

AP[®] STATISTICS
2008 SCORING GUIDELINES

Question 5

Intent of Question

The primary goals of this question were to assess a student's ability to (1) state the appropriate hypotheses; (2) identify and compute the appropriate test statistic; (3) make a conclusion in the context of the problem; and (4) compare two sets of proportions to identify the preferred habitat.

Solution

Part (a):

Step 1: States a correct pair of hypotheses.

H_0 : Moose have no preference for habitat type.

H_a : Moose have a preference for habitat type.

OR

H_0 : The number of moose in each habitat type is proportional to the amount of acreage of that habitat type.

H_a : The number of moose in at least one habitat type is not proportional to the amount of acreage of that habitat type.

OR

H_0 : $p_1 = 0.340, p_2 = 0.101, p_3 = 0.104, p_4 = 0.455$, where p_i = the proportion of moose in habitat type i .

H_a : At least one of these proportions is incorrect.

Step 2: Identifies a correct test (by name or formula) and checks appropriate conditions.

- Chi-square goodness-of-fit test (or test for more than two proportions)

$$\chi^2 = \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}}$$

- The stem of the problem stated that conditions for inference are met.

Step 3: Correct mechanics, including the value of the test statistic, df, and p -value (or rejection region).

- The test statistic, with $df = 4 - 1 = 3$, is

$$\chi^2 = \frac{(25 - 39.780)^2}{39.780} + \frac{(22 - 11.817)^2}{11.817} + \frac{(30 - 12.168)^2}{12.168} + \frac{(40 - 53.235)^2}{53.235} = 43.6893.$$

- The p -value is $P(\chi_3^2 \geq 43.6893) < 0.0005$ (a calculator gives the p -value as 1.7569×10^{-9}).

AP[®] STATISTICS
2008 SCORING GUIDELINES

Question 5 (continued)

Step 4: States a correct conclusion in the context of the problem, using the result of the statistical test.

The data are not consistent with the researchers' expectation. Because the p -value is less than $\alpha = 0.05$, we reject H_0 . There is strong evidence that moose have a preference for habitat type.

OR

The data are not consistent with the researchers' expectation. If the null hypothesis is true and the number of moose in each of the habitat types is proportional to the acreage in that habitat type, then we would observe a test statistic of 43.69 or one more extreme less than 0.05 percent of the time. There is strong evidence that moose have a preference for habitat type.

Part (b):

The moose seem to prefer habitat types 2 and 3. Relative to the proportion of total acreage, a higher proportion of moose were observed in each of these habitat types than expected. In habitat types 1 and 4, the observed proportion of moose was less than the expected proportion of moose, indicating that these two habitat types are less desirable.

OR

Habitat type 3 seems to be the most preferred—it has a positive difference between the observed (30) and expected (12.168) counts of moose and the largest contribution to the chi-square statistic (26.1325). Alternatively, habitat type 3 has the largest positive difference between the observed proportion of moose (0.256) and the expected proportion of moose (0.104).

Scoring

This problem is scored in four sections. Section 1 consists of part (a), step 1. Section 2 consists of part (a), steps 2 and 3. Section 3 consists of part (a), step 4. Section 4 consists of part (b). Sections 1, 2, and 3 are scored as essentially correct (E) or incorrect (I), and section 4 is scored as essentially correct (E), partially correct (P), or incorrect (I).

If an inappropriate inference procedure is used in part (a), then all three sections must be scored as incorrect (I).

Section 1 [part (a), step 1]: States a correct pair of hypotheses.

- Hypotheses must be given in context—which includes some reference to moose and the different habitat types—to earn an E. Hypotheses that clearly address sample data (like “observed number of moose”) are incorrect.

AP[®] STATISTICS
2008 SCORING GUIDELINES

Question 5 (continued)

Section 2 [part (a), steps 2 and 3]: Identifies a correct test and checks appropriate conditions. Mechanics are correct.

- A discussion of conditions for inference should generally be treated as extraneous. However, if the response includes inappropriate conditions—like normality or independent samples—the response cannot receive a score of 4.
- An inappropriate method of calculating df will result in these combined steps being scored incorrect.

Section 3 [part (a), step 4]: States a correct conclusion in the context of the problem.

- If an incorrect p -value in steps 2 and 3 is obtained from a chi-square goodness-of-fit test, but the conclusion is consistent with this p -value, step 4 can be considered correct.
- If both an α and a p -value are given together, the linkage between the p -value and the conclusion is implied. If no α is given, the solution must be explicit about the linkage by giving a correct interpretation of the p -value *OR* explaining how the conclusion follows from the p -value.

Section 4 [part (b)] is scored as follows:

Essentially correct (E) if habitat types 2 and 3 are identified as the preferred habitat types with a justification that indicates there is a higher proportion (or a higher number) of moose than expected relative to the proportion of total acreage in those areas. One way to do this is to compare the observed density of moose across the four habitat types. Note that habitat types 2 and 3 also happen to make the largest contribution to the chi-square statistic.

OR

Habitat type 3 is identified as the most preferred because it has a higher proportion (or higher number) of moose than expected and the largest chi-square contribution *OR* the largest positive difference in observed and expected proportions *OR* the highest density of moose.

Partially correct (P) if habitat types 2 and 3 (or habitat type 3 alone) are identified with an incomplete justification. For example, a student might select habitat type 3 as most preferred based on the fact that it yields the largest contribution to the chi-square statistic but not indicate that there is a higher proportion (or higher number) of moose than expected in these areas.

Incorrect (I) if habitat types 2 and 3 (or just habitat type 3) are identified with no or incorrect justification *OR* habitat types 1 or 4 are identified.

Each essentially correct (E) response counts as 1 point, and a partially correct (P) response in part (b) counts as $\frac{1}{2}$ point.

- | | |
|----------|-----------------------------|
| 4 | Complete Response |
| 3 | Substantial Response |
| 2 | Developing Response |

AP[®] STATISTICS
2008 SCORING GUIDELINES

Question 5 (continued)

1 Minimal Response

If a response is between two scores (for example, $2\frac{1}{2}$ points), use a holistic approach to determine whether to score up or down, depending on the strength of the response and communication.