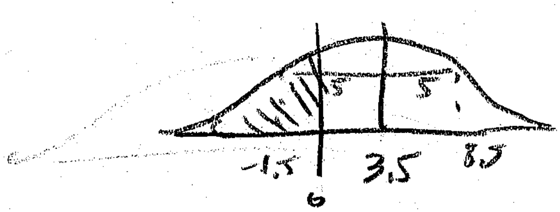
$\mu_D = 68 - 64.5 = 3.5$ in

variances add: $\sigma_D^2 = \sigma_M^2 + \sigma_F^2$

$\sigma_D^2 = (4)^2 + (3)^2$

$\sigma_D = \sqrt{(4)^2 + (3)^2} = 5$ in

Height difference, D

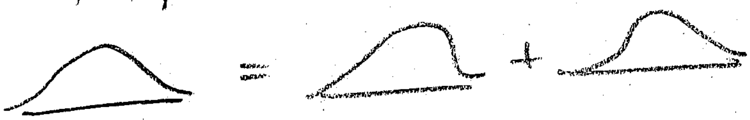


P(Female taller) = P(D<0) = normalcdf(−999, 0, 3.5, 5)

$= \boxed{0.242}$

#5) At your house in Hawaii (which has little variation in outside temperature throughout the year) you pay an electric bill and a natural gas bill at your home. Random variations month by month cause both of these bills to go up or down and the cost of the bills are independent of one another. If the electric bill cost mean is $40 with standard deviation of $12 and the gas bill cost mean is $25 with standard deviation of $8, what is the mean and standard deviation of total cost of these two bills?

total, T = electric, E + gas, G



algebra: $T = E + G$

means: $\mu_T = \mu_E + \mu_G$

$\mu_T = 40 + 25$

$\mu_T = \boxed{\$65}$

variances add: $\sigma_T^2 = \sigma_E^2 + \sigma_G^2$

$\sigma_T^2 = (12)^2 + (8)^2$

$\sigma_T = \sqrt{(12)^2 + (8)^2}$

$\sigma_T = \boxed{\$14.42}$

#6) A couple plans to have children until they get a girl, but they agree that they will not have more than three children even if all are boys. (Assume boys and girls are equally likely.)
a) Create a probability model for the number of children they'll have.
b) Find the expected number of children.
c) Find the expected number of boys they'll have.
d) Find the standard deviation of the number of children the couple may have.

a) $S = \{ G \quad BG \quad BBG \quad BBB$

\# children $X = 1 \quad 2 \quad 3 \quad 3$

\# boys $Y = 0 \quad 1 \quad 2 \quad 3$

X	1	2	3
P	$\boxed{\frac{1}{2}}$	$\frac{1}{2}\cdot\frac{1}{2}$ $\boxed{\frac{1}{4}}$	$\frac{1}{2}\frac{1}{2}\frac{1}{2}+\frac{1}{2}\frac{1}{2}\frac{1}{2}$ $\boxed{\frac{1}{4}}$

b) $E(X) = (1)(\frac{1}{2}) + (2)(\frac{1}{4}) + (3)(\frac{1}{4})$
$= \boxed{1.75 \text{ children}}$

c) Let $Y =$ ~~#~~ boys:

Y	0	1	2	3
P	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$

$E(Y) = (0)(\frac{1}{2}) + (1)(\frac{1}{4}) + (2)(\frac{1}{8}) + (3)(\frac{1}{8})$
$= \boxed{0.875 \text{ boys}}$

d) $\sigma_X = \sqrt{(1-1.75)^2(\frac{1}{2}) + (2-1.75)^2(\frac{1}{4}) + (3-1.75)^2(\frac{1}{4})}$
(or 1varstats)
$= \boxed{.829 \text{ children}}$

#7) You roll a die. If it comes up a 6, you win \$100. If not, you get to roll again. If you get a 6 the second time, you win \$50. If not, you lose.
a) Create a probability model for the amount you win at this game.
b) Find the expected amount you'll win.
c) How much would you be willing to pay to play this game?
d) Find the standard deviation of the amount you might win rolling a die.

a) $S = \{ 6 \quad \overline{6}6 \quad \overline{6}\overline{6}\}$

winnings X: \$100 \quad \$50 \quad \$0

X	\$100	\$50	\$0
P	$\frac{1}{6}$ $\frac{6}{36}$	$\frac{5}{6}\cdot\frac{1}{6}$ $\frac{5}{36}$	$\frac{5}{6}\cdot\frac{5}{6}$ $\frac{25}{36}$

b) $E(X) = (100)(\frac{6}{36}) + (50)(\frac{5}{36}) + (0)(\frac{25}{36})$
$= \boxed{\$23.61}$

c) $\boxed{\$23.61}$ is the "fair" value

d) $\sigma_X = \sqrt{(100-23.61)^2(\frac{6}{36}) + (50-23.61)^2(\frac{5}{36}) + (0-23.61)^2(\frac{25}{36})}$
$= \boxed{\$38.16}$

#8. Given independent random variables *X* and *Y* with means and standard deviation as shown, find the mean and standard deviation of each of the following new variables:

a) $3X$
b) $Y + 6$
c) $X + Y$
d) $X - Y$
e) $X_1 + X_2$

	Mean	SD
X	10	2
Y	20	5

Transforming one distribution

a) $\mu = 3\mu_X = 3(10) = \boxed{30}$

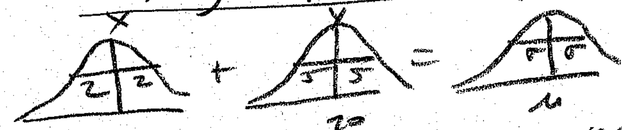
$\sigma = 3\sigma_X = 3(2) = \boxed{6}$

b) $\mu = \mu_Y + 6 = (20) + 6 = \boxed{26}$

$\sigma = \sigma_Y = 5$

(adding doesn't affect spread)

combining multiple distributions

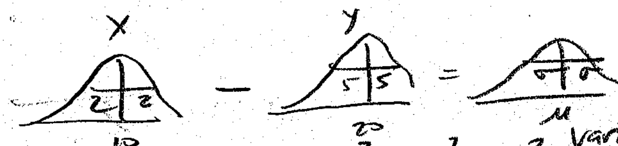
c) 

$\mu = \mu_X + \mu_Y = 10 + 20 = \boxed{30}$

$\sigma^2 = \sigma_X^2 + \sigma_Y^2$ variances add

$= (2)^2 + (5)^2$

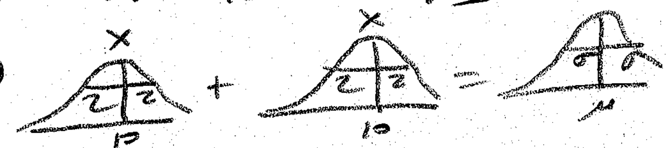
$\sigma = \sqrt{4+25} = \boxed{\sqrt{29}}$

d) 

$\mu = \mu_X - \mu_Y = 10 - 20 = \boxed{-10}$

$\sigma^2 = \sigma_X^2 + \sigma_Y^2$ variance still add

$\sigma = \boxed{\sqrt{29}}$

e) 

$\mu = \mu_X + \mu_X = 10 + 10 = \boxed{20}$

$\sigma^2 = \sigma_X^2 + \sigma_X^2 = (2)^2 + (2)^2$

$\sigma = \sqrt{4+4} = \boxed{\sqrt{8}}$

#9. An insurance policy costs \$100 and will pay policyholders \$10,000 if they suffer a major injury (resulting in hospitalization) or \$3,000 if they suffer a minor injury (resulting in lost time from work). The company estimates that each year 1 in every 2000 policyholders may have a major injury, and 1 in 500 a minor injury.
a) Create a probability model for the profit on a policy.
b) What's the company's expected profit on this policy?
c) What's the standard deviation?

a)

profit X	−\$9900	−\$2900	+\$100
p	$\frac{1}{2000}$	$\frac{1}{500}$	$\frac{399}{400}$ (what's left)

b) $E(X) = (-9900)\left(\frac{1}{2000}\right) + (-2900)\left(\frac{1}{500}\right) + (100)\left(\frac{399}{400}\right) = \boxed{\$89}$ (or $\bar{x}$ for 1var stats X,P)

c) $\sigma_X = \sqrt{(-9900-89)^2\left(\frac{1}{2000}\right) + (-2900-89)^2\left(\frac{1}{500}\right) + (100-89)^2\left(\frac{399}{400}\right)}$

$= \boxed{\$260.54}$ (or σ_x for 1var stats X,P)

#10. Your company bids for two contracts. You believe the probability you get contract #1 is 0.8. If you get contract #1, the probability you also get contract #2 will be 0.2, and if you do not get #1, the probability you get #2 will be 0.3.

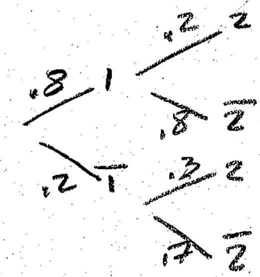
a) Are the two contracts independent? Explain.

b) Find the probability you get both contracts.

c) Find the probability you get no contract.

d) Let X be the number of contracts you get. Find the probability model for X.

e) Find the expected value and standard deviation of X.



a) $P(2|1) = .2 \neq P(2|\bar{1}) = .3$

so $\boxed{\text{not independent}}$

b) $P(1 \cap 2) = P(1) \cdot P(2|1) = (.8)(.2) = \boxed{.16}$

c) $P(\bar{1} \cap \bar{2}) = P(\bar{1}) \cdot P(\bar{2}|\bar{1}) = (.2)(.7) = \boxed{.14}$

d) $S = \{12,\ 1\bar{2},\ \bar{1}2,\ \bar{1}\bar{2}\}$

#contracts	2	1	1	0

X	0	1	2
P	$(.2)(.7)$	$(.8)(.8)+(.2)(.3)$	$(.8)(.2)$
	.14	.7	.16

e) $E(X) = (0)(.14) + (1)(.7) + (2)(.16)$

$= \boxed{1.02 \text{ contracts}}$

$\sigma_X = \sqrt{(0-1.02)^2(.14) + (1-1.02)^2(.7) + (2-1.02)^2(.16)}$

$= \boxed{0.547 \text{ contracts}}$

#11. Bicycles arrive at a bike shop in boxes. Before they can be sold, they must be unpacked, assembled, and tuned (lubricated, adjusted, etc.). Based on past experience, the shop manager makes the following assumptions about how long this may take:

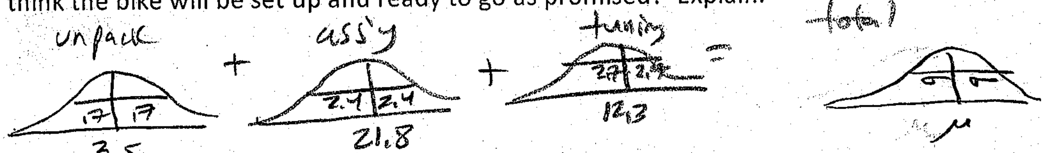
- The times for each setup phase are independent.
- The times for each phase follow a Normal model.
- The means and standard deviations of the times (in minutes) are as shown:

Phase	Mean	SD
Unpacking	3.5	0.7
Assembly	21.8	2.4
Tuning	12.3	2.7

a) What are the mean and standard deviation for the total bicycle setup time?

b) A customer decides to buy a bike like one of the display models but wants a different color. The shop has one, still in the box. The manager says they can have it ready in half an hour. Do you think the bike will be set up and ready to go as promised? Explain.

a)

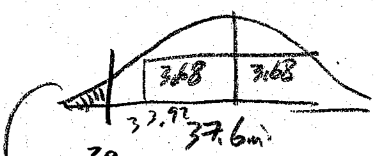


$\mu = \mu_U + \mu_A + \mu_T = 3.5 + 21.8 + 12.3 = \boxed{37.6 \text{ min}}$

$\sigma^2 = \sigma_U^2 + \sigma_A^2 + \sigma_T^2 = (.7)^2 + (2.4)^2 + (2.7)^2 = 13.54$

$\sigma = \sqrt{13.54} = \boxed{3.68 \text{ min}}$

b)



$P(\text{time} < 30 \text{ min}) = \text{Normalcdf}(-999, 30, 37.6, 3.68)$

$= \boxed{.019}$

Chapter 16 Practice Quiz

AP Statistics Quiz B – Chapter 16 **Name**____________________

A fast food restaurant just leased a new freezer and food fryer for three years. The service contract for the freezer offers unlimited repairs for a fee of $125 a year plus a $35 service charge for each repair needed. The restaurant's research suggested that during a given year 80% of these freezers need no repairs, 11% needed to be serviced once, 5% twice, 4% three times, and none required more than three repairs.

1. Find the expected number of repairs this kind of freezer is expected to need each year. Show your work.

$E = (0)(.8) + (1)(.11) + (2)(.05) + (3)(.04)$ —or—

$= \boxed{.33 \text{ repairs}}$

X	0	1	2	3
P	.8	.11	.05	.04

1var stats $\bar{x} = 0.33$ ← must show this too

2. Find the standard deviation of the number of repairs each year.

X	0	1	2	3
P	.8	.11	.05	.04

1var stats $S = 0.749$ repairs —or— $S = \sqrt{(0-.33)^2(.8)+(1-.33)^2(.11)+(2-.33)^2(.05)+(3-.33)^2(.04)}$

$= .749$ repairs

3. What are the mean and standard deviation of the restaurant's annual expense for the service contract?

$E = 125 + 35R$

$\mu_{Exp} = 125 + 35(.33) = \boxed{\$136.55}$ $\sigma_{Exp} = 35(.749) = \boxed{\$26.215}$

4. How many times should the restaurant expect to have to get this freezer repaired over the three-year term of the lease?

$\mu_{3yrs} = \mu + \mu + \mu = 3(.33) = \boxed{0.99 \text{ repairs}}$

5. What is the standard deviation of the number of repairs that may be required during the three-year term of the lease? On what assumption does your calculation rest? Do you think this assumption is reasonable?

$\sigma_{3y}^2 = \sigma_y^2 + \sigma_y^2 + \sigma_z^2$

$\sigma_{3y} = \sqrt{(.749)^2+(.749)^2+(.749)^2} = \boxed{1.297 \text{ repairs}}$

Years must be independent

6. The yearly service contract for the food fryer estimates a mean annual cost of $140 with a standard deviation of $40. What is the expected value and standard deviation of the total cost for the service contracts for the freezer and the food fryer?

$\mu_{sum} = \mu_{freezer} + \mu_{fryer} = \$136.55 + \$140 = \boxed{\$276.55}$

$\sigma_{sum}^2 = \sigma_{freezer}^2 + \sigma_{fryer}^2$ $\sigma_{sum} = \sqrt{(26.215)^2 + (40)^2} = \boxed{\$47.82}$

7. Which service contract should the restaurant expect to cost more each year? How much more? With what standard deviation?

$\boxed{\text{Fryer}}$ costs more : $\$140 - \$136.55 = \boxed{\$3.45 \text{ more}}$

$\sigma_{difference} = \boxed{\$47.82}$ (same as #6, variances always add)