AP Statistics Ch 10 - Required Practice **Name:** Solutions **Per:** ____

#1. **Light Intensity**

Distance	Candlepower
2 feet	531.2
5	84.3
8	33.6
10	21.1
15	9.5
20	5.3
25	3.4

(a) Straighten the data shown in the table. Does the data follow an exponential or power model (check original data and both models)? Write out the LSRL and justify your decision (you must use residual plots).

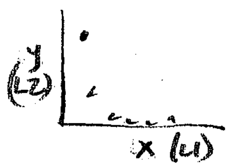
$y = k^x$

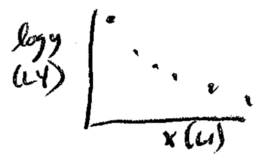
$\log y = x \log k$

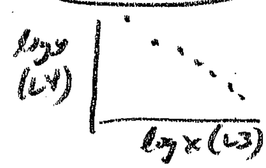
Original data | Exponential Model | Power Model

$y = x^k$

$\log y = k \log x$







$\hat{y} = 277.5 - 14.754x$

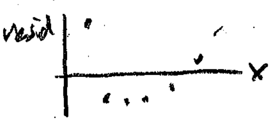
$r^2 = .4006$

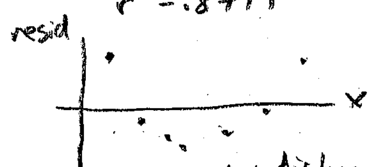
$\log \hat{y} = 2.4323 - .0858x$

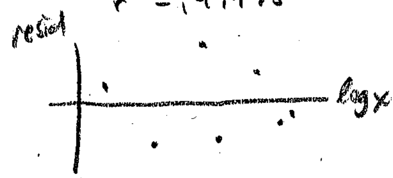
$r^2 = .8779$

$\log \hat{y} = 3.3265 - 1.9991 \log x$

$r^2 = .99998$







Best LSRL: $\log(\hat{y}) = 3.3265 - 1.9991 \log(x)$

x: distance (ft)

y: light intensity (candlepower)

(b) Using the best model, find the light intensity at the following distances:

1 foot:

$\log(\hat{y}) = 3.3265 - 1.9991 \log(1)$

$\log_{10}(\hat{y}) = 3.3265$

$\hat{y} = 10^{3.3265}$

$\boxed{\hat{y} = 2120.8 \text{ candlepower}}$

12 feet:

$\log(\hat{y}) = 3.3265 - 1.9991 \log(12)$

$\log_{10}(\hat{y}) = 1.1691\ldots$

$\hat{y} = 10^{1.1691\ldots}$

$\boxed{\hat{y} = 14.76 \text{ candlepower}}$

30 feet:

$\log(\hat{y}) = 3.3265 - 1.9991 \log(30)$

$\log_{10}(\hat{y}) = 0.37358\ldots$

$\hat{y} = 10^{0.37358\ldots}$

$\boxed{\hat{y} = 2.36 \text{ candlepower}}$

Alligators

Scientists collect information on many kinds of wildlife, and for a variety of reasons. Through their research they learn about the animals' habits, populations, and locations. Such information can help them learn more about the animals, protect endangered species, detect changes that may signal environmental problems, or keep track of animals that may present risks to humans.

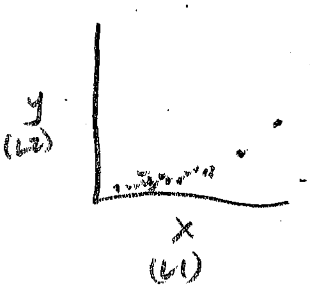
In central Florida, where alligators and humans live in close proximity, it is important to track the locations and sizes of alligators. The animals may be spotted from the air, from a boat, or on land. Wildlife experts can accurately estimate the alligator's length, but they usually want to know the animal's weight as well. That's a little harder to determine, unless you'd like to be the one who picks the gator up to step on the scale…

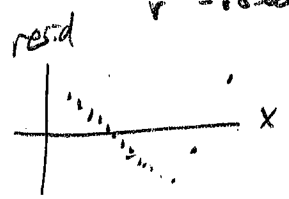
To develop a way to estimate the weight of an alligator, the wildlife researchers measured the lengths and weights of several captured alligators. Then they used those data to develop a model enabling them to estimate an alligator's weight from its length – something they can guess from a safe distance! Officials hope to use this model to identify alligators that should be relocated because they have grown so large as to pose a threat to humans.

Length (inches)	Weight (pounds)
86	83
88	70
72	61
74	54
61	44
90	106
89	84
68	39
76	42
114	197
90	102
78	57
94	130
74	51
147	640
58	28
86	80
94	110
63	33
86	90
69	36
72	38
128	366
85	84
82	80

#2. Create a linear model for the original, un-straightened, data and discuss the accuracy of this model.

$\hat{y} = -393.2640 + 5.9024x$ x: length (in)
y: weight (lbs)
$r^2 = .8361$

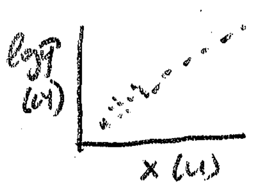




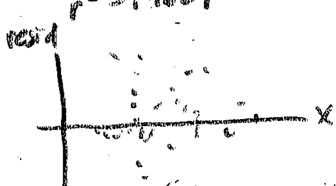
Although r^2 is fairly high, there is a pattern in the residuals. The data is non-linear, so this model should not be used.

#3. Now straighten the data to create a stronger predictive model. Write the LSRL of the improved model and include evidence showing why this model is superior to the original model.

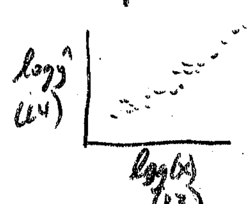
exp



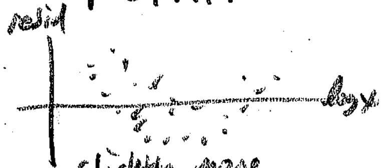
$\log\hat{y} = 0.5799 + 0.01538x$
$r^2 = .9601$



power



$\log\hat{y} = -4.4187 + 3.286x$
$r^2 = .9449$



slightly more pattern

The exponential model has the least pattern in the residuals:
$\log(\hat{y}) = 0.5799 + 0.01538x$
x: length (in)
y: weight (lbs)
$r^2 = .9601$
This is also stronger than the original data model, predicting 96% of the variation in weight (compared to 84% unstraightened)

#4. **Mortgages**

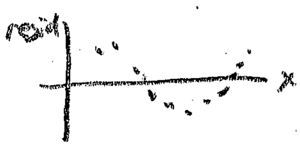
Year	Million $
1970	1.2
1972	2.5
1974	2.9
1976	3.1
1978	5.8
1980	8.3
1982	10.8
1984	14.7
1986	21.8
1988	29.7

(a) Straighten the data shown in the table. Does the data follow an exponential or power model (check original data and both models)? Write out the LSRL and justify your decision (you must use residual plots).

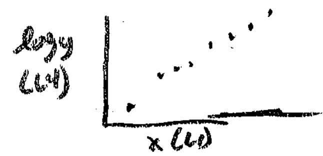
Original data

$\hat{y} = -2845.7 + 1.443x$

$r^2 = .8866$

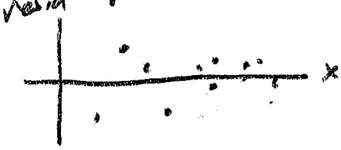


Exponential Model

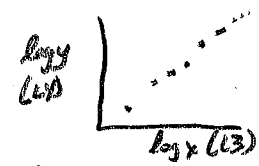


$\log \hat{y} = -145.7 + .074x$

$r^2 = .9850$

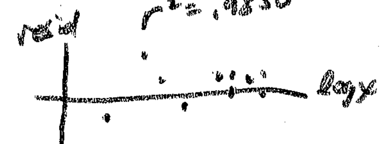


Power Model



$\log \hat{y} = -111.33 + 337.325 \log x$

$r^2 = .9850$



Exponential and power look equally good. I'll choose exponential, it is a little simpler

Best LSRL: $\log \hat{y} = -145.7 + .074x$

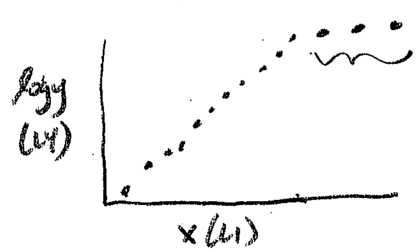
x: year

y: mortgages ($ millions)

(b) Here are three additional data points for post-1990 mortgages. Add these points to your lists then clear and recreate L3 and L4, but <u>do not</u> re-straighten the data. Examine the scatterplot for your straightening method.

Year	Million $
1990	32.4
1995	39.5
2000	49.7

Comment on how these post-1990 mortgages differ from the previous trend.



Until 1990, mortgages were increasing exponentially, but beginning in 1990 mortgages were still increasing each year, but not as quickly as before 1990.

1. Models. For each of the models listed below, predict y when $x = 2$.

a) $\hat{y} = 1.2 + 0.8x$

b) $\ln \hat{y} = 1.2 + 0.8x$

c) $\sqrt{\hat{y}} = 1.2 + 0.8x$

d) $\frac{1}{\hat{y}} = 1.2 + 0.8x$

e) $\hat{y} = 1.2x^{0.8}$

a) $\hat{y} = 1.2 + 0.8(2)$

$\boxed{\hat{y} = 2.8}$

b) $\ln(\hat{y}) = 1.2 + 0.8(2)$

$\ln(\hat{y}) = 2.8$

$\log_e(\hat{y}) = 2.8$

$\hat{y} = e^{2.8} = \boxed{16.444}$

c) $\sqrt{\hat{y}} = 1.2 + 0.8(2)$

$\sqrt{\hat{y}} = 2.8$

$(\sqrt{y})^2 = (2.8)^2$

$\hat{y} = \boxed{7.84}$

d) $\frac{1}{\hat{y}} = 1.2 + 0.8(2)$

$\frac{1}{\hat{y}} \times \frac{2.8}{1}$

$2.8\hat{y} = 1$

$\hat{y} = \frac{1}{2.8}$

$\hat{y} = \boxed{0.3571}$

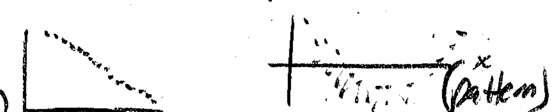
e) $\hat{y} = 1.2(2)^{0.8}$

$\hat{y} = \boxed{2.0893}$

28. Orange production. The table below shows that as the number of oranges on a tree increases, the fruit tend to get smaller. Create a model for this relationship, and express any concerns you may have.

Number of Oranges/Tree	Average Weight/Fruit (lb)
50	0.60
100	0.58
150	0.56
200	0.55
250	0.53
300	0.52
350	0.50
400	0.49
450	0.48
500	0.46
600	0.44
700	0.42
800	0.40
900	0.38

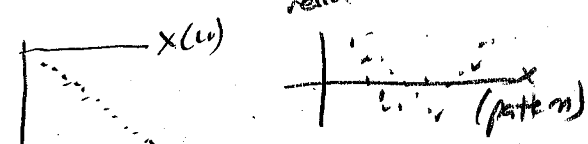
Orig



$\hat{y} = .5987 - (2.5 \cdot 10^{-4})x$

$r^2 = .9878$

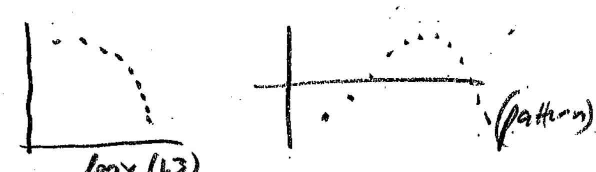
exp



$\log\hat{y} = -.21556 - (2.313 \cdot 10^{-4})x$

$r^2 = .9973$

power



$\log\hat{y} = .0869 - .1589x$

$r^2 = .8812$

All have pattern in residuals, pattern is most random for exponential, and it has highest r^2, But LSRL slope is $2.313 \cdot 10^{-4} = .0002313$ (extremely sensitive). Not sure I would trust this model.

Chapter 10 Practice Quiz

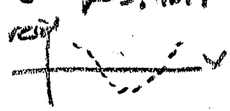
The average movie ticket prices in selected years since 1948 are listed in the table below.

Year	Movie Ticket Price
1948	$0.36
1954	$0.49
1958	$0.68
1963	$0.86
1967	$1.22
1971	$1.65
1974	$1.89
1975	$2.03
1976	$2.13
1977	$2.23
1978	$2.34
1979	$2.47
1980	$2.69
1981	$2.78
1982	$2.94
1983	$3.15
1984	$3.36
1985	$3.55
1986	$3.71
1987	$3.91
1988	$4.11

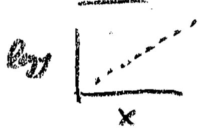
a. Use re-expressed data to create a model that predicts ticket prices.

orig

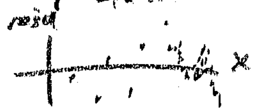
$\hat{y} = -190.1 + .0995x$
$r^2 = .9819$



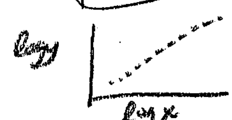
exp



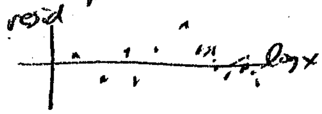
$\log \hat{y} = -52.869 + .0269x$
$r^2 = .9968$



Power (slightly better r^2)



$\log y = -401.955 + 122.055 \log x$
$r^2 = .9970$



b. Find the movie ticket price this model predicts for 2004.

power: $\log(\hat{y}) = -401.955 + 122.055 \log(x)$

$\log(\hat{y}) = -401.955 + 122.055 \log(2004)$

$\log_{10}(\hat{y}) = 1.0581\ldots$

$\hat{y} = 10^{1.0581\ldots} = \boxed{\$11.43}$

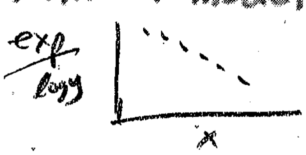
2. During a chemistry lab, students were asked to study a radioactive element which decays over time. The results are in the table.

(it's going to be exponential)

Time (in days)	0.5	2	4	6	8	10
Element (in grams)	320	226	160	115	80	57

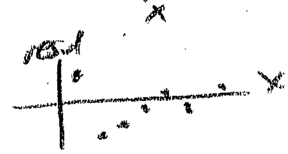
a. Find a model for the data.

exp



$\log \hat{y} = 2.5246 - .0775x$

x: time (days)
y: element (g)



b. Find the predicted amount of the element remaining after thirty minutes.

$30\text{min} \cdot \frac{\text{hr}}{60\text{ min}} \cdot \frac{\text{day}}{24\text{ hr}} = .02083\text{ days}$

$\log_{10}(\hat{y}) = 2.5246 - .0775(.02083)$

$\log_{10}(\hat{y}) = 2.52298\ldots$

$\hat{y} = 10^{2.52298\ldots} = \boxed{333.42 \text{ grams}}$