

AP[®] STATISTICS
2006 SCORING GUIDELINES

Question 4

Intent of Question

The primary goals of this question are to evaluate a student's ability to: (1) identify and compute an appropriate confidence interval, after checking the necessary conditions; (2) interpret the interval in the context of the question; and (3) use the confidence interval to conduct an appropriate test of significance.

Solution

Part (a):

Step 1: Identifies the appropriate confidence interval by name or formula and checks appropriate conditions.

Two sample t interval for $\mu_A - \mu_S$, the difference in mean waiting times, or

$$(\bar{x}_A - \bar{x}_S) \pm t_{df}^* \sqrt{\frac{s_A^2}{n_A} + \frac{s_S^2}{n_S}} \quad [\text{See the next page for possible values of } df.]$$

- Conditions:
1. Independent random samples
 2. Large samples or normal population distributions

One sample of 150 patients divided into two groups after sampling does not meet the condition of two independent random samples with fixed sample sizes. Nevertheless, it is reasonable to assume that mode of transportation splits the patients into two independent groups. Secondly, use of the two sample t interval is reasonable because each sample size is large (e.g., $n_A = 77 > 30$ and $n_S = 73 > 30$).

Alternatively, we could assume that the waiting times are (at least approximately) normally distributed, but we have no way to check this assumption with the information provided.

Step 2: Correct Mechanics

Degrees of freedom = $\min\{(77 - 1), (73 - 1)\} = 72$.

$$\begin{aligned} & (6.04 - 8.30) \pm 2.6459 \sqrt{\frac{4.30^2}{77} + \frac{5.16^2}{73}} \\ & -2.26 \pm 2.6459 \cdot (0.7777) \\ & -2.26 \pm 2.0577 \\ & (-4.3177, -0.2023) \end{aligned}$$

Step 3: Interpretation

Based on this sample, we are 99 percent confident that the true difference in the populations' mean waiting times (ambulance – self) is between -4.3177 minutes and -0.2023 minutes.

Equivalently,

With 99 percent confidence, the true mean wait time for those who arrive by ambulance is shorter than those who are self transported by somewhere between 0.2 and 4.3 minutes.

Part (b):

Since zero is not in the 99 percent confidence interval of plausible values for the true difference in means, we can reject $H_0 : \mu_A - \mu_S = 0$ in favor of the alternative $H_a : \mu_A - \mu_S \neq 0$ at the $\alpha = .01$ significance level.

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Question 4 (continued)

Thus, we have statistically significant evidence that there is a difference in the mean wait times for the two groups.

Scoring

Part (a) is scored according to the number of correct steps. Each step is scored as essentially correct (E) or incorrect (I). Part (b) is scored as essentially correct (E) or incorrect (I).

Step 1: Identification of method and check of conditions.

A score of essentially correct (E) requires each of the following; otherwise, the score is I.

- A correct confidence interval procedure should be named or a correct formula given.
- Normality must be assessed by either checking that *EACH* sample size is sufficiently large (e.g., >30) OR by stating that each population needs to be normally distributed but that the data are not given to check that.
- It must be stated that the two groups are assumed to be independent random samples.

Step 2: Mechanics

An identifiable minor error in Step 2 will not necessarily change a score from essentially correct to incorrect. The following confidence intervals are all scored E.

Solutions to Step 2			
Procedure	d.f.	t*	Confidence Interval
Unequal Variances	140.37	2.61140	(-4.2910, -0.2291)
Large samples	∞	2.576	(-4.26, -0.26)
Pooled variance	148	2.6095	(-4.2797, -0.2404)
Conservative Approach	72	2.6459	(-4.3177, -0.2023)
Unequal Variances	100 (table)	2.626	(-4.302, -0.218)
Unequal Variances	1000 (table)	2.581	(-4.267, -0.253)
Conservative Approach	60 (table)	2.660	(-4.329, -0.191)
Conservative Approach	80 (table)	2.639	(-4.312, -0.208)

Step 3: Interpretation of Confidence Interval

For a score of essentially correct (E), the interpretation must be about a difference of population means AND be in context AND mention the 99 percent level of confidence.

Note: If the interpretation is correct and also explains the meaning of “confidence level,” then that extra explanation must be correct for a score of E. If it is not, the score is I.

Note: The correct interpretation of the confidence interval for Step 3 of part (a) may be found in part (b).

Part (b) is essentially correct (E) if the student concludes that the mean times differ because zero is not contained in the 99 percent confidence interval. The confidence level or the significance level must be stated and the conclusion must be stated in context.

Part (b) is incorrect (I) if the student only says the mean wait time for patients transported via ambulance is significantly lower without indicating the significance level adjustment needed for a one-sided test.

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Question 4 (continued)

4 Complete Response

All three steps of the confidence interval in part (a) are essentially correct and part (b) is essentially correct.

3 Substantial Response

All three steps of the confidence interval in part (a) are essentially correct and part (b) is incorrect.

OR

Two steps of the confidence interval in part (a) are essentially correct and part (b) is essentially correct.

2 Developing Response

Two steps of the confidence interval in part (a) are essentially correct and part (b) is incorrect.

OR

One step of the confidence interval in part (a) is essentially correct and part (b) is essentially correct.

1 Minimal Response

One step of the confidence interval in part (a) is essentially correct and part (b) is incorrect.

OR

Part (b) is essentially correct.