AP Stats Ch 14,15 - Extra Practice **Name:** solutions **Per:** ____

2. Rain. The weather reporter on TV makes predictions such as a 25% chance of rain. What do you think is the meaning of such a phrase?

Historically, over many similar days, on 1 out of every 4 days it rained.

4. Snow. After an unusually dry autumn, a radio announcer is heard to say, "Watch out! We'll pay for these sunny days later on this winter." Explain what he's trying to say, and comment on the validity of his reasoning.

He's implying that in any short run of days we should expect the average number of sunny days. This is not true — expected average # of sunny days is in the long run, anything can happen in the short term.

6. Crash. Commercial airplanes have an excellent safety record. Nevertheless, there are crashes occasionally, with the loss of many lives. In the weeks following a crash, airlines often report a drop in the number of passengers, probably because people are afraid to risk flying.

a) A travel agent suggests that, since the law of averages makes it highly unlikely to have two plane crashes within a few weeks of each other, flying soon after a crash is the safest time. What do you think?

b) If the airline industry proudly announces that it has set a new record for the longest period of safe flights, would you be reluctant to fly? Are the airlines due to have a crash?

a) no such thing as a "law of averages". Probability is only long term. All weeks have some likelihood of a crash.

b) No, to the contrary, longest period of safe flights = lower overall probability of a crash in any week.

9. Spinner. The plastic arrow on a spinner for a child's game stops rotating to point at a color that will determine what happens next. Are the following probability assignments possible? Why or why not?

	Probabilities of ...			
	Red	Yellow	Green	Blue
a)	0.25	0.25	0.25	0.25
b)	0.10	0.20	0.30	0.40
c)	0.20	0.30	0.40	0.50
d)	0	0	1.00	0
e)	0.10	0.20	1.20	−1.50

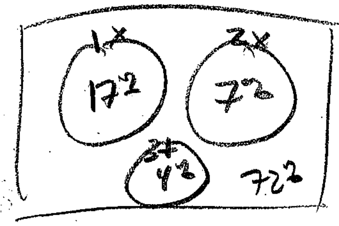
a) ok
b) ok
c) not ok (P(sum) > 1)
d) ok
e) not ok (P(x) can't be negative)

11. Car repairs. A consumer organization estimates that over a 1-year period 17% of cars will need to be repaired once, 7% will need repairs twice, and 4% will require three or more repairs. What is the probability that a car chosen at random will need

a) no repairs? $\boxed{0.72}$

b) no more than one repair? $0.72 + .17 = \boxed{0.89}$

c) some repairs? $0.17 + .07 + .04 = \boxed{0.28}$



13. More repairs. Consider again the auto repair rates described in Exercise 11. If you own two cars, what is the probability that

a) neither will need repair?

b) both will need repair?

c) at least one car will need repair?

independent events

$P(A \cap B) = P(A) \cdot P(B)$

a) $(.72)(.72) = \boxed{0.5184}$

b) $(.28)(.28) = \boxed{.0784}$ or $1 - P(\text{no repair})$

c) YN or NY or YY

$(.28)(.72) + (.72)(.28) + (.28)(.28) = \boxed{0.4816}$

$.2016 + .2016 + .0784$

15. Repairs, again. You used the Multiplication Rule to calculate repair probabilities for your cars in Exercise 13.

a) What must be true about your cars in order to make that approach valid?

b) Do you think this assumption is reasonable? Explain.

independent events

Yes, but not absolutely certain. Owner may neglect or treat well both cars, or may choose more or less reliable cars both times.

12. Stats projects. In a large Introductory Statistics lecture hall, the professor reports that 55% of the students enrolled have never taken a Calculus course, 32% have taken only one semester of Calculus, and the rest have taken two or more semesters of Calculus. The professor randomly assigns students to groups of three to work on a project for the course. What is the probability that the first groupmate you meet has studied

no calc .55
1sem .32
2+Sem (.13)

one student

a) two or more semesters of Calculus? $\boxed{.13}$
b) some Calculus? $.32+.13=\boxed{.45}$
c) no more than one semester of Calculus? $.55+.32=\boxed{.87}$

14. Another project. You are assigned to be part of a group of three students from the Intro Stats class described in Exercise 12. What is the probability that, of your other two groupmates,

a) neither has studied Calculus? $(.55)(.55)=\boxed{.3025}$
b) both have studied at least one semester of Calculus? $(.32+.13)(.32+.13)=\boxed{.2025}$
c) at least one has had more than one semester of Calculus?

$= 1 - P(\text{neither has more than 1 sem})$

$1-(.55+.32)(.55+.32)$

$\boxed{.2431}$

16. Final project. You used the Multiplication Rule to calculate probabilities about the Calculus background of your Statistics groupmates in Exercise 14.

a) What must be true about the groups in order to make that approach valid?

events must be independent

b) Do you think this assumption is reasonable? Explain.

yes. Students were assigned to groups randomly so whether or not have taken calculus should be independent from student to student

27. Dice. You roll a fair die three times. What is the probability that
a) you roll all 6's?
b) you roll all odd numbers?
c) none of your rolls gets a number divisible by 3?
d) you roll at least one 5?
e) the numbers you roll are not all 5's?

a) $\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6}$ b) $\frac{3}{6} \cdot \frac{3}{6} \cdot \frac{3}{6}$ c) $\frac{4}{6} \cdot \frac{4}{6} \cdot \frac{4}{6}$ d) $1 - P(\text{no } 5)$ e) $1 - P(\text{all } 5)$

$1 - \frac{5}{6}\frac{5}{6}\frac{5}{6}$ $1 - \frac{1}{6}\frac{1}{6}\frac{1}{6}$

.00463 0.125 .296 .4213 .9954

33. Tires. You bought a new set of four tires from a manufacturer who just announced a recall because 2% of those tires are defective. What is the probability that at least one of yours is defective?

$1 - P(\text{none defective})$

$1 - (.98)(.98)(.98)(.98)$

.0776

34. Pepsi. For a sales promotion, the manufacturer places winning symbols under the caps of 10% of all Pepsi bottles. You buy a six-pack. What is the probability that you win something?

$1 - P(\text{no wins})$ or $1W + 2W + 3W + 4W + 5W + 6W$

$1 - (.9)^6$ $(.1)^1(.9)^5 + (.1)^2(.9)^4 + (.1)^3(.9)^3 + (.1)^4(.9)^2$

$+ (.1)^5(.9)^1$

$+ (.1)^6$

.4686

Textbook Ch15 (p.362) #4, 5, 6, 10, 11, 16, 17, 20, 22, 25, 29, 34, 43.

4. Travel. Suppose the probability that a U.S. resident has traveled to Canada is 0.18, to Mexico is 0.09, and to both countries is 0.04. What's the probability that an American chosen at random has
a) traveled to Canada but not Mexico?
b) traveled to either Canada or Mexico?
c) not traveled to either country?

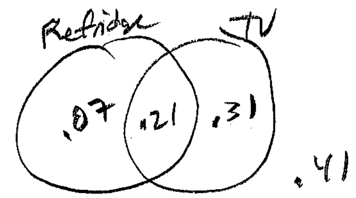
(a) $\boxed{.14}$

(b) $.14+.04+.05=\boxed{.23}$

$(.18)+(.09)-(.04)$

(c) $1-.23=\boxed{.77}$

5. Amenities. A check of dorm rooms on a large college campus revealed that 38% had refrigerators, 52% had TVs, and 21% had both a TV and a refrigerator. What's the probability that a randomly selected dorm room has
a) a TV but no refrigerator?
b) a TV or a refrigerator, but not both?
c) neither a TV nor a refrigerator?

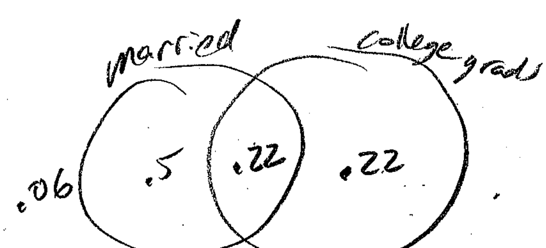


a) $\boxed{.31}$

b) $.07+.31=\boxed{.38}$

c) $\boxed{.41}$

6. Workers. Employment data at a large company reveal that 72% of the workers are married, that 44% are college graduates, and that half of the college grads are married. What's the probability that a randomly chosen worker
a) is neither married nor a college graduate?
b) is married but not a college graduate?
c) is married or a college graduate?



$P(M)=.72$

$P(C)=.44$

$P(M|C)=0.5$

$P(C)$ e $P(M|C)$ M $P(M \cap C)$

$P(M \cap C)=P(C)\cdot P(M|C)$

$=(.44)(.5)$

$=.22$

a) $\boxed{.06}$

b) $\boxed{.5}$

c) $.5+.22+.22=\boxed{.94}$

10. Pets. In its monthly report, the local animal shelter states that it currently has 24 dogs and 18 cats available for adoption. Eight of the dogs and 6 of the cats are male. Find each of the following conditional probabilities if an animal is selected at random:

a) The pet is male, given that it is a cat.
b) The pet is a cat, given that it is female.
c) The pet is female, given that it is a dog.

	Dog	Cat	
M	8	6	14
F	16	12	28
	24	18	42

a) $P(M|C) = \frac{6}{18} = \boxed{.33}$

b) $P(C|F) = \frac{12}{28} = \boxed{.4286}$

c) $P(F|D) = \frac{16}{24} = \boxed{.667}$

11. Health. The probabilities that an adult American man has high blood pressure and/or high cholesterol are shown in the table.

		Blood Pressure		
		High	OK	
Cholesterol	High	0.11	0.21	.32
	OK	0.16	0.52	.68
		.27	.73	1

a) What's the probability that a man has both conditions?
b) What's the probability that he has high blood pressure?
c) What's the probability that a man with high blood pressure has high cholesterol?
d) What's the probability that a man has high blood pressure if it's known that he has high cholesterol?

a) $P(BP \cap C) = \frac{.11}{1} = \boxed{.11}$

b) $P(BP) = \frac{.27}{1} = \boxed{.27}$

c) $P(C|BP) = \frac{.11}{.27} = \boxed{.4074}$

d) $P(BP|C) = \frac{.11}{.32} = \boxed{.3438}$

16. Sick cars. Twenty percent of cars that are inspected have faulty pollution control systems. The cost of repairing a pollution control system exceeds \$100 about 40% of the time. When a driver takes her car in for inspection, what's the probability that she will end up paying more than \$100 to repair the pollution control system?

$P(\text{faulty}) = 0.2$

$P(>\$100 \mid \text{faulty}) = 0.4$

$P(\text{faulty} \wedge >\$100) = P(\text{faulty}) \cdot P(>\$100 \mid \text{faulty})$

$= 0.2 \cdot 0.4$

$= \boxed{.08}$ (83)

17. Cards. You are dealt a hand of three cards, one at a time. Find the probability of each of the following.

a) The first heart you get is the third card dealt.
b) Your cards are all red (that is, all diamonds or hearts).
c) You get no spades.
d) You have at least one ace.

a) not a heart

X X H

$\frac{39}{52} \cdot \frac{38}{51} \cdot \frac{13}{50}$

$\boxed{.1453}$

b) R R R

$\frac{26}{52} \cdot \frac{25}{51} \cdot \frac{24}{50}$

$\boxed{.1176}$

c) not a spade

X X X

$\frac{39}{52} \cdot \frac{38}{51} \cdot \frac{37}{50}$

$\boxed{.4135}$

d) $P(\text{at least } 1 A)$

$= 1 - P(\text{no } A)$

$= 1 - \frac{48}{52} \cdot \frac{47}{51} \cdot \frac{46}{50}$

$= \boxed{.2174}$

20. Shirts. The soccer team's shirts have arrived in a big box, and people just start grabbing them, looking for the right size. The box contains 4 medium, 10 large, and 6 extra-large shirts. You want a medium for you and one for your sister. Find the probability of each event described.

20 total

a) The first two you grab are the wrong sizes.
b) The first medium shirt you find is the third one you check.
c) The first four shirts you pick are all extra-large.
d) At least one of the first four shirts you check is a medium.

a) $\bar{M} \cdot \bar{M}$

$\frac{16}{20} \cdot \frac{15}{19}$

$\boxed{.6316}$

b) $\bar{M}\bar{M}M$

$\frac{16}{20} \cdot \frac{15}{19} \cdot \frac{4}{18}$

$\boxed{.1404}$

c) X X X X

$\frac{6}{20} \cdot \frac{5}{19} \cdot \frac{4}{18} \cdot \frac{3}{17}$

$\boxed{.0031}$

d) $P(\text{at least } 1 M) = 1 - P(\text{none})$

$1 - \bar{M}\bar{M}\bar{M}\bar{M}$

$1 - \frac{16}{20} \cdot \frac{15}{19} \cdot \frac{14}{18} \cdot \frac{13}{17}$

$\boxed{.6244}$

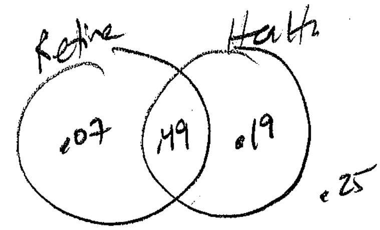
22. Benefits. Fifty-six percent of all American workers have a workplace retirement plan, 68% have health insurance, and 49% have both benefits. We select a worker at random.

a) What's the probability he has neither employer-sponsored health insurance nor a retirement plan?

b) What's the probability he has health insurance if he has a retirement plan?

c) Are having health insurance and a retirement plan independent events? Explain.

d) Are having these two benefits mutually exclusive? Explain.



a) $\boxed{.25}$

b) $P(H|R) = \dfrac{.49}{.56} = \boxed{.875}$

c) $P(H) = .68$ $P(H \cap R) = .49$ test: $P(H \cap R) \stackrel{?}{=} P(H) \cdot P(R)$

$P(R) = .56$ $.49 \stackrel{?}{=} (.68)(.56)$ $\boxed{\text{not independent}}$

$.49 \neq .3808$

d) $\boxed{\text{NO}}$ $P(H \cap R) \neq 0$ $\boxed{\text{there is overlap}}$

-or-

$P(H) = .68$

$P(H|R) = .875$ $\boxed{\text{not equal so not independent}}$

25. Cards. If you draw a card at random from a well-shuffled deck, is getting an ace independent of the suit? Explain.

$P(A) = \frac{4}{52}$

$P(\text{Hearts}) = \frac{13}{52}$

$P(A|\text{Hearts}) = \frac{1}{13}$

$P(A) \stackrel{?}{=} P(A|\text{Hearts})$

$\frac{4}{52} \stackrel{?}{=} \frac{1}{13}$

$\frac{1}{13} = \frac{1}{13}$

$\boxed{\text{independent}}$

29. Men's health, again. Given the table of probabilities from Exercise 11, are high blood pressure and high cholesterol independent? Explain.

		Blood Pressure	
		High	OK
Cholesterol	High	0.11	0.21
	OK	0.16	0.52

% across rows (or columns)
not equal
so $\boxed{\text{not independent}}$

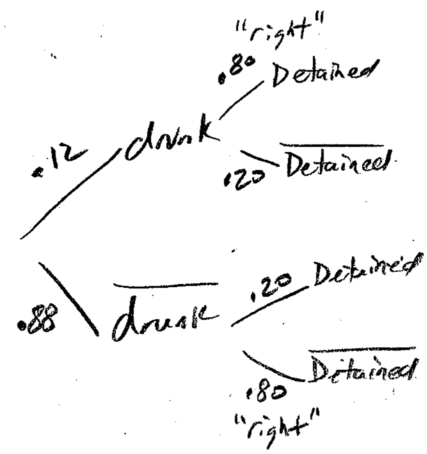
34. Cars. A random survey of autos parked in student and staff lots at a large university classified the brands by country of origin, as seen in the table. Is country of origin independent of type of driver?

Origin	Driver: Student	Driver: Staff	
American	(55%) 107	(64%) 105	212
European	(17%) 33	(7%) 12	45
Asian	(28%) 55	(29%) 47	102
	195	164	359

$\boxed{\text{not independent}}$ higher percentage of staff drive American. higher percentage of students drive European.

43. Drunks. Police often set up sobriety checkpoints—roadblocks where drivers are asked a few brief questions to allow the officer to judge whether or not the person may have been drinking. If the officer does not suspect a problem, drivers are released to go on their way. Otherwise, drivers are detained for a Breathalyzer test that will determine whether or not they are arrested. The police say that based on the brief initial stop, trained officers can make the right decision 80% of the time. Suppose the police operate a sobriety checkpoint after 9 p.m. on a Saturday night, a time when national traffic safety experts suspect that about 12% of drivers have been drinking.

a) You are stopped at the checkpoint and, of course, have not been drinking. What's the probability that you are detained for further testing?
b) What's the probability that any given driver will be detained?
c) What's the probability that a driver who is detained has actually been drinking?
d) What's the probability that a driver who was released had actually been drinking?



a) $P(\text{detain} \mid \overline{\text{drunk}}) = \boxed{0.20}$

b) $P(\text{detain}) = (.12)(.80) + (.88)(.20)$

$= \boxed{.272}$

c) $P(\text{drunk} \mid \text{detain}) = \dfrac{(.12)(.80)}{(.12)(.20)+(.88)(.80)}$

$= \boxed{.353}$

d) $P(\text{drunk} \mid \overline{\text{detain}}) = \dfrac{(.12)(.20)}{(.12)(.20)+(.88)(.80)}$

$= \boxed{.033}$

Challenge problem (requires some tricky counting strategies):
In a game of poker, you are dealt 5 cards at random from a standard deck of 52 cards. What is the probability that your hand is a full house (a full house consists of a pair of cards and three-of-a-kind of a different card).

all 5 card hands are equally likely, so;

$$P(\text{fullhouse}) = \frac{\#\text{ways to get a full house}}{\#\text{5 card hands}}$$

#5 card hands: drawing from a set of distinct objects without replacement (using them up) is a combination (in this case, order doesn't matter, cards just end up in the hand.)

$$_{52}C_5 = 2598960$$

#ways to get a fullhouse: full house = 3 of a kind + pair

4 'choices' (use multiplication principle, but fill each box using a combination)

$$\underset{\text{\# cards to have a 3 of a kind of e.g. "K"}}{\underline{13}} \cdot \underset{\text{\#ways to get 3 of these cards}}{\underline{_4C_3}} \cdot \underset{\text{\#cards to have a pair of e.g. "8"}}{\underline{12}} \cdot \underset{\text{\#ways to get 2 of these cards}}{\underline{_4C_2}}$$

$$\underline{13} \cdot \underline{4} \cdot \underline{12} \cdot \underline{6}$$

$$= 3744$$

$$\text{So } P(\text{full house}) = \frac{3744}{2598960} = \boxed{.0014405762}$$