

AP[®] STATISTICS
2009 SCORING GUIDELINES

Question 4

Intent of Question

The primary goals of this question were to evaluate a student's ability to (1) identify and compute an appropriate confidence interval after checking the necessary conditions; (2) interpret the interval in the context of the question; and (3) use the confidence interval to make an inference about whether or not a council member's belief is supported.

Solution

Part (a):

Step 1: Identify the appropriate confidence interval by name or formula and check for appropriate conditions.

The two-sample t interval for $\mu_N - \mu_S$, the difference in population mean response times, is

$$(\bar{x}_N - \bar{x}_S) \pm t^* \sqrt{\frac{s_N^2}{n_N} + \frac{s_S^2}{n_S}}$$

where μ_N denotes the mean response for calls from the northern fire station and μ_S denotes the mean response for calls from the southern fire station.

Conditions: 1. Independent random samples
 2. Large samples or normal population distributions

A random sample of 50 calls was selected from the northern fire station, independent of the random sample of 50 calls selected from the southern fire station.

The use of the two-sample t interval is reasonable because both sample sizes are large ($n_N = 50 > 30$ and $n_S = 50 > 30$), and by the central limit theorem, the sampling distributions for the two sample means are approximately normal. Therefore the sampling distribution of the difference of the sample means $\bar{x}_N - \bar{x}_S$ is approximately normal.

Step 2: Correct Mechanics

Unequal variances: Degrees of freedom = 96.

$$\begin{aligned} & (4.3 - 5.3) \pm 1.985 \sqrt{\frac{3.7^2}{50} + \frac{3.2^2}{50}} \\ & -1.0 \pm 1.985 \times .6918 \\ & (-2.37, 0.37) \end{aligned}$$

Step 3: Interpretation

Based on these samples, one can be 95 percent confident that the difference in the population mean response times (northern - southern) is between -2.37 minutes and 0.37 minutes.

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Question 4 (continued)

Part (b):

Zero is within the 95 percent confidence interval of plausible values for the difference in population means. Therefore this confidence interval does not support the council member's belief that there is a difference in mean response times for the two fire stations.

Scoring

This problem is scored in four sections. Section 1 consists of part (a), step 1; section 2 consists of part (a), step 2; section 3 consists of part (a), step 3; and section 4 consists of part (b). Sections 1, 2, and 3 are scored as essentially correct (E) or incorrect (I), and section 4 is scored as essentially correct (E), partially correct (P), or incorrect (I).

Section 1: Identify the appropriate confidence interval by name or formula and check for appropriate conditions.

Section 1 is scored as follows:

Essentially correct (E) if the student does both of the following:

- Indicates that the appropriate procedure is a t confidence interval for difference in means, either by name or formula.
- Verifies that the sample sizes are large enough to use this procedure by referencing a number (such as 25 or 30) *OR* by stating that 50 is large enough that the central limit theorem applies.

Incorrect (I) if the student omits at least one of the two elements above *OR* names or gives a formula for a two-sample z confidence interval instead of a two-sample t confidence interval.

Section 2: Correct Mechanics

Section 2 is scored as follows:

Essentially correct (E) if the student shows a correct confidence interval, either by displaying numbers in the formula or by writing the numerical interval.

Note: The following are acceptable solutions for section 2:

- The following degrees of freedom, t^* values, and confidence intervals are all acceptable.

Solutions to Step 2			
Procedure	Degrees of freedom	t^*	Confidence interval for $\mu_N - \mu_S$
Unequal variances	96	1.985	(-2.37, 0.37)
Conservative df	40 (using table)	2.021	(-2.40, 0.40)
Conservative df	49	2.010	(-2.39, 0.39)
Conservative df	50 (using table)	2.009	(-2.39, 0.39)
Pooled variance	98	1.984	(-2.37, 0.37)

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Question 4 (continued)

- Students do not need to state explicitly the degrees of freedom used.
- It is acceptable if a student reports a confidence interval for $\mu_S - \mu_N$, in which case the signs are reversed.
- An identifiable minor arithmetic error will not necessarily change a score on section 2 from essentially correct to incorrect.

Incorrect (I) if the numerical interval given is incorrect because of a substantive statistical error, such as failing to square the standard deviations *OR* if the interval is missing or completely wrong.

Section 3: Interpretation

Section 3 is scored as follows:

Essentially correct (E) if the student correctly interprets the confidence interval (not the confidence level) in context. The interpretation must indicate that the interval is for a difference in mean response times. If the student gives an interpretation of *both* the confidence level and the confidence interval, both must be correct to be scored as essentially correct (E).

Incorrect (I) if a student gives an interpretation of the confidence level *instead of* the confidence interval *OR* if the interpretation of the confidence interval is wrong.

Note: The correct interpretation of the confidence interval for step 3 of part (a) may be found in part (b).

Section 4

Section 4 is scored as follows:

Essentially correct (E) if the student makes a correct conclusion in context, supported by the fact that zero is contained within the 95 percent confidence interval.

Partially correct (P) if the student makes a correct conclusion supported by the fact that zero is contained in the confidence interval, but one or both of the following occurs:

- The student omits the context.
- The student makes a statistically incorrect statement in the explanation, such as stating that the council member's belief is wrong. Such a statement is equivalent to accepting the null hypothesis of no difference in population mean response times.

Incorrect (I) if the student gives the incorrect conclusion *OR* makes no reference to a confidence interval.

Notes:

- The answer in part (b) needs to be consistent with the confidence interval given in part (a). If the interval does not cover zero, then part (b) must state that the interval does support the council member's belief because the interval does not contain zero.
- The response for part (b) may be found in the space allocated to part (a).

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Question 4 (continued)

Each essentially correct (E) response counts as 1 point, and a partially correct (P) response in part (b) counts as $\frac{1}{2}$ point.

- 4 Complete Response**
- 3 Substantial Response**
- 2 Developing Response**
- 1 Minimal Response**

If a response is between two scores (for example, $1\frac{1}{2}$ points), use a holistic approach to determine whether to score up or down, depending on the strength of the response and communication.