AP Stats Ch 19 - Extra Practice **Name:** Solutions **Per:** ___

28. Legal Music. A random sample of 168 students were asked how many songs were in their digital music library and what fraction of them were legally purchased. Overall, they reported having a total of 117,079 songs, of which 23.1% were legal. The music industry would like a good estimate of the fraction of songs in students' digital music libraries that are legal.

a) Think carefully. What is the parameter being estimated? What is the population? What is the sample size?

b) Check the conditions for making a confidence interval.

c) Construct a 95% confidence interval for the fraction of legal digital music.

d) Explain what this interval means. Do you believe that you can be this confident about your result? Why or why not?

a) p = proportion of songs in all students' music libraries which are legal

Sample size = 117079

$\hat{p} = .231$

b) ✓ $np = (117079)(.231) = 27045 \geq 10$

$nq = (117079)(.769) = 90034 \geq 10$

✓ – SRS (really a cluster sample, assume representative) ??

✓ – 117079 < 10% of all songs in libraries

✓ – indep (assumed) ??

c) $CI = \hat{p} \pm z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = .231 \pm (1.96)\sqrt{\frac{(.231)(.769)}{117079}}$

$= .231 \pm .0024$

$= \boxed{(.2286, .2334)}$

d) We are 95% ~~confident that the true~~ fraction of songs which are legal in all students' libraries is between 22.86% and 23.34%.

This is ridiculously narrow due to the extremely large sample size. Accuracy is highly dependent upon how representative this sample is to the population (which is suspect).

32. Hiring. In preparing a report on the economy, we need to estimate the percentage of businesses that plan to hire additional employees in the next 60 days.

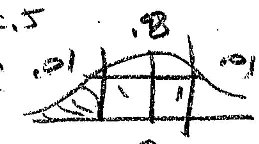
a) How many randomly selected employers must we contact in order to create an estimate in which we are 98% confident with a margin of error of 5%?

b) Suppose we want to reduce the margin of error to 3%. What sample size will suffice?

c) Why might it not be worth the effort to try to get an interval with a margin of error of only 1%?

a) use $\hat{p} = .5$

z^* for 98%:



$z^* = \text{invNorm}(.01, 0, 1)$

$= \cancel{-}2.326$

$z^* \sqrt{\frac{\hat{p}\hat{q}}{n}} = .05$

$(2.326)\sqrt{\frac{(.5)(.5)}{n}} = .05$

⋮

$n = \frac{(.5)(.5)}{\left(\frac{.05}{2.326}\right)^2} = 541.029$

$\boxed{542}$

b) $n = \frac{(.5)(.5)}{\left(\frac{.03}{2.326}\right)^2} = 1502.85$

$\boxed{1503}$

c) $n = \frac{(.5)(.5)}{\left(\frac{.01}{2.326}\right)^2} = \boxed{13526}$

Surveying over 13 thousand businesses would be much more costly and time-consuming.

5. Conclusions. A catalog sales company promises to deliver orders placed on the Internet within 3 days. Follow-up calls to a few randomly selected customers show that a 95% confidence interval for the proportion of all orders that arrive on time is 88% ± 6%. What does this mean? Are these conclusions correct? Explain.

a) Between 82% and 94% of all orders arrive on time. NO, not certain

b) 95% of all random samples of customers will show that 88% of orders arrive on time. No, not a specific probability

c) 95% of all random samples of customers will show that 82% to 94% of orders arrive on time. No, the proportion is .82 to .94, 95% is the level of confidence, not a proportion

d) We are 95% sure that between 82% and 94% of the orders placed by the customers in this sample arrived on time. NO, we know for certain that 88% of orders for this sample are on time. uncertainty is for the inference to all orders.

e) On 95% of the days, between 82% and 94% of the orders will arrive on time. NO, 95% not the proportion.

7. Confidence intervals. Several factors are involved in the creation of a confidence interval. Among them are the sample size, the level of confidence, and the margin of error. Which statements are true?

a) For a given sample size, higher confidence means a smaller margin of error.

$ME = z^* SE$

False. n fixed. so SE fixed. higher confidence, $z^* \uparrow$ so $ME \uparrow$

b) For a specified confidence level, larger samples provide smaller margins of error.

True confid. level fixed, z^* fixed $ME = z^* SE$ if $n \uparrow$ $SE \downarrow$ so $ME \downarrow$

c) For a fixed margin of error, larger samples provide greater confidence.

True ME fixed $ME = z^* SE$ if $n \uparrow$ $SE \downarrow$, then $z^* \uparrow$ = greater confidence.

d) For a given confidence level, halving the margin of error requires a sample twice as large.

False confid. level fixed, z^* fixed

$ME \to \frac{1}{2} ME$ $\quad \underset{1/2}{ME} = z^* \underset{1/2}{SE}$

$SE \to \frac{1}{2} SE$ $\quad SE = \sqrt{\frac{pq}{n}}$

$\frac{1}{2} = \sqrt{\frac{1}{n}}$

$\frac{1}{4} = \frac{1}{n}$

$n = 4$

Sample would need to b 4X as large.

23. Only child. In a random survey of 226 college students, 20 reported being "only" children (with no siblings). Estimate the proportion of students nationwide who are only children.

a) Check the conditions (to the extent you can) for constructing a confidence interval.
b) Construct a 95% confidence interval.
c) Interpret your interval.
d) Explain what "95% confidence" means in this context.

a) SRS? ✓
Indep? ✓
$n < 10\%$ ✓
success/failure ✓

b) $z^* = 1.96$ $\hat{p} = \frac{20}{226} = .0885$

$SE_{\hat{p}} = \sqrt{\frac{(.0885)(.9115)}{226}} = .0189$

$ME = 1.96(.0189) = .0370$

$CI = .0885 \pm .0370$

$\boxed{(.0515, .1255)}$

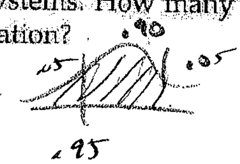
c) We believe with 95% confidence that between 5.1% and 12.6% of all college students are 'only' children.

d) If we selected random groups of 226 students, 95% of the confidence intervals would contain the true proportion of 'only' children.

35. Pilot study. A state's environmental agency worries that many cars may be violating clean air emissions standards. The agency hopes to check a sample of vehicles in order to estimate that percentage with a margin of error of 3% and 90% confidence. To gauge the size of the problem, the agency first picks 60 cars and finds 9 with faulty emissions systems. How many should be sampled for a full investigation?

$\hat{p} = \frac{9}{60} = .15$

90% CL



$z^* = \text{invNorm}(.95, 0, 1) = 1.645$

$SE_{\hat{p}} = \sqrt{\frac{(.15)(.85)}{n}}$

$ME = z^* SE_{\hat{p}}$

$.03 = 1.645\sqrt{\frac{(.15)(.85)}{n}}$

$\left(\frac{.03}{1.645}\right)^2 = \frac{(.15)(.85)}{n}$

$n = \frac{(1.645)^2(.15)(.85)}{(.03)^2}$

$n = 383.35$

$\boxed{n = 384 \text{ cars}}$