AP Stats Ch 17 - Extra Practice **Name:** Solutions **Per:** ___

33. Hot hand. A basketball player who ordinarily makes about 55% of his free throw shots has made 4 in a row. Is this evidence that he has a "hot hand" tonight? That is, is this streak so unusual that it means the probability he makes a shot must have changed? Explain.

NO. This is still a small number of trials ... anything can happen. The probability of making the next shot is still .55,

How unlikely is making 4 in a row?

$P(YYYY) = (.55)^4 = .0915$, happens randomly 9% of the time, on the border of being unusual (establish rule of thumb is <5% chance)

35. Hotter hand. Our basketball player in Exercise 33 has new sneakers, which he thinks improve his game. Over his past 40 shots, he's made 32—much better than the 55% he usually shoots. Do you think his chances of making a shot really increased? In other words, is making at least 32 of 40 shots really unusual for him? (Do you think it's his sneakers?)

Let's find $P(x \geq 32)$

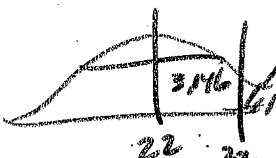
w/ binomial: $x \mid 0\ 1\ 2\ 3 \ldots 31\ \underbrace{32\ 33 \ldots 40}$

$1 - \text{binomcdf}(40, .55, 31) = .00088$

w/ normal approx:

$\mu = np = (40)(.55) = 22$ (& $nq = (40)(.45) = 18$ both ≥ 10) so valid to use

$\sigma = \sqrt{npq} = \sqrt{40(.55)(.45)} = 3.146$



normalcdf(32, 999, 22, 3.146) = .00074

or just z-score:

$z = \frac{x-\mu}{\sigma} = \frac{32-22}{3.146} = 3.18$ std devs above mean

pick any of these as justification to say:

It is very unlikely he would make 32 shots in 40 if his chance of making was still 55%. This is evidence that his chances of making a shot have increased

(But, this isn't an experiment, so we can't say it is due to his sneakers.)

1. Bernoulli. Can we use probability models based on Bernoulli trials to investigate the following situations? Explain.

a) We roll 50 dice to find the distribution of the number of spots on the faces.

no (more than 2 outcomes)

b) How likely is it that in a group of 120 the majority may have Type A blood, given that Type A is found in 43% of the population?

yes

c) We deal 5 cards from a deck and get all hearts. How likely is that?

no (trials are not independent)

d) We wish to predict the outcome of a vote on the school budget, and poll 500 of the 3000 likely voters to see how many favor the proposed budget.

no ($n < 10\%$ pop not true, sampling w/o replacing is changing the probability)

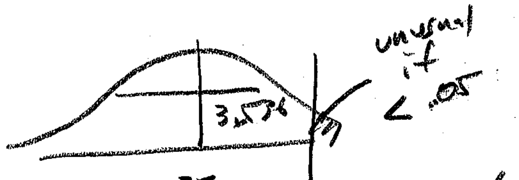
e) A company realizes that about 10% of its packages are not being sealed properly. In a case of 24, is it likely that more than 3 are unsealed?

yes

32. True-False. A true-false test consists of 50 questions. How many does a student have to get right to convince you that he is not merely guessing? Explain.

$np = (50)(.5) = 25$
$nq = (50)(.5) = 25$ ≥ 10 so we can use a normal approximation

$\mu = np = (50)(.5) = 25$
$\sigma = \sqrt{npq} = \sqrt{(50)(.5)(.5)} = 3.536$



unusual if $< .05$

? invNorm(.95, 25, 3.536) = 30.8
would need to get at least 31 correct

— or —

$z = \frac{x - \mu}{\sigma}$

$z = \frac{x - 25}{3.536}$ $\ldots x = 32.07$ would need around 32 or more correct to be "unusual"

7. Hoops. A basketball player has made 80% of his foul shots during the season. Assuming the shots are independent, find the probability that in tonight's game he
a) misses for the first time on his fifth attempt.
b) makes his first basket on his fourth shot.
c) makes his first basket on one of his first 3 shots.

a) X | 1st 2 3 4 5 6 7 . . .

geometpdf(.20, 5)

$= \boxed{.01892}$ prob. of miss

or

Y Y Y Y N
(.8)(.8)(.8)(.8)(.2)

b) X | 1st 2 3 4 5 6 . .

geometpdf(.80, 4)

$= \boxed{.0064}$

or

N N N Y
(.2)(.2)(.2)(.8)

c) X | $\underbrace{1^{st}\ 2\ 3}$ 4 5 6 7 . . .

geometcdf(.80, 3)

$= \boxed{.992}$

9. More hoops. For the basketball player in Exercise 7, what's the expected number of shots until he misses? (p = .20)

mean

geometric

$\mu = \frac{1}{p} = \frac{1}{.20} = \boxed{5 \text{ shots}}$

13. Lefties. Assume that 13% of people are left-handed. If we select 5 people at random, find the probability of each outcome described below.

a) The first lefty is the fifth person chosen.
b) There are some lefties among the 5 people.
c) The first lefty is the second or third person.
d) There are exactly 3 lefties in the group.
e) There are at least 3 lefties in the group.
f) There are no more than 3 lefties in the group.

a) (position) X | 1st 2 3 4 5

geometpdf(.13, 5) $= \boxed{.0745}$

b) (number) X | 0 1 2 3 4 5

1 − binompdf(5, .13, 0) $= \boxed{.5016}$

c) (position) X | 1st 2 3 4 5

geometpdf(.13, 2) + geometpdf(.13, 3) $= \boxed{.2115}$

d) (number) X | 0 1 2 3 4 5

binompdf(5, .13, 3) $= \boxed{.0166}$

e) (number) X | 0 1 2 3 4 5

1 − binomcdf(5, .13, 2) $= \boxed{.0179}$

f) (number) X | 0 1 2 3 4 5

binomcdf(5, .13, 3) $= \boxed{.9987}$

15. Lefties redux. Consider our group of 5 people from Exercise 13.

a) How many lefties do you expect? (number)
b) With what standard deviation?
c) If we keep picking people until we find a lefty, how long do you expect it will take? (geometric)

a) expected value of binomial $= \mu = np = (5)(.13) = \boxed{.65 \text{ lefties}}$

b) $\sigma = \sqrt{npq} = \sqrt{5(.13)(.87)} = \boxed{.752 \text{ lefties}}$

c) expected value of geometric $= \mu = \frac{1}{p} = \frac{1}{.13} = \boxed{7.69 \text{ people}}$

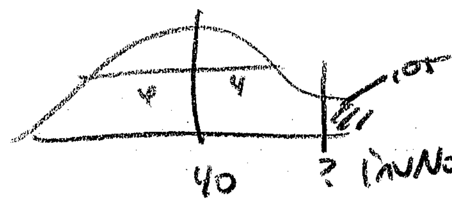
31. ESP. Scientists wish to test the mind-reading ability of a person who claims to "have ESP." They use five cards with different and distinctive symbols (square, circle, triangle, line, squiggle). Someone picks a card at random and thinks about the symbol. The "mind reader" must correctly identify which symbol was on the card. If the test consists of 100 trials, how many would this person need to get right in order to convince you that ESP may actually exist? Explain.

$p = 0.2$ if guessing $\left(\frac{1}{5}\right)$

assume normal approx $\mu = np = (100)(.2) = 40$

$\sigma = \sqrt{npq} = \sqrt{100(.2)(.8)} = 4$

Using 5% as unusual:



invNorm(.95, 40, 4) = 26.6

If the person gets 27 or more correct that would be too unusual to happen by chance.