#12. The amount of cereal that can be poured into a small bowl varies with a mean of 1.5 ounces and a standard deviation of 0.3 ounces. A large bowl holds a mean of 2.5 ounces with a standard deviation of 0.4 ounces. You open a new box of cereal and pour one large and one small bowl.
a) How much more cereal do you expect to be in the large bowl?
b) What's the standard deviation of this difference?
c) If the difference follows a Normal model, what's the probability the small bowl contains more cereal than the large one?
d) What are the mean and standard deviation of the total amount of cereal in the two bowls?
e) If the total follows a Normal model, what's the probability you poured out more than 4.5 ounces of cereal in the two bowls together?
f) The amount of cereal the manufacturer puts in the boxes is a random variable with a mean of 16.3 ounces and a standard deviation of 0.2 ounces. Find the expected amount of cereal left in the box, and the standard deviation.

a) expected = mean $\mu = \mu_L - \mu_S = 2.5 - 1.5 = \boxed{1.0 \text{ oz}}$
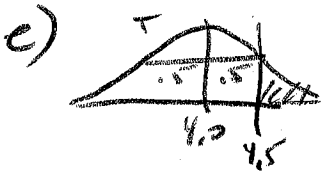
b) $\sigma^2 = \sigma_L^2 + \sigma_S^2$ (variances always add)

$\sigma = \sqrt{(.3)^2 + (.4)^2} = \boxed{0.5 \text{ oz}}$
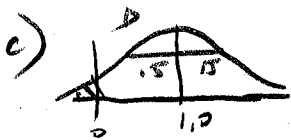
c) $P(\text{small has more}) = P(D < 0) = \text{normalcdf}(-999, 0, 1.0, .5) = \boxed{.023}$

d) $\mu_T = \mu_L + \mu_S = 2.5 + 1.5 = \boxed{4.0 \text{ oz}}$ $\sigma_T = \boxed{0.5 \text{ oz}}$ (same as b)

e) $P(T > 4.5) = \text{normalcdf}(4.5, 999, 4.0, 0.5) = \boxed{0.159}$

f) what's Left, $X = \text{Initial amt}_I - \text{Total poured out}_T$

so $\mu_X = \mu_I - \mu_T = 16.3 - 4.0 = \boxed{12.3 \text{ oz}}$

$\sigma_X^2 = \sigma_I^2 + \sigma_T^2 = (.2)^2 + (.5)^2$

$\sigma_X = \sqrt{(.2)^2 + (.5)^2} = \boxed{0.5385 \text{ oz}}$

#13. A casino knows that people play the dollar slot machines in hopes of hitting the jackpot, but that most of them lose their dollar. Suppose a certain machine pays out an average of $0.92, with a standard deviation of $120.

a) Why is the standard deviation so large?

b) If you play 5 times, what are the mean and standard deviation of the casino's profit?

c) If gamblers play this machine 1000 times in a day, what are the mean and standard deviation of the casino's profit?

d) Do you think the casino is likely to be profitable? Explain.

a) payment distribution:
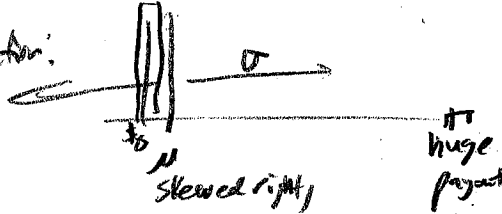
σ is large because there is a large difference between payouts.

skewed right, huge payout

b) profit = 1 − payout

profit = 1 − .92 = .08 per play

play 5X,

total, $T = P + P + P + P + P$

$T = 5P = 5(.08) = \boxed{\$0.40}$

$\sigma_T^2 = \sigma_P^2 + \sigma_P^2 + \sigma_P^2 + \sigma_P^2 + \sigma_P^2$
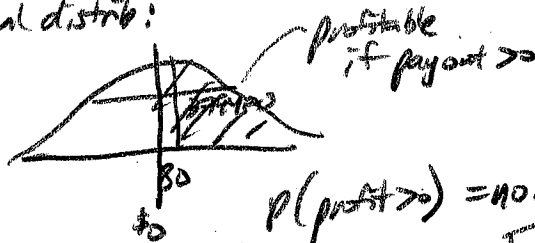
$\sigma_T^2 = 5\sigma_P^2$

$\sigma_T = \sqrt{5\sigma_P^2} = \sqrt{5}(120) = \boxed{\$268.33}$

c) Same as b but with 1000 in place of 5:

$\mu_T = 1000(.08) = \boxed{\$80}$

$\sigma_T = \sqrt{1000}(120) = \boxed{\$3794.73}$

d) assuming normal distrib:

profitable if payout > 0

$P(\text{profit} > 0) = \text{normalcdf}(0, 99999, 80, 3794.73)$

$= \boxed{.5085}$

yes, each machine has just over 50% chance of being profitable, but the casino will have many slot machines, so overall definitely profitable.

19. Contest. You play two games against the same opponent. The probability you win the first game is 0.4. If you win the first game, the probability you also win the second is 0.2. If you lose the first game, the probability that you win the second is 0.3.
a) Are the two games independent? Explain your answer.
b) What's the probability you lose both games?
c) What's the probability you win both games?
d) Let random variable X be the number of games you win. Find the probability model for X.
e) What are the expected value and standard deviation of X?

a) not independent
$$P(w|w) = .2$$
$$P(w|L) = .3 \neq$$

b)

$$P(LL) = (.6)(.7) = .42$$

c) $P(ww) = (.4)(.2) = .08$

d)
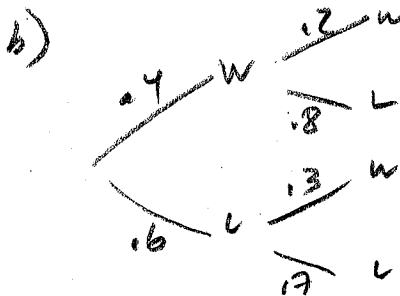
S	ww	WLorLW	LL
X	2	1	0
P	.08	.5 (1−.08−.42)	.42

$$E = (2)(.08) + (1)(.5) + (0)(.42)$$
$$= 0.66 \text{ games}$$

e) calc $\sigma = 0.62$ games

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21. Batteries. In a group of 10 batteries, 3 are dead. You choose 2 batteries at random.
a) Create a probability model for the number of good batteries you get.
b) What's the expected number of good ones you get?
c) What's the standard deviation?

a)

S	GG	GB or BG	BB
X	2	1	0
P	$\frac{7}{10} \cdot \frac{6}{9}$	$\frac{7}{10} \cdot \frac{3}{9} + \frac{3}{9} \cdot \frac{7}{10}$	$\frac{3}{10} \cdot \frac{2}{9}$
	≈.467	≈.467	.067

b) $E = (2)(.467) + (1)(.467) + (0)(.067) = 1.401 \text{ batteries}$

c) $\sigma = .61 \text{ batteries}$

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25. Random variables. Given independent random variables with means and standard deviations as shown, find the mean and standard deviation of each of these variables:
a) 0.8Y
b) 2X − 100
c) X + 2Y
d) 3X − Y
e) $Y_1 + Y_2$

	Mean	SD
X	120	12
Y	300	16

a) $\mu = 0.8(300) = 240$
$\sigma = 0.8(16) = 12.8$

b) $\mu = 2(120) - 100 = 140$
$\sigma = 2(12) = 24$

c) $\mu = 120 + 2(300) = 720$
$\sigma^2 = (12)^2 + (2 \cdot 16)^2$
$\sigma = \sqrt{1168} = 34.176$

d) $\mu = 3(120) - 300 = 60$
$\sigma^2 = (3 \cdot 12)^2 + (16)^2$
$\sigma = \sqrt{1552} = 39.395$

e) $\mu = 300 + 300 = 600$
$\sigma^2 = 16^2 + 16^2$
$\sigma = \sqrt{512} = 22.63$

27. Eggs. A grocery supplier believes that in a dozen eggs, the mean number of broken ones is 0.6 with a standard deviation of 0.5 eggs. You buy 3 dozen eggs without checking them.
  a) How many broken eggs do you expect to get?
  b) What's the standard deviation?
  c) What assumptions did you have to make about the eggs in order to answer this question?

3x

a) $\mu = 3(0.6) = \boxed{1.8 \text{ eggs}}$

b) $\sigma^2 = (.5)^2 + (.5)^2 + (.5)^2$
   $\sigma = \sqrt{.75} = \boxed{.866}$

c) probability of egg breakage carton to carton is independent.

31. Fire! An insurance company estimates that it should make an annual profit of $150 on each homeowner's policy written, with a standard deviation of $6000.
  a) Why is the standard deviation so large?
  b) If it writes only two of these policies, what are the mean and standard deviation of the annual profit?
  c) If it writes 10,000 of these policies, what are the mean and standard deviation of the annual profit?
  d) Do you think the company is likely to be profitable? Explain.
  e) What assumptions underlie your analysis? Can you think of circumstances under which those assumptions might be violated? Explain.

a) Some (small #) of profit is very low due to payout, most is $150.

b) $\mu = 2(150) = \$300$
   $\sigma^2 = (6000)^2$
   $\sigma = \sqrt{.720000} = \boxed{8485.28}$

c) $\mu = 10000(150) = \boxed{\$1,500,000}$
   $\sigma = \sqrt{10000(6000)^2} = \boxed{\$600,000}$

d) payout per individual is independent.
   If a hurricane hits, many homeowners all affected. or forest fire.