AP Stats Ch 14,15 - Required Practice **Name:** Solutions **Per:** ____

#1. For each of the following, list the sample space and tell whether you think the outcomes are equally likely:
a) Roll two dice; record the sum of the numbers.
b) A family has 3 children; record the genders in order of birth.
c) Toss four coins; record the number of tails.
d) Flip a coin until you get a head or 3 consecutive tails.

a) $S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\}$ not equally likely

b) $S = \{BBB, BBG, BGB, GBB, GGB, GBG, BGG, GGG\}$ equally likely

c) $S = \{0, 1, 2, 3, 4\}$ not equally likely

d) $S = \{H, TH, TTH, TTT\}$ not equally likely

#2. The American Red Cross says that 40% of the U.S. population has type A blood, 11% have type B blood, 4% have type AB blood, and the rest of the population has type O blood. If a person from the U.S. is chosen at random, what is the probability that they will have type O blood?

$P(\text{not } O) = .40 + .11 + .04 = \boxed{.55}$

$P(O) = 1 - P(\text{not } O) = 1 - .55 = \boxed{.45}$

#3. If a single die is rolled one time, find the probabilities of getting:
a) a 4
b) an even number
c) a number greater than 4
d) a number less than 7
e) a number greater than 0

$S = \{1, 2, 3, 4, 5, 6\}$ equally likely

a) $P(4) = \boxed{\frac{1}{6}}$

b) $P(\text{even}) = \boxed{\frac{3}{6}} = \frac{1}{2}$

c) $P(>4) = \boxed{\frac{2}{6}} = \frac{1}{3}$

d) $P(<7) = \frac{6}{6} = \boxed{1}$

e) $P(>0) = \frac{6}{6} = \boxed{1}$

#4. Researchers are interested in the relationship between cigarette smoking and lung cancer. Suppose an adult male is randomly selected from a particular population. Assume that the following table shows some probabilities involving the compound event that the individual does or does not smoke and the person is or is not diagnosed with cancer:

Event	Probability
smokes and gets cancer	0.05
smokes and does not get cancer	0.20
does not smoke and gets cancer	0.03
does not smoke and does not get cancer	0.72

	cancer	no cancer	
smoker	.05	.20	.25
nonsmoker	.03	.72	.75
	.08	.92	1.00

a) Find the probability that the individual gets cancer, given that he is a smoker.
b) Find the probability that the individual does not get cancer, given that he is a smoker.
c) Find the probability that the individual gets cancer, given that he does not smoke.
d) Find the probability that the individual does not get cancer, given that he does not smoke.

a) $P(\text{cancer} \mid \text{smoker}) = \frac{.05}{.25} = \boxed{.2}$

b) $P(\overline{\text{cancer}} \mid \text{smoker}) = \frac{.20}{.25} = \boxed{.8}$

c) $P(\text{cancer} \mid \text{nonsmoker}) = \frac{.03}{.75} = \boxed{.04}$

d) $P(\overline{\text{cancer}} \mid \text{nonsmoker}) = \frac{.72}{.75} = \boxed{.96}$

#5. The table shows the political affiliation of American voters and their positions on the death penalty.
a) Find the probability that a randomly chosen voter favors the death penalty?
b) Find the probability that a Republican favors the death penalty?
c) Find the probability that a voter who favors the death penalty is a Democrat?

Party	Death Penalty: Favor	Oppose	
Republican	0.26	0.04	.3
Democrat	0.12	0.24	.36
Other	0.24	0.10	.34
	.62	.38	1.00

a) $P(\text{favor}) = \frac{.62}{1} = .62$

b) $P(\text{favor} \mid R) = \frac{.26}{.3} = .87$

c) $P(D \mid \text{favor}) = \frac{.12}{.62} = .19$

#6. If a single die is rolled one time, find the probabilities of getting:
a) a number greater than 3 or an odd number.
b) an even number or a 5.

a) 1 2 3 4 5 6

$P(>3 \text{ or odd}) = P(>3) + P(\text{odd}) - P(3 \cap \text{odd})$
$= \frac{3}{6} + \frac{3}{6} - \frac{1}{6} = \boxed{\frac{5}{6}}$

b) 1 2 3 4 5 6

$P(\text{even} \cup 5) = P(\text{even}) + P(5) - P(\text{even} \cap 5)$
$= \frac{3}{6} + \frac{1}{6} - \frac{0}{6}$
$= \frac{4}{6} = \boxed{\frac{2}{3}}$

#7. A Gallup Poll in June 2004 asked 1005 U.S. adults how likely they were to read Bill Clinton's autobiography *My Life*.
The table shows how they responded.
If we select a person at random from this sample of 1005 adults,
a) What is the probability that the person responded "Will definitely not read it"?
b) What is the probability that the person will probably or definitely read it?

Response	Number
Will definitely read it	90
Will probably read it	211
Will probably not read it	322
Will definitely not read it	382
Total	1005

a) $P(\text{def not read it}) = \boxed{\frac{382}{1005}}$

b) $P(\text{prob} \cup \text{def}) = P(\text{prob}) + P(\text{def}) - P(\text{prob} \cap \text{def})$
$= \frac{211}{1005} + \frac{90}{1005} - \frac{0}{1005}$
$= \frac{301}{1005}$

#8. If a single die is rolled one time, find the probability of getting
a) an odd number.
b) a number greater than 3 or an odd number.
c) a number greater than 3 and ~~an odd~~ number.

$S = \{1, 2, 3, 4, 5, 6\}$

a) $P(\text{odd}) = \frac{3}{6} = \boxed{\frac{1}{2}}$

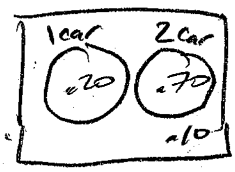
b) $P(>3 \cup \text{odd}) = P(>3) + P(\text{odd}) - P(>3 \cap \text{odd})$ or
$= \frac{3}{6} + \frac{3}{6} - \frac{1}{6} = \boxed{\frac{5}{6}}$

1, 2, 3, 4, 5, 6

c) $P(>3 \cap \text{odd}) = P(>3) \cdot P(\text{odd} \mid >3)$ and
$= \frac{3}{6} \cdot \frac{1}{3} = \frac{3}{18} = \boxed{\frac{1}{6}}$

$P(\text{odd} \mid >3)$
1, 2, 3, $\boxed{4, 5, 6}$

#9. In building new homes, a contractor finds that the probability of a home buyer selecting a two-car garage is 0.70 and selecting a one-car garage is 0.20. (Note that the builder will not build a three-car or larger garage).
a) What is the probability that the buyer will select either a one-car or a two-car garage?
b) What is the probability that the buyer will select no garage?
c) What is the probability that the buyer will not want a two-car garage?



a) $P(1 \text{ or } 2) = P(1) + P(2) - P(1 \text{ AND } 2)$
$= .20 + .70 - 0 = \boxed{.90}$

b) $P(\text{no garage}) = 1 - P(1 \text{ or } 2) = 1 - .90 = \boxed{.10}$

c) $P(\text{not } 2) = 1 - P(2) = 1 - .70 = \boxed{0.3}$

#10. The table shows the political affiliation of American voters and their positions on the death penalty.
a) Find the probability that a randomly chosen voter favors the death penalty?
b) Find the probability that a Republican favors the death penalty? (AND or conditional?)
c) Find the probability that a voter who favors the death penalty is a Democrat?
d) A candidate thinks she has a good chance of gaining the votes of anyone who is a Republican or in favor of the death penalty. What portion of the voters is that?
e) Are party affiliation and position on the death penalty independent? Explain.

Party	Death Penalty: Favor	Death Penalty: Oppose	
Republican	0.26	0.04	.3
Democrat	0.12	0.24	.36
Other	0.24	0.10	.34
	.62	.38	1.0

(a) $P(\text{favor}) = \frac{.62}{1} = \boxed{.62}$

(b) $P(\text{favor}|R) = \frac{.26}{.3} = \boxed{.87}$

if AND: $P(\text{favor} \cap R) = \boxed{.26}$

(c) $P(D|\text{favor}) = \frac{P(D \cap \text{favor})}{P(\text{favor})} = \frac{.12}{.62} = \boxed{.19}$

d) $P(R \cup \text{favor}) = P(R) + P(\text{favor}) - P(R \cap \text{favor})$

$= .3 + .62 - .26$

$= \boxed{.66}$

e) $P(\text{favor}) = .62$

$P(\text{favor}|R) = .87$

$P(\text{favor}) \neq P(\text{favor}|R)$

so not independent

one draw

#11. You draw a card at random from a standard deck of 52 cards. Find each of the following conditional probabilities:
a) The card is a heart, given that it is red.
b) The card is red, given that it is a heart.
c) The card is an ace, given that it is red.
d) The card is a queen, given that it is a face card (jack, queen, or king).

$\frac{c(\text{red hearts})}{c(\text{red cards})}$

a) writing out all rules:

$P(H|R) = \frac{P(H \cap R)}{P(R)} = \frac{P(H) \cdot P(R|H)}{P(R)} = \frac{\left(\frac{13}{52}\right)\left(\frac{13}{13}\right)}{\left(\frac{26}{52}\right)} = \frac{13/52}{26/52} = \frac{13}{26} = \boxed{\frac{1}{2}}$

b) $P(R|H) = \frac{P(R \cap H)}{P(H)} = \frac{c(R \cap H)}{c(H)} = \frac{13}{13} = \boxed{1}$

c) $P(\text{Ace}|R) = \frac{P(\text{Ace} \cap R)}{P(R)} = \frac{c(\text{Ace} \cap R)}{c(R)} = \frac{2}{26} = \boxed{\frac{1}{13}}$

d) $P(Q|\text{Face}) = \frac{P(Q \cap \text{Face})}{P(\text{Face})} = \frac{4/52}{12/52} = \frac{4}{12} = \boxed{\frac{1}{3}}$

#12 Ivy conducted a taste test for four different brands of chocolate chip cookies. Below is a two-way table that describes which cookie each subject preferred and their gender.

	Cookie Brand A	B	C	D	Totals
Female	4	6	13	13	36
Male	22	11	11	14	58
Totals	26	17	24	27	94

Suppose one subject from this experiment is selected at random.

(a) Find the probability that the selected subject preferred Brand C.

$P(C) = \frac{24}{94} = \boxed{.255}$

(b) Find the probability that the selected subject preferred Brand C, given that she is female.

$P(C \mid \text{Female}) = \frac{13}{36} = \boxed{.361}$

(c) Are the events "preferred Brand C" and "female" independent? Explain.

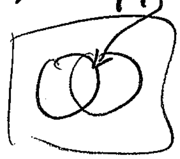
$P(C) \neq P(C \mid \text{Female})$ so "C" and "female"

$.255 \neq .361$ are not independent

(d) Are the events "preferred Brand C" and "female" mutually exclusive? Explain.

$P(C \cap F) = \frac{13}{94} \neq 0$ so "C" and "female"

are not mutually exclusive



(e) If a random sample of two subjects is selected, what is the probability that neither preferred Brand A?

$P(1^{st}\text{ not }A \cap 2^{nd}\text{ not }A) = P(1^{st}\text{ not }A) \cdot P(2^{nd}\text{ not }A \mid 1^{st}\text{ not }A)$

$= \frac{68}{94} \cdot \frac{67}{93}$

$= \boxed{.521}$

A	not A	
26	17+24+27 (68)	94

#13 $P(A)=0.5,\ P(B)=0.2$ *A and B are independent events*

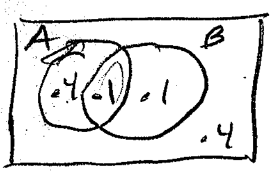
a) *Find* $P(A \cap B)$
b) *Find* $P(A \cup B)$
c) Are A and B disjoint (mutually exclusive)? Why or why not?
d) Draw a Venn Diagram for events A and B and label all probabilities.
e) *Find* $P(A \mid B)$
f) *Find* $P(B \mid A)$
g) *Find* $P(A^C \cap B^C)$

(a) $P(A \wedge B) = P(A) \cdot P(B|A)$ (indep, so $P(B|A) = P(B)$) $\Rightarrow P(A \wedge B) = P(A) \cdot P(B) = (0.5)(0.2) = \boxed{0.1}$

(b) $P(A \cup B) = P(A) + P(B) - P(A \wedge B) = 0.5 + 0.2 - 0.1 = \boxed{0.6}$

(c) $P(A \wedge B) = 0.1 \neq 0$ so $\boxed{\text{not disjoint}}$

(d)



e) $P(A|B) = \dfrac{P(A \wedge B)}{P(B)} = \dfrac{0.1}{0.2} = \boxed{0.5}$

f) $P(B|A) = \dfrac{P(B \wedge A)}{P(A)} = \dfrac{0.1}{0.5} = \boxed{0.2}$

g) $P(A^c \wedge B^c) =$ "not in A" and "not in B"
$= \boxed{0.4}$ (by Venn diagram)

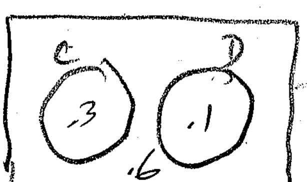
#14 $P(C)=0.3,\ P(D)=0.1$ *A and B are disjoint (mutually exclusive) events*

a) *Find* $P(C \cap D)$
b) *Find* $P(C \cup D)$
c) Draw a Venn Diagram for events C and D and label all probabilities.
d) *Find* $P(\overline{C})$
e) *Find* $P(C \mid D)$
f) Are C and D independent? Why or why not?
g) *Find* $P(\overline{C} \cap \overline{D})$

(a) $P(C \wedge D) = \boxed{0}$ (definition of disjoint)

(b) $P(C \cup D) = P(C) + P(D) - P(C \wedge D) = 0.3 + 0.1 - 0 = \boxed{0.4}$

(c)



d) $P(C) = \boxed{0.3}$ (given)

e) $P(C|D) = \dfrac{P(C \wedge D)}{P(D)} = \dfrac{0}{0.1} = \boxed{0}$

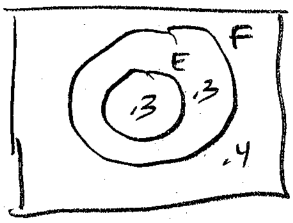
f) $P(C) \neq P(C|D)$
$0.3 \neq 0$
so $\boxed{\text{not independent}}$
(disjoint events with non zero probabilities can never be disjoint)

g) $P(\overline{C} \wedge \overline{D}) =$ "not C" AND "not D"
$= \boxed{0.6}$ (by Venn diagram)

#15 $P(E)=0.3$, $P(F)=0.6$ *E is a subset of F*

a) Draw a Venn Diagram for events E and F and label all probabilities.
b) Find $P(E\cap F)$
c) Find $P(E\cup F)$
d) Find $P(F|E)$
e) Find $P(E|F)$
f) Are E and F independent? Why or why not?
g) Are E and F disjoint? Why or why not?

a)



b) $P(E\cap F)=P(E)\cdot P(F|E)=P(E)\cdot\frac{P(F\cap E)}{P(E)}$
$=(0.3)\left(\frac{0.3}{0.3}\right)=\boxed{0.3}$

(c) $P(E\cup F)=P(E)+P(F)-P(E\cap F)=0.3+0.6-.3=\boxed{0.6}$

(d) $P(F|E)=\frac{P(F\cap E)}{P(E)}=\frac{0.3}{0.3}=\boxed{1}$

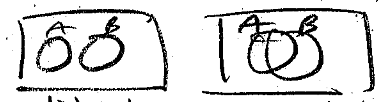
(e) $P(E|F)=\frac{P(E\cap F)}{P(F)}=\frac{0.3}{0.6}=\boxed{0.5}$

(f) $P(E)\neq P(E|F)$ so $\boxed{\text{not independent}}$
$0.3\neq 0.5$

(g) $P(E\cap F)\neq 0$ so $\boxed{\text{not disjoint}}$
$0.3\neq 0$

#16. Which of the following pairs of events A and B are disjoint? Which are independent?
a) A single fair coin is tossed once. A="heads", B="tails".
b) A single fair coin is tossed twice. A="tails 1st flip", B="tails 2nd flip".
c) Two cards are drawn from a deck, with replacement. A="1st card is red", B="2nd card is red".
d) Two cards are drawn from a deck, without replacement. A="1st card is red", B="2nd card is red".
e) One card is drawn from a deck. A="Card is red", B="Card is a spade".
f) One card is drawn from a deck. A="Card is a heart", B="Card is a 3".

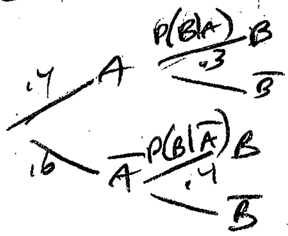
Disjoint has to do with "overlap"

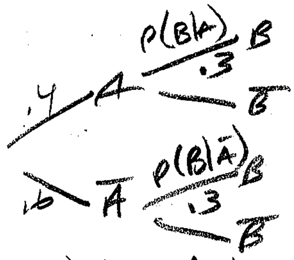


to test for disjoint, check $P(A \cap B) \stackrel{?}{=} 0$

If $P(A \cap B) = 0$ then disjoint

Independence has to do with whether one event's outcome affects the other event's probability:





and $P(B)$ would be $(.4)(.3) + (.6)(.3) = .3$

not indep. if $P(B) \neq P(B|A) \neq P(B|\bar{A})$

independent if $P(B) = P(B|A) = P(B|\bar{A})$

a) single fair coin tossed once, A = heads, B = tails

test for disjoint

$P(A \cap B) = P(H \cap T) = 0$

can't be both at once

so $\boxed{\text{disjoint}}$

test for independence

$P(A) = P(H) = 0.5 \quad P(B) = P(T) = 0.5, \quad P(A \cap B) = 0$

$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0}{0.5} = 0$

$P(B) \neq P(B|A)$ so $\boxed{\text{not indep}}$

$0.5 \neq 0$

b) single fair coin tossed twice, A = tail 1st flip, B = tails 2nd flip

test for disjoint

$P(A \cap B) = P(1^{st}T \cap 2^{nd}T)$

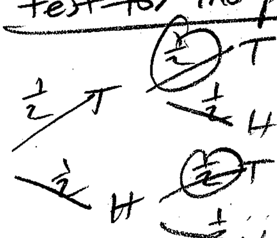
$= P(1^{st}T) \cdot P(2^{nd}T \mid 1^{st}T)$

$= (0.5)(0.5)$

$= 0.25$

so $\boxed{\text{not disjoint}}$

test for independence



$P(T|T) = P(T|H)$

so $\boxed{\text{independent}}$

(continued $\Rightarrow$)

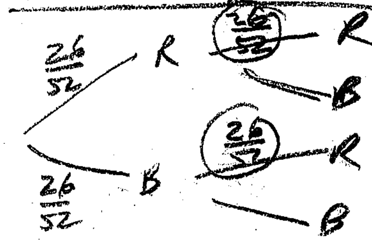
c) Two cards are drawn from a deck $\boxed{\text{with}}$ replacement $A = 1^{st}$ is red, $B = 2^{nd}$ is red

test for disjoint

$P(A \cap B) = P(\text{both are red}) \neq 0$ (is possible)

so $\boxed{\text{not disjoint}}$

test for independence



$P(R|R) = P(R|B) = P(R\ 2^{nd})$

so $\boxed{\text{independent}}$

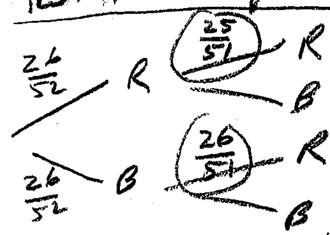
d) Same as (c) but $\boxed{\text{without}}$ replacement.

test for disjoint

$P(A \cap B) = P(\text{both red}) \neq 0$

so $\boxed{\text{not disjoint}}$

test for independence



$P(R|R) \neq P(R|B)$ so $\boxed{\text{not independent}}$

e) One card drawn from deck, A = card is red, B = card is a spade

test for disjoint

$P(A \cap B) = P(\text{card is a red spade}) = 0$ (all spades are black)

so $\boxed{\text{disjoint}}$

test for independence

$P(B|A) \stackrel{?}{=} P(B)$

$P(\text{spade}|\text{red}) \stackrel{?}{=} P(\text{spade})$

$\frac{0 \text{ are spades}}{26 \text{ red cards}} \neq \frac{13 \text{ are spades}}{52 \text{ cards}}$

not equal (probability of spade is changing)

so $\boxed{\text{not independent}}$

f) One card drawn from deck, A = card is a heart, B = card is a 3

test for disjoint

$P(A \cap B) = P(\text{card is a 3 and a heart}) \neq 0$ (is possible)

so $\boxed{\text{not disjoint}}$

test for independence

let's use test #2 this time:

If $P(A \cap B) = P(A) \cdot P(B)$ then indep.

$P(A \cap B) = P(3) \cdot P(\text{heart} | 3)$

$= \frac{4}{52} \cdot \frac{1}{4} = \frac{1}{52}$

$P(A) = P(3) = \frac{4}{52}$, $P(B) = P(\text{heart}) = \frac{1}{4}$

$P(A \cap B) \stackrel{?}{=} P(A) \cdot P(B)$

$\frac{1}{52} \stackrel{?}{=} \frac{4}{52} \cdot \frac{1}{4}$

$\frac{1}{52} = \frac{1}{52}$ ✓ so $\boxed{\text{independent}}$

#17

a) A batter who had failed to get a hit in seven consecutive times at bat then hits a game-winning home run. When talking to reporters afterward, he says he was very confident that last time at bat because he knew he was "due for a hit". Comment on his reasoning.

b) You flip four coins 32 times. Are you guaranteed to get four "heads" twice? Explain.

a) Each at bat is independent — no connection to other at bats, "due for a hit" not valid.

b) No. We can predict frequency with large number of trials, but 32 is not that large. Anything can happen with a small number of trials.

#18. Abby, Barbara, Carla, Dan, and Ernie work in a firm's public relations office. Their employer must choose two of them to attend a conference in Chicago. To avoid unfairness, the choice will be made by drawing two names from a hat.

a) List the sample space (write down all possible choices of two of the five names). For convenience, use only the first letter of their names.

b) What is the probability of each of these choices?

c) What is the probability that neither of the two men (Dan and Ernie) is chosen?

a) $S = \{AB, AC, AD, AE, BC, BD, BE, CD, CE, DE\}$

b) $\frac{1}{10}$

c) $\frac{3}{10}$

#19. A couple plans to have three children. Find the probability that the children are:

(children gender is independent)

a) all boys

b) all girls

c) two boys or two girls

d) at least one child of each sex

a) $P(BBB) = (\frac{1}{2})(\frac{1}{2})(\frac{1}{2}) = \boxed{\frac{1}{8}}$

b) $P(GGG) = (\frac{1}{2})(\frac{1}{2})(\frac{1}{2}) = \boxed{\frac{1}{8}}$

c) $P(2B \text{ or } 2G)$ $\quad S = \{BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG\}$ (8)

$E = \{BBG, BGB, GBB, BGG, GBG, GGB\}$ (6) equally likely

$P(2B \text{ or } 2G) = \frac{6}{8} = \boxed{\frac{3}{4}}$

d) $P(\text{at least one child of each sex})$ same event as E $\quad \boxed{\frac{3}{4}}$

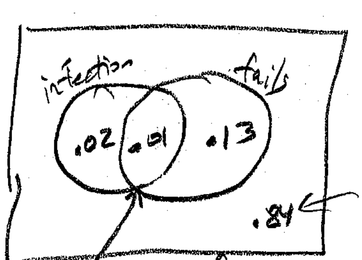
or $P(\text{at least one of each}) = 1 - P(\text{all boys or all girls})$

$= 1 - \frac{2}{8}$

$= \frac{6}{8}$

$= \frac{3}{4}$

#20. Suppose that you have torn a tendon and are facing surgery to repair it. The orthopedic surgeon explains the risks to you. Infection occurs in 3% of such operations, the repair fails in 14%, and both infection and failure occurs together in 1%. What percent of these operations succeed and are free from infection?



fill in venn from most overlapped first

$P(\text{not infection AND not fail}) = \boxed{0.84}$

#21. Two cards are dealt, one after the other, from a shuffled 52-card deck.

Why is it wrong to say that the probability of getting two red cards is $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right) = \frac{1}{4}$?

What is the correct probability of this event?

this probability changes due to 1st card (w/o replacement card draws are not independent)

$P(R \cap R) = P(R) \cdot P(2^{nd} R \mid 1^{st} R)$

$= \frac{1}{2} \cdot \frac{25}{51} = \boxed{\frac{25}{102}}$

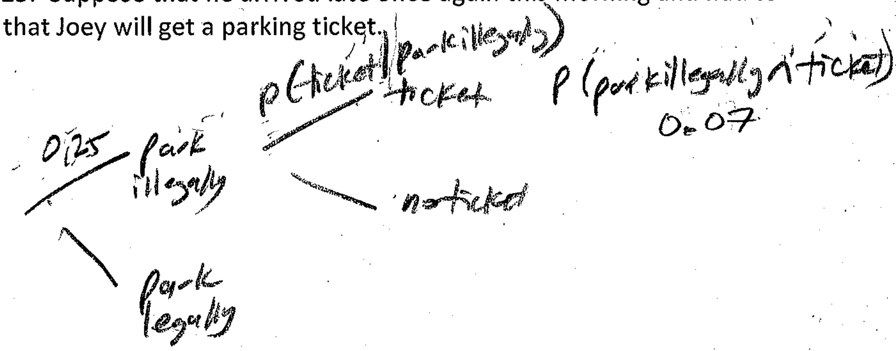
$\approx .24509$

#22. Parking for students at Central High School is very limited, and those who arrive late have to park illegally and take their chances of getting a ticket. Joey has determined that the probability that he has to park illegally and that he gets a parking ticket is .07. He has kept data from last year and found that because of his perpetual tardiness, the probability that he will have to park illegally is .25. Suppose that he arrived late once again this morning and had to park in a no-parking zone. Find the probability that Joey will get a parking ticket.

$P(\text{park illegally} \cap \text{ticket}) = .07$

$P(\text{park illegally}) = .25$

Find $P(\text{ticket} \mid \text{park illegally})$



$P(\text{park illegally} \cap \text{ticket}) = P(\text{park illegally}) \cdot P(\text{ticket} \mid \text{park illegally})$

$0.07 = 0.25 \cdot P(\text{ticket} \mid \text{park illegally})$

$\boxed{0.28} = \frac{0.07}{0.25} = P(\text{ticket} \mid \text{park illegally})$

#23. Before the introduction of purple, M&M's color distribution was as follows: 20% yellow, 20% red, 10% each for orange, blue, and green, and the rest were brown.
a) If you pick an M&M at random, what is the probability that it is yellow or orange?
b) If you pick three M&M's in a row, what is the probability that the third one is the first one that's red?
c) If you pick three M&M's in a row, what is the probability that none are yellow?
d) If you pick three M&M's in a row, what is the probability that at least one is green?

a) $P(Y \cup O) = 0.2 + 0.1 = \boxed{0.3}$ (no subtract – disjoint)

b) $P(R) = 0.20$
$P(\bar{R}) = 0.80$
$P(\bar{R}\bar{R}R) = (0.8)(0.8)(0.2) = \boxed{0.128}$

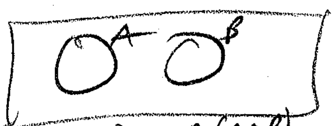
c) $P(Y) = 0.2$
$P(\bar{Y}) = 0.8$
$P(\bar{Y}\bar{Y}\bar{Y}) = (0.8)(0.8)(0.8) = \boxed{0.512}$

assuming container is large enough for draws to be independent (wouldn't assume this if they told us the total number in container)

d) $P(\text{at least 1 } G) = 1 - P(\text{no Green})$
$= 1 - P(\bar{G}\bar{G}\bar{G}) = 1 - (.9)(.9)(.9)$
$= 1 - .729 = \boxed{.271}$

#24. Can disjoint events ever be independent? Explain. No
Can independent events ever be disjoint? Explain. No

disjoint events

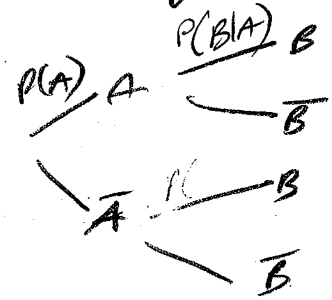


(A occurring forces the $P(B) = 0$)

$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{0}{P(A)} = 0$

so $P(B|A) \neq P(B)$
$0 \neq P(B)$ (as long as $P(B) \neq 0$)

independent events



$P(A \cap B) = P(A) \cdot P(B|A)$

unless one of event probabilities is zero
$P(A \cap B) \neq 0$
so not disjoint

#25. A junk box in your room contains a dozen old batteries, five of which are totally dead. You start picking batteries one at a time and testing them. Find the probability that:
a) The first two you choose are both good.
b) At least one of the first three works.
c) The first four you pick all work.
d) You have to pick 5 batteries in order to find one that works.

12 batteries
5 dead (bad) 7 good

(a) $P(GG) = \left(\frac{7}{12}\right)\left(\frac{6}{11}\right) = \frac{42}{132} = \boxed{.318}$

(b) $P(\text{at least 1 } G \text{ in } 3) = 1 - P(3 \text{ bad}) = 1 - \left(\frac{5}{12}\right)\left(\frac{4}{11}\right)\left(\frac{3}{10}\right) = \boxed{0.955}$

(c) $P(GGGG) = \left(\frac{7}{12}\right)\left(\frac{6}{11}\right)\left(\frac{5}{10}\right)\left(\frac{4}{9}\right) = \boxed{0.071}$

(d) $P(BBBBG) = \left(\frac{5}{12}\right)\left(\frac{4}{11}\right)\left(\frac{3}{10}\right)\left(\frac{2}{9}\right)\left(\frac{7}{8}\right) = \boxed{0.009}$

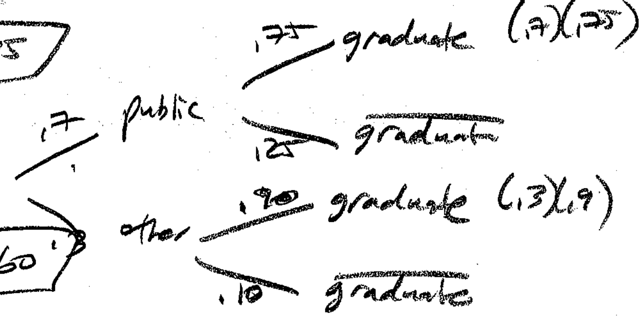
#26. A private college report contains these statistics:
70% of incoming freshman attended public schools
75% of public school students who enroll as freshmen eventually graduate
90% of other freshman eventually graduate
a) Is there any evidence that a freshman's chances to graduate may depend upon what kind of high school the student attended? Explain.
b) What percentage of freshmen eventually graduate?
c) What percentage of students who graduate from the college attended a public high school?

a) $\boxed{\text{yes}}$ $P(\text{grad} \mid \text{public}) = 0.75 \neq P(\text{grad} \mid \text{other}) = 0.90$

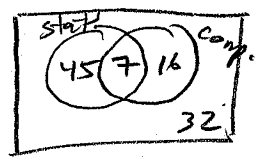
b) $P(\text{grad}) = (.7)(.75) + (.3)(.9) = \boxed{.795}$

c) $P(\text{public} \mid \text{graduate}) = \dfrac{P(\text{public} \cap \text{graduate})}{P(\text{graduate})}$

$= \dfrac{(.7)(.75)}{(.7)(.75)+(.3)(.9)} = \boxed{.660}$



#27. A university requires its biology majors to take a course called BioResearch. The prerequisite for this course is that students must have taken either a Statistics course or a Computer course. By the time they are juniors, 52% of the Biology majors have taken Statistics, 23% have had a Computer course, and 7% have done both.
a) What percentage of the junior Biology majors are ineligible for BioResearch?
b) What is the probability that a junior Biology major who has taken Statistics has also taken a Computer course.
given
c) Are taking these two courses disjoint events? Explain.
d) Are taking these two courses independent events? Explain.



(a) $\boxed{.32}$

(b) $P(\text{comp} \mid \text{stats}) = \dfrac{7}{52} = \boxed{.1346}$

(c) $\boxed{\text{No}}$ not disjoint $P(\text{stats} \cap \text{comp}) = .07 \ (\neq 0)$

(d) $P(\text{comp}) = 0.23$
$P(\text{comp} \mid \text{stats}) = .1346$ $\neq$ so $\boxed{\text{not independent}}$

#28. Suppose that 23% of adults smoke cigarettes. It's known that 57% of smokers and 13% of nonsmokers develop a certain lung condition by age 60.
a) Explain how these statistics indicate that lung condition and smoking are not independent.
b) What is the probability that a randomly selected 60 year old has this lung condition?
c) What is the probability that someone with the lung condition was a smoker?

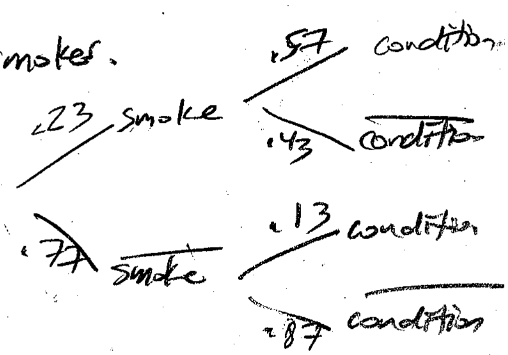
(a) percentage of people w/ lung condition is different (depends upon) whether a person smokes.

(b) $P(\text{condition}) = (.23)(.57) + (.77)(.13) = \boxed{.2312}$

(c) $P(\text{smoker} \mid \text{condition}) = \dfrac{P(\text{smoker} \cap \text{condition})}{P(\text{condition})}$

$= \dfrac{(.23)(.57)}{(.23)(.57)+(.77)(.13)}$

$= \boxed{.567}$



#29. The table below is a probability model for the number of cars in a randomly-selected household in the United States. (Based on U.S. Census 2000 data).

Number of cars	0	1	2	3	4	5 or more
Probability	0.07	0.19	0.47	?	0.06	0.02

(.19)

(a) What is the probability that a randomly selected household has three cars? (That is, fill in the space marked with a "?") Show your work.

$P(3) = 1 - P(\text{other}) = 1 - .81 = \boxed{.19}$

(b) What is the probability that a randomly-selected household has at least 2 cars? Show your work.

$P(\text{at least } 2) = P(2) + P(3) + P(4) + P(5 \text{ or more})$

$= .47 + .19 + .06 + .02$

$= \boxed{.74}$ (OR = add, no need to subtract these are all disjoint)

Chapter 14,15 Practice Quizzes

AP Statistics Quiz A – Chapter 14 **Name**________________________

1. Five multiple choice questions, each with four possible answers, appear on your history exam. What is the probability that if you just guess, you $P(R) = \frac{1}{4},\ P(W) = \frac{3}{4}$
   a. get none of the questions correct?
   WWWWW
   $\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4}\cdot\frac{3}{4} = \boxed{.2373}$
   b. get all of the questions correct? RRRRR
   $\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4} = 9.7656 \cdot 10^{-4} = \boxed{.000976}$
   c. get at least one of the questions wrong?
   $P(\text{at least 1 wrong}) = 1 - P(\text{all correct}) = 1 - 9.7656 \cdot 10^{-4} = \boxed{.999023}$
   d. get your first incorrect answer on the fourth question? RRRW
   $\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{1}{4}\cdot\frac{3}{4} = \boxed{.01171875}$

2. The Masterfoods company manufactures bags of Peanut Butter M&M's. They report that they make 10% each brown and red candies, and 20% each yellow, blue, and orange candies. The rest of the candies are green.

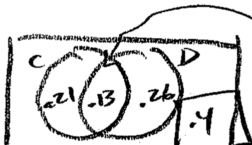
   Brown 0.1
   Red 0.1
   Yellow 0.2
   Blue 0.2
   Orange 0.2
   Green 0.2

   a. If you pick a Peanut Butter M&M at random, what is the probability that
      i. it is green?
      $P(\text{green}) = 1 - (\text{Sum}) = \boxed{.2}$
      ii. it is a primary color (red, yellow, or blue)?
      $P(R \cup Y \cup B) = 0.1 + 0.2 + 0.2 = \boxed{0.5}$
      iii. it is not orange?
      $P(\overline{\text{orange}}) = 1 - P(\text{orange}) = 1 - .2 = \boxed{0.8}$

   b. If you pick four M&M's in a row, what is the probability that
      i. they are all blue? BBBB
      $(.2)(.2)(.2)(.2) = \boxed{.0016}$
      ii. none are green?
      $P(\overline{G}) = 1 - .2 = .8$
      $\overline{G}\ \overline{G}\ \overline{G}\ \overline{G}$
      $(.8)(.8)(.8)(.8) = \boxed{.4096}$
      iii. at least one is red?
      $P(\text{at least 1 red}) = 1 - P(\text{no red})$ $P(\overline{R}) = 1 - .1 = .9$
      $\overline{R}\ \overline{R}\ \overline{R}\ \overline{R}$
      $(.9)(.9)(.9)(.9) = .6561$
      $P(\text{at least 1 R}) = 1 - .6561 = \boxed{.3439}$
      iv. the fourth one is the first one that is brown?
      $P(\text{Brown}) = .1$
      $P(\overline{\text{Brown}}) = 1 - .1 = .9$
      $\overline{B}\ \overline{B}\ \overline{B}\ B$
      $(.9)(.9)(.9)(.1) = \boxed{.0729}$

   c. After picking 10 M&M's in a row, you still have not picked a red one. A friend says that you should have a better chance of getting a red candy on your next pick since you have yet to see one. Comment on your friend's statement.
   No, probability is expected result over the long run.
   Small number of trials can result in any possible outcome
   P(red) on next is still 10%.

1. According to the American Pet Products Manufacturers Association (APPMA) 2003-2004 National Pet Owners Survey, 39% of U.S. households own at least one dog and 34% of U.S. households own at least one cat. Assume that 60% of U.S. households own a cat or a dog.
   a. What is the probability that a randomly selected U.S. household owns neither a cat nor a dog?



$P(C \cup D) = P(C) + P(D) - P(C \cap D)$

$.60 = .34 + .39 - P(C \cap D)$

$P(C \cap D) = \boxed{.13}$ so $P(\bar{C} \cap \bar{D}) = 1 - .21 - .13 - .26 = \boxed{.4}$

   b. What is the probability that a randomly selected U.S. household owns both a cat and a dog?

$P(C \cap D) = \boxed{.13}$

   c. What is the probability that a randomly selected U.S. household owns a cat if the household has a dog?

$P(C \mid D) = \frac{.13}{.39} = \boxed{.333}$

2. A manufacturing firm orders computer chips from three different companies: 10% from Company A; 20% from Company B; and 70% from Company C. Some of the computer chips that are ordered are defective: 4% of chips from Company A are defective; 2% of chips from Company B are defective; and 0.5% of chips from Company C are defective. A worker at the manufacturing firm discovers that a randomly selected computer chip is defective. What is the probability that the computer chip came from Company B? Show your work.

.10 A: .04 defect $(.10)(.04)$; $\cancel{.96}$ no defect $(.10)(.96)$

.20 B: .02 defect $(.20)(.02)$; .98 no defect $(.20)(.98)$

.70 C: .005 defect $(.70)(.005)$; .995 no defect $(.70)(.995)$

conditional sample space

$$P(B \mid \text{defect}) = \frac{(.20)(.02)}{(.10)(.04) + (.20)(.02) + (.70)(.005)} = \boxed{.3478}$$

3. A survey of an introductory statistics class in Autumn 2003 asked students whether or not they ate breakfast the morning of the survey. Results are as follows:

		Breakfast		
		Yes	No	Total
Sex	Male	66	66	132
	Female	125	74	199
	Total	191	140	331

a. What is the probability that a randomly selected student is female?

$P(F) = \frac{199}{331} = \boxed{.6012}$

b. What is the probability that a randomly selected student ate breakfast?

$P(B) = \frac{191}{331} = \boxed{.577}$

c. What is the probability that a randomly selected person is a female and ate breakfast?

$P(F \cap B) = \frac{125}{331} = \boxed{.378}$

d. What is the probability that a randomly selected student is female, given that the student ate breakfast?

$P(F|B) = \frac{125}{191} = \boxed{.654}$

e. What is the probability that a randomly selected student ate breakfast, given that the student is female?

$P(B|F) = \frac{125}{199} = \boxed{.628}$

f. Does it appear that whether or not a student ate breakfast is independent of the student's sex? Explain.

$P(B) = \frac{191}{331} = .577$ $\quad P(B|F) = \frac{125}{199} = .628$ $\quad P(B|M) = \frac{66}{132} = .5$

(check any two of these)

No; these variables (events) are $\boxed{\text{not independent}}$

The probability of eating breakfast depends upon sex,

— or —

$P(F \cap B) \stackrel{?}{=} P(F) \cdot P(B)$

$\frac{125}{331} \stackrel{?}{=} \frac{199}{331} \cdot \frac{191}{331}$

$.3776 \neq .3469$ (not independent because the simplified AND formula doesn't work)