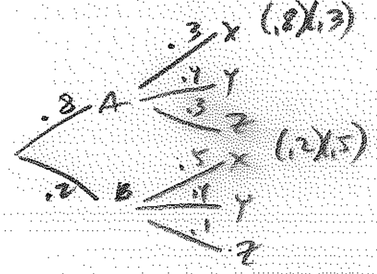
Even more probability review — SOLUTIONS

practice problems (p.160)

Multiple-Choice

① $P(A \cap E) = \boxed{\frac{12}{125}}$ $P(C|E) = \boxed{\frac{28}{63}}$ $\boxed{C}$

② 

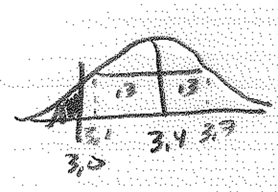
$P(B|X) = \dfrac{(.2)(.5)}{(.8)(.3)+(.2)(.5)} = \boxed{\frac{5}{17}}$ $\boxed{B}$

③ Total = Stats + Spanish

$\mu_T = \mu_{Stats} + \mu_{Spanish} = 2.4 + 2.65 = 5.05$

If we were told these vary independently, then:

$\sigma_T^2 = \sigma_{Stats}^2 + \sigma_{Spanish}^2$, but we aren't told so $\boxed{E}$

④ 

normalcdf(−999, 3.0, 3.4, .3) = .091211 $\boxed{\text{4 is closest}}$

⑤

W	L	A	B	O
.46	.32	.11	.07	.04

$P(A \cup L) = P(A) + P(L) - P(A \cap L)$

0 they said to assume mutually-exclusive

$= .11 + .32$

$= \boxed{.43}$ $\boxed{D}$

Free-Response (p.162)

①

X	2	3	4	← L1
P	1/3	5/12	1/4	← L2

1-var stats L1, L2

$\boxed{\mu_X = \bar{x} = 2.9167 \quad \sigma_X = \sigma = .7592}$

or by hand →

$\mu = \sum x \cdot p(x)$

$= 2\left(\frac{1}{3}\right) + 3\left(\frac{5}{12}\right) + 4\left(\frac{1}{4}\right) = \boxed{2.916\overline{6}}$

$\sigma = \sum (x-\mu)^2 p(x)$

$= (2-2.916\overline{6})^2\left(\frac{1}{3}\right) + (3-2.91666)^2\left(\frac{5}{12}\right) + (4-2.916\overline{6})^2\left(\frac{1}{4}\right)$

$= \boxed{.5764}$ (rounding)

(2) $P(A) = .6$
$P(B) = .3$
$P(B|A) = .5$

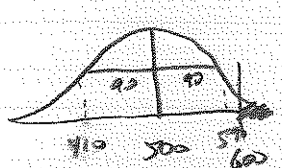
a) $P(A \cap B) = P(A) \cdot P(B|A)$
$= (.6)(.5) = \boxed{.3}$

b) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$= (.6) + (.3) - (.3) = \boxed{.6}$

c) independent? $P(B) = .3 \neq P(B|A) = .5$
so A & B are $\boxed{\text{not independent}}$
(probability of B changes if A happens)

(3)

a)



normalcdf(600, 99999, 500, 90) = $\boxed{.1333}$

b)

normalcdf(450, 600, 500, 90) = .5775 × (9000) (n = 9000)
= 5197.3 ≈ $\boxed{5197 \text{ scores}}$

c)

? = invNorm(.90, 500, 90) = 615.3 ≈ $\boxed{615}$

(4)

$\mu_X = 3$
$\sigma_X^2 = .25$ so $\sigma_X = \sqrt{.25} = .5$

then $\mu_{3+6X} = 3 + 6(3) = \boxed{21}$
$\sigma_{3+6X} = 6(.5) = \boxed{3}$
(+3 shift doesn't affect spread)

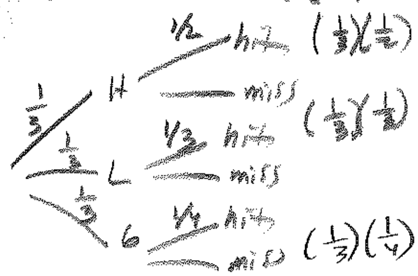
(5)

H	L	G
1/2	1/3	1/4

(Sum = 1.08$\overline{3}$) ← this is not a sample space

instead $P(H) = \frac{1}{2}$
$P(L) = \frac{1}{3}$
$P(G) = 1/4$

"take turns" implies they each shot the same number of times, so



$$P(L \mid \text{hit}) = \frac{(\frac{1}{3})(\frac{1}{3})}{(\frac{1}{3})(\frac{1}{2}) + (\frac{1}{3})(\frac{1}{3}) + (\frac{1}{3})(\frac{1}{4})}$$

$\approx .30769 = \boxed{\frac{4}{13}}$

(6) $\mu_x = 3$ $\mu_y = 5$ $\mu_{x+y} = \mu_x + \mu_y = 3+5 = \boxed{8}$

$\sigma_x^2 = 1$ $\sigma_y^2 = 1.3$ $\sigma_{x+y}^2 = \sigma_x^2 + \sigma_y^2$

$\sigma_{x+y} = \sqrt{(1)+(1.3)} = \boxed{1.5166}$

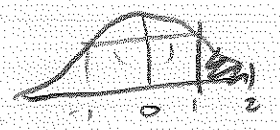
(7) I.

normalcdf$(-2, 2, 0, 1) = .9544998\cancel{76}$ $\boxed{\text{False}}$ (not exactly 95%)

95.44998762

II. $\boxed{\text{True}}$ (Normal distributions are perfectly symmetrical)

III.



normalcdf$(1, 2, 0, 1)$ $= .135905$ $>$ normalcdf$(2, 3, 0, 1)$ $= .021400$ $\boxed{\text{TRUE}}$

(8) a) A: draw face card on 1^{st} draw } indep?

B: draw face card on 2^{nd} draw

1^{st} FC	2^{nd} FC
$\frac{12}{52}$	$\frac{11}{51}$

$P(B|A) = P(2^{nd}\text{ Face card} \mid 1^{st}\text{ Face card}) = \frac{11}{51}$

$P(B|\bar{A}) = P(2^{nd}\text{ Face card} \mid 1^{st}\text{ not Face card}) = \frac{12}{51}$

1^{st} not FC	2^{nd} FC
$\frac{40}{52}$	$\frac{12}{51}$

Because probability of B is changing if A happens, $\boxed{\text{A and B are not independent}}$

b) A: draw face card on 1^{st} draw

C: 1^{st} card drawn is a diamond

w/in 12 FC's, 3 are diamond

$P(C|A) = P(1^{st}\text{ card diamond} \mid 1^{st}\text{ card FC}) = \frac{3}{12} = \frac{1}{4}$

(12 face cards)

(10 of these are diamond)

$P(C|\bar{A}) = P(1^{st}\text{ card diamond} \mid 1^{st}\text{ card not FC}) = \frac{10}{40} = \frac{1}{4}$

(40 non-FC)

Because probability of C is not changing whether or not A happens, $\boxed{\text{A and C are independent}}$

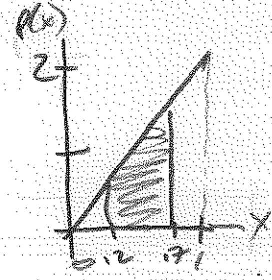
(9)

$X =$ invNorm$(.4, 700, 50) = \boxed{687.33}$

(10)

$P(\text{laptop} \mid \text{desktop}) = \dfrac{P(\text{laptop} \cap \text{desktop})}{P(\text{desktop})} = \dfrac{.3}{.8} = \boxed{.375 = \frac{3}{8}}$

(11)



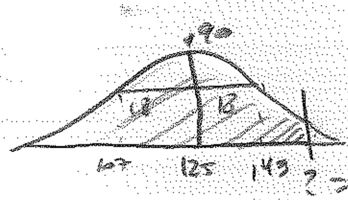
$P(.2 \le x \le .7) = A_\Delta - A_\Delta$

$\frac{1}{2}(.7)(1.4) - \frac{1}{2}(.2)(.4)$

$.49 - .04$

$= \boxed{.45}$

(12)



$? = \text{invNorm}(.90, 125, 18) = \boxed{148 \text{ lbs}}$

(13)

X	2	3	4
P(x)	.3	.5	$\boxed{.2}$

← adds to 1.0

Y	3	4	5	6
P(Y)	$\boxed{.15}$	.1	$\boxed{}$	.4

given $P(X=4 \text{ and } Y=3) = .03$

$P(A \cap B) = P(A) \cdot P(B|A)$ but if independent $P(B|A) = P(B)$

$= P(A) \cdot P(B)$ ← (problem says X & Y are indep)

here $P(X=4 \cap Y=3) = P(X=4) \cdot P(Y=3 \mid \cancel{X=4})$

$P(X=4 \cap Y=3) = P(X=4) \cdot P(Y=3)$

$(.03) = (.2) \cdot P(Y=3)$

so $P(Y=3) = \dfrac{.03}{.2} = .15$

now $P(Y=5) = 1 - (.15 + .1 + .4)$

$= 1 - .65$

$= \boxed{.35}$

(14)

multiples of 24: 24, 48, 72, 96, ~~120~~

multiples of 36: 36, 72, ~~108~~

$S = \{1 - - - - - - 100\}$ (equally-likely)

$E = \{24, 36, 48, 72, 96\}$

$P(24 \text{ or } 36) = \boxed{\dfrac{5}{100}}$

(15) component: find gender of 1 student at the law school

outcomes: M (50%) F (50%)

model: single digit

0—4 M, 5—9 F

trial ends: simulate class of 12 (look at 12 students)

trial #1: 79692 51707 73|274
FFFFM FMFMF FM|
8 are female

trial #2: 41957 21607 51|248
MMFFF MMFMF FM|
6 are female

trial #3: 07094 31250 69|26
MFMFM MMFFM FF|
6 are female

trial #4: 59365 43671 12|704
FFMFF MMFFM MM|
6 are female

trial #5: 91547 03977 92|309
FMFMF MMFMF FM|
6 are female

$$P(X \le 4 \text{ out of } 12) = \frac{0}{5} = 0$$
(experimental)

Simulation shows having as few as 4 or fewer women in 12 should likely never happen, so there IS evidence females are underrepresented.

(16) component: gender of one child

outcomes: F (.6) M (.4)

model: single digit

0—5 F, 6—9 M

trial ends: when we have 3 girls

trial 1: 79692 51|702 73271 12548
MMMMF FF|
6 kids

trial 2: 4195|7 21607 51248 54772
FFMF|
4 kids

trial 3: 07094| 31250 69426
FMFM4|
5 kids

trial 4: 59365 |43671 12904
FMFMF|
5 kids

trial 5: 9154|7 03977 92309
MFFF|
4 kids

kids to get 3 girls: 6, 4, 5, 5, 4

$\bar{x} = 4.8$ kids

It is estimated to take 4.8 kids, on average, to get 3 girls on this planet.

(17) (a) $P(x \le 22) = P(20) + P(21) + P(22)$
$= .2 + .3 + .2$
$= \boxed{.7}$

(b) $P(x > 21) = P(22) + P(23) + P(24)$
$= .2 + .1 + .2$
$= \boxed{.5}$

(c) $P(21 \le x < 24) = P(21) + P(22) + P(23)$
$= .3 + .2 + .1$
$= \boxed{.6}$

(d) $P(x \le 21 \text{ or } x > 23) = P(20) + P(21) + P(24)$
$= .2 + .3 + .2$
$= \boxed{.7}$

(18) (a) $P(R) = \dfrac{18}{38} = \boxed{.4737}$

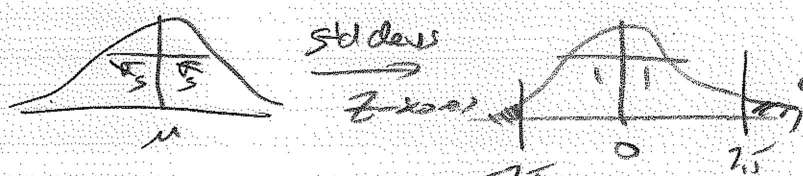
(b) $S = \{\text{red, black, green}\}$

X return (profit)	+\$1	-\$1	-\$1
P	$\frac{18}{38}$	$\frac{18}{38}$	$\frac{2}{38}$

EV = μ of 1-var stats

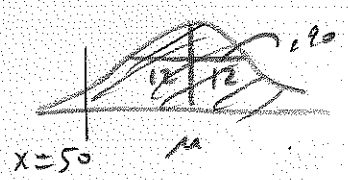
$= (1)\left(\frac{18}{38}\right) + (-1)\left(\frac{18}{38}\right) + (-1)\left(\frac{2}{38}\right)$
$= \boxed{-\$.0526}$ (lose about 5 cents, on average, each time you play because of the green slots)

(19)



normalcdf(2.5, 999, 0, 1)
= .006209...
$\times 2 = \boxed{.0124}$

(20)



$z = \dfrac{x - \mu}{\sigma}$

$z = \text{invNorm}(.1, 0, 1) = -1.28155$

$X = 50$ assoc. w/ $z = -1.28155$

$(-1.28155) = \dfrac{(50) - \mu}{12}$

Solve for μ:

$12(-1.28155\ldots) = 50 - \mu$

$\mu = 50 - 12(-1.28155\ldots)$
$= \boxed{65.3786}$

Cumulative review problems (p.165)

① I) False  mean is pulled away from median towards the tail

II) True

III) False

② divide into 'strata' by ethnicity:

Asian	African Amer.	Latino	Caucasian
(25%)	(8%)	(12%)	(55%)
SRS	SRS	SRS	SRS
sample 25 Asians	8 African Amer.	12 Latinos	55 Caucasians

Sample needed = 100 to match proportion:

This would be stratified random sampling (take an SRS from each ethnic group, adjust the sample sizes to match %)

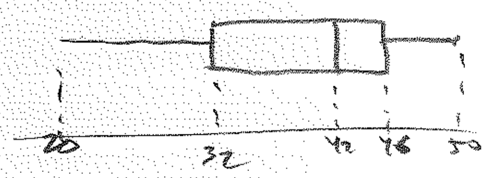
③ (calculator) (varstats) min = 20

Q1 = 32

MED = 42

Q3 = 46

MAX = 50



PRACTICE PROBLEMS (next page)

Multiple-choice

① binomial $\mu = np = (60)(.4) = \boxed{24}$

$\sigma = \sqrt{npq} = \sqrt{(60)(.4)(.6)} = \boxed{3.7947}$ $\boxed{a.}$

② probability occurring on 2nd trial for 1st time is geometric model: NY

So $P(N) \cdot P(Y) = .25$ → $p - p^2 = .25$ quadratic formula

$(1-p)(p) = .25$ $p^2 - p + .25 = 0$ $p = \frac{-(-1) \pm \sqrt{(1)^2 - 4(1)(.25)}}{2(1)}$

so $\boxed{p = \frac{1}{2}}$ $\boxed{a.}$ $p = \frac{1 \pm \sqrt{0}}{2} = \frac{1}{2}$

③ a) $np \geq 5$, $n(1-p) \geq 5$ – success/fail condition is ≥ 10 so would also be ≥ 5 (required)

b) trials indep. (required for all Binomials, even if not normal)

→ c) sample size too large to permit doing on a calculator (normal is nice for this, but this doesn't have to be true)

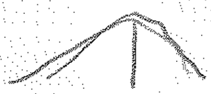
d) pop size > 10 sample size → sample < 10% pop (required, otherwise can't assume probability is constant from trial to trial)

e) All true (no)

(4)

$\mu = 50$

samples $n = 9$



$\mu_{\bar{x}} = \mu = 50$

$\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}} = \frac{12}{\sqrt{9}} = \frac{12}{3} = 4$

Central limit theorem $n \geq 25$

(this sample too small to say it is normal, but is likely more normal than the population

so (b)

FREE RESPONSE

(1) $p = .025$ 1 can

(binomial) X # cans detect 0 1 2 3 --- 48

$1 - \text{binomcdf}(48, .025, 1) = \boxed{.3383}$

(2) (a) for $n = 3$ distribution will still be skewed, but slightly less skewed than the population.

(b) for $n = 30 \geq 25$, by the central limit theorem, the sampling distribution will be approximately normal.

(3) X # heads (binomial) 0 ——— 1,000,000

w/ $np = (1{,}000{,}000)(.5) = 500000$

$nq = 500000$

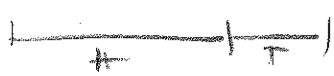
≥ 10 so normal approx is ok

$\mu = np = 500{,}000$

$\sigma = \sqrt{npq} = \sqrt{1000000(.5)(.5)}$

$= 500$

>1000 more heads than tails



$H + T = 1{,}000{,}000$

$H = T + 1000 \quad T = H - 1000$

$H + (H - 1000) = 1{,}000{,}000$

$2H - 1000 = 1000000$

$2H = 999000$

$H = 499500$

$P(H > 499500) = \text{normalcdf}(499500, 9.10^{99}, 500000, 500)$

$= \boxed{.8413}$

(5) X number of 3s: 0 1 2 - 9, $\underbrace{10 - - - 49\ 50}$

$P(X \geq 10) = 1 - \text{binomcdf}(50, \frac{1}{12}, 9)$

$= 1 - [\text{binompdf}(50, \frac{1}{12}, 0) + \text{binompdf}(50, \frac{1}{12}, 1) \cdots + \text{binompdf}(50, \frac{1}{12}, 9)]$

$\text{binompdf}(n, p, x) = {}_nC_x (p)^x (1-p)^{n-x}$

so

$= 1 - [{}_{50}C_0 (.083)^0 (.917)^{50} + {}_{50}C_1 (.083)^1 (.917)^{49} + \ldots {}_{50}C_9 (.083)^9 (.917)^{41}$

(d) (except there should be parenthese around the right part)

(4) $S = \{G, BG, BBG, BBBG, BBBBG, BBBBBG, BBBBBBG, \dots\}$

X: $\frac{1}{1}$, $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$ ...

random variable for % of kids in a family which are girls

P: $\frac{1}{2}$, $\frac{1}{2}\frac{1}{2} = \frac{1}{4}$, $\frac{1}{2}\frac{1}{2}\frac{1}{2} = \frac{1}{8}$, $\frac{1}{16}$, $\frac{1}{32}$, $\frac{1}{64}$, $\frac{1}{128}$ ...

so $EV_{\%\text{ of girls}} = (1)(\frac{1}{2}) + (\frac{1}{2})(\frac{1}{4}) + (\frac{1}{3})(\frac{1}{8}) + (\frac{1}{4})(\frac{1}{16}) + (\frac{1}{5})(\frac{1}{32}) + (\frac{1}{6})(\frac{1}{64}) + (\frac{1}{7})(\frac{1}{128}) + \dots$

$= .5 + .125 + .041666 + .015625 + .00625 + .0026041667 + .0011160714$

$\approx .69226+$ isn't exact, but proportion of girls would definitely increase above 50% to over 69%.

(5)
- multiple trials
- Y/N Y = graduate
- $p = .20$ (constant)
- # out of 55

Binomial

X (# who graduate out of 55): 0 1 2 3 ... 55

$\mu = np = (55)(.20) = \boxed{11}$

$\sigma = \sqrt{npq} = \sqrt{(55)(.2)(.8)} = \boxed{2.9665}$

SKIP PROBLEMS 6, 7, 8, 10, 12, 15, 16, 20

(about sampling distributions, which we haven't covered yet)

(9) Y/N Y = defective brakes

$p = .15$ (constant)

$n = 20$

X (# defects out of 20): 0 1 2 3 4 5 ... 20 (binomial)

$P(3 \text{ out of } 20) = \text{binompdf}(20, .15, 3)$

$= \boxed{.2428}$

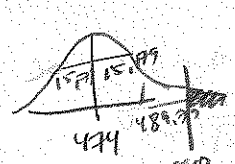
(or ${}_{20}C_3 (.15)^3 (1-.15)^{17}$)

(11) $np = (1000)(.474) = 474 \geq 10$ ✓

$nq = 1000 - 474 = 526 \geq 10$ ✓

Normal approx justified

half games $= \frac{1}{2}(1000) = 500$ (boundary)



$\mu = np = (1000)(.474) = 474$

$\sigma = \sqrt{npq} = \sqrt{1000(.474)(1-.474)} = 15.789997$ (15.79)

$P(\text{win more than } \frac{1}{2}) = \text{normalcdf}(500, 99999, 474, 15.79) = \boxed{.0498}$ (quite unlikely)

(13) Bernoulli?
- Y/N? Y = reason
- multiple trials? (each person is a trial)
- $p = .7$ constant

\# out of total — Binomial

a) X 0 1 2 3 4 5 6 7 8

\# team

$P(x=5) = \text{binompdf}(8, .7, 5) = \boxed{.2541}$

b) X 0 1 2 3 4 5 6 7 8

$P(x=8) = \text{binompdf}(8, .7, 8) = \boxed{.0576}$

c) X 0 $\underbrace{1\ 2\ 3\ 4\ 5\ 6\ 7\ 8}$

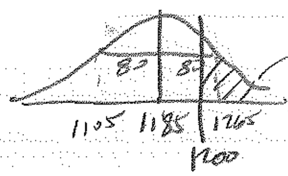
$P(x \geq 1) = 1 - P(x=0)$

$= 1 - \text{binompdf}(8, .7, 0)$

$= 1 - 6.561 \times 10^{-5}$

$= \boxed{.9999}$

(14) Can assume this is normal (by Central Limit Theorem $n \geq 25$)



$P(x > 1200) = \text{normalcdf}(1200, 99999, 1185, 80) = \boxed{.4256}$

(17) X 0 $\underbrace{1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12}$

\# fails

$P(x \geq 1) = 1 - P(x=0)$

$= 1 - \text{binompdf}(12, .003, 0)$

$= \boxed{.0354}$ (low, but, unfortunately, not impossible :( )

(18) $S = \{P1, P2, P3, P4, P5\}$

P	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

a) $P(P3) = \frac{1}{5}$ "next" when success occurs (geometric)

1st 2nd 3rd 4th ...

$P(\text{next is } P3) = \text{geometpdf}(\frac{1}{5}, 1) = \boxed{.2}$ $(\frac{1}{5})$ just (p)

b) avg = EV = $\frac{1}{p}$ (this is geometric)

$= \frac{1}{(\frac{1}{5})} = \boxed{5 \text{ boxes}}$

(19) assuming $B(40, .8)$ means binomial $n = 40$, $p = .8$

$np = (40)(.8) = 32 \geq 10$ ✓

$nq = (40)(.2) = 8 \geq 10$ ✗

so we should not use a normal approx.

CUMULATIVE REVIEW PROBLEMS

① by hand:

$S = \{ \overset{(0)}{TTT}, \overset{(1)}{HTT}, \overset{(1)}{THT}, \overset{(1)}{TTH}, \overset{(2)}{THH}, \overset{(2)}{HTH}, \overset{(2)}{HHT}, \overset{(3)}{HHH} \}$

$P \quad \frac{1}{2}\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8} \quad \frac{1}{8}$

X (# heads)	0	1	2	3
P	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{3}{8}$	$\frac{1}{8}$

using binomial:

X	0	1	2	3
	binompdf$(3, \frac{1}{2}, 0) = \frac{1}{8}$	binompdf$(3, \frac{1}{2}, 1) = \frac{3}{8}$	binompdf$(3, \frac{1}{2}, 2) = \frac{3}{8}$	binompdf$(3, \frac{1}{2}, 3) = \frac{1}{8}$

② If the students were not randomly assigned to the classes this would not be an SRS, but because students are randomly assigned to classes and then SRS w/in each class it is an SRS so (B)

SRS requires – sampling from entire population (true)
– every individual has same chance to be selected (true)
– every combination of individuals has same chance (true)

③ $\widehat{\text{reaction time}} = -.2 + .8(\text{mg})$

$.8 \frac{\text{secs}}{\text{mg drug}}$ for every 1 additional mg of drug, reaction time increases by 0.8 seconds, on average.

④ $P(A) = .5$
$P(B) = .3$
$P(A \cup B) = .65$

for indep. check $P(B) \overset{?}{=} P(B|A)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$.65 = .5 + .3 - P(A \cap B)$
so $P(A \cap B) = .15$
and $P(A \cap B) = P(A) \cdot P(B|A)$
$.15 = .5 \cdot P(B|A)$
$\frac{.15}{.5} = P(B|A) = .3$
so $P(B) = .3 = P(B|A) = .3$
not changing,
So A & B are independent

—or—

if independent the simpler version of ~~AND formula~~ will be true:
$P(A \cap B) = P(A) \cdot P(\cancel{B|A})$ → $P(B)$
$P(A \cap B) \overset{?}{=} P(A) \cdot P(B)$
$.15 = (.5)(.3)$
$.15 = .15$
yes
so are independent

(5) a) ht in inches (quantitative, numerical)
b) color of shirt (qualitative, categorical)
c) Hort (qualitative, categorical)
d) value of change in pocket (quantitative, numerical)
e) thin, normal, heavy (qualitative, categorical)
f) pulse rate (quantitative, numerical)
g) religion (qualitative, categorical)