Answer: The sample space for having three children is {BBB, BBG, BGB, GBB, BGG, GBG, GGB, GGG} Of these, there are seven outcomes that have at least one boy. Of these, three have two boys and one girl. Thus, P(the couple has exactly two boys|they have at least one boy) = 3/7.

3. Does the following table represent the probability distribution for a discrete random variable?

X	1	2	3	4
p(X)	.2	.3	.3	.4

Answer: No, because

$$\sum p_i = 1.2$$

4. In a standard normal distribution, what is $P(z > .5)$?

Answer: From the table, we see that $P(z < .5) = .6915$. Hence, $P(z > .5) = 1 - .6915 = .3085$. By calculator, *normalcdf*(.5,100) = .3085375322.

5. A random variable X has N(13,.45). Describe the distribution of 2 – 4X (that is, each datapoint in the distribution is multiplied by 4, and that value is subtracted from 2)

Answer: We are given that the distribution of X is normal with $\mu_X = 13$ and $\sigma_X = .45$. Because $\mu_{a \pm bX} = a \pm b\mu_X$, $\mu_{2-4X} = 2 - 4\mu_X = 2 - 4(13) = -50$. Also, because $\sigma_{a \pm bX} = b\sigma_X$, $\sigma_{2-4X} = 4\sigma_X = 4(.45) = 1.8$

PRACTICE PROBLEMS

Multiple Choice

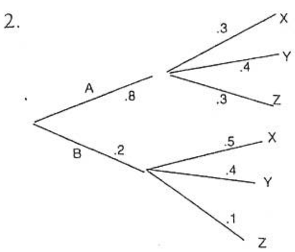
1.

	D	E	Total
A	15	12	27
B	15	23	38
C	32	28	60
Total	62	63	125

In the table above what are P(A and E) and P(C|E)?

a. 12/125, 28/125
b. 12/63, 28/60
c. 12/125, 28/63
d. 12/125, 28/60
e. 12/63, 28/63

2.



For the tree diagram pictured above, what is P(B|X)?

a. 1/4
b. 5/17
c. 2/5
d. 1/3
e. 4/5

3. It turns out that 25 seniors at Fashionable High School took both the AP Statistics exam and the AP Spanish Language exam. The mean score on the statistics exam for the 25 seniors was 2.4 with a standard deviation of 0.6 and the mean score on the Spanish Language exam was 2.65 with a standard deviation of 0.55. We want to combine the scores into a single score. What are the correct mean and standard deviation of the combined scores?

a. 5.05; 1.15
b. 5.05; 1.07
c. 5.05; 0.66
d. 5.05; 0.81
e. 5.05; you cannot determine the standard deviation from this information.

4. The GPA (grade point average) of students who take the AP Statistics exam are approximately normally distributed with a mean of 3.4 with a standard deviation of 0.3. What is the probability that a student selected at random from this group has a GPA lower than 3.0?

a. 0.0918
b. 0.4082
c. 0.9082
d. –.0918
e. 0

5. The 2000 Census identified the ethnic breakdown of the state of California to be approximately as follows: White: 46%, Latino: 32%, Asian: 11%, Black: 7%, and Other: 4%. Assuming that these are mutually exclusive categories (not a realistic assumption, by the way),

what is the probability that a random selected person from the state of California is of Asian or Latino descent?

a. 46%
b. 32%
c. 11%
d. 43%
e. 3.5%

Free Response

1. Find μ_X and σ_X for the following discrete probability distribution:

X	2	3	4
$p(X)$	1/3	5/12	1/4

2. Given that $P(A) = 0.6$, $P(B) = 0.3$, and $P(B|A) = 0.5$.
   (a) $P(A \text{ and } B) = ?$
   (b) $P(A \text{ or } B) = ?$
   (c) Are events A and B independent?

3. Consider a set of 9000 scores on a national test that is know to be approximately normally distributed with a mean of 500 and a standard deviation of 90.
   (a) What is the probability that a randomly selected student has a score greater than 600?
   (b) How many scores are there between 450 and 600?
   (c) Rachel needs to be in the top 1% of the scores on this test to qualify for a scholarship. What is the minimum score Rachel needs?

4. Consider a random variable X with $\mu_X = 3$, $\sigma_X^2 = 0.25$ Find
   (a) μ_{3+6X}
   (b) σ_{3+6X}.

5. Harvey, Laura, and Gina take turns throwing spit-wads at a target. Harvey hits the target 1/2 the time, Laura hits it 1/3 of the time, and Gina hits the target 1/4 of the time. Given that somebody hit the target, what is the probability that it was Laura?

6. Consider two discrete, independent, random variables X and Y with $\mu_X = 3$, $\sigma_X^2 = 1$, $\mu_Y = 5$, and $\sigma_Y^2 = 1.3$. Find μ_{X+Y} and σ_{X+Y}.

7. Which of the following statements are true of a normal distribution?
   I. Exactly 95% of the data are within 2 standard deviations of the mean.

   II. The mean = the median = the mode
   III. The area under the normal curve between $z = 1$ and $z = 2$ is greater than the area between $z = 2$ and $z = 3$.

8. Consider the experiment of drawing two cards from a standard deck of 52 cards. Let event A = "draw a face card on the first draw," B = "draw a face card on the second draw," C = "the first card drawn is a diamond"
   (a) Are the events A and B *independent?*
   (b) Are the events A and C *independent?*

9. A normal distribution has mean 700 and standard deviation 50. The probability is .6 that a randomly selected term from this distribution is above x. What is x?

10. Suppose 80% of the homes in Lakeville have a desktop computer and 30% have both a desktop computer and a laptop computer. What is the probability that a randomly selected home will have a laptop computer given that they have a desktop computer?

11. Consider a probability density curve defined by the line $y = 2x$ on the interval [0,1] (the area under $y = 2x$ on [0,1] is 1). Find $P(.2 \leq X \leq .7)$.

12. Half Moon Bay, California, has an annual pumpkin festival at Halloween. A prime attraction to this festival is a "largest pumpkin" contest. Suppose that the weights of these giant pumpkins is approximately normally distributed with a mean of 125 pounds and a standard deviation of 18 pounds. Farmer Harv brings a pumpkin that is at the 90% percentile of all the pumpkins in the contest. What is the approximate weight of Harv's pumpkin?

13. Consider the following two probability distributions for independent discrete random variable X and Y:

X	2	3	4
$P(X)$	.3	.5	?

Y	3	4	5	6
$P(Y)$	?	.1	?	.4

If $P(X = 4 \text{ and } Y = 3) = .03$, what is $P(Y = 5)$?

14. A contest is held to give away a free pizza. Contestants pick an integer at random from the integers 1 through 100. If the picked number is divisibly by 24 or by 36, the contestant wins the pizza. What is the probability that a contestant wins a pizza?

Use the following excerpt from a random number table for questions 15 and 16:

79692	51707	73274	12548	91497	11135	81218	79572	06484	87440
41957	21607	51248	54772	19481	90392	35268	36234	90244	02146
07094	31750	69426	62510	90127	43365	61167	53938	03694	76923
59365	43671	12704	87941	51620	45102	22785	07729	40985	92589
91547	03927	92309	10589	22107	04390	86297	32990	16963	09131

15. Men and women are about equally likely to earn degrees at City U. However, there is some question whether or not women have equal access to the prestigious School of Law. This year, only 4 of the 12 new students are female. Describe and conduct five trials of a simulation to help determine if this is evidence that women are under represented in the School of Law.

16. Suppose that, on a planet far away, the probability of a girl being born is .6, and it is socially advantageous to have three girls. How many children would a family have to have, on average, until they have three girls? Describe and conduct five trials of a simulation to help answer this question.

17. Consider a random variable X with the following probability distribution:

x	20	21	22	23	24
$p(x)$	.2	.3	.2	.1	.2

(a) Find $P(x \leq 22)$
(b) Find $P(x > 21)$
(c) Find $P(21 \leq x < 24)$
(d) Find $P(x \leq 21 \text{ or } x > 23)$

18. In the casino game of roulette, a ball is rolled around the rim of a circular bowl while a wheel containing 38 slots into which the ball can drop is spun in the opposite direction from the rolling ball; 18 of the slots are red, 18 are black, and 2 are green. A player bets a set amount, say \$1, and wins \$1 if the ball falls into the color slot the player has wagered on. Assume a player decides to bet that the ball will fall into one of the red slots.

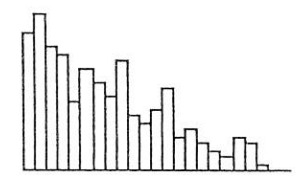
(a) What is the probability that the player will win.
(b) What is the expected return on a single bet of \$1 on red?

19. A random variable X is normally distributed with mean μ, and standard deviation s (that is, X has $N(\mu,s)$). What is the probability that a term selected at random from this population will be more than 2.5 standard deviations from the mean?

20. The normal random variable X has a standard deviation of 12. We also know that $P(x > 50) = .90$. Find the mean μ of the distribution.

CUMULATIVE REVIEW PROBLEMS

1. Consider the following histogram:



Which of the following statements is true and why?

I. The mean and median are approximately the same value
II. The mean is probably greater than the median
III. The median is probably greater than the mean.

2. You are going to do an opinion survey in your school. You can sample 100 students and desire that the sample accurately reflects the ethnic composition of your school. The school data clerk tells you that the student body is 25% Asian, 8% African American, 12% Latino, and 55% Caucasian. How could you sample the student body so that your sample of 100 would reflect this composition and what is such a sample called?

3. The following data represent the scores on a 50-point AP Statistics quiz:

46, 36, 50, 42, 46, 30, 46, 32, 50, 32, 40, 42, 20, 47, 39, 32, 22, 43, 42, 46, 48, 34, 47, 46, 27, 50, 46, 42, 20, 23, 42

Determine the 5-number summary for the quiz and draw a box plot of the data.

example: Harold fails to study for his statistics final. The final has 100 multiple choice questions, each with 5 choices. Harold has no choice but to guess randomly at all 100 questions. What is the probability that Harold will get at least 30% on the test?

solution: Because 100(.2) and 100(.8) are both greater than 10, we can use the normal approximation to the sampling distribution of $\hat{p}$. Because $p = .2$, the sampling distribution of $\hat{p}$ has = .2 and

$$\sigma_{\hat{p}} = \sqrt{\frac{.2(1-.2)}{100}} = .04.$$

Therefore,

$$P(\hat{p} > .3) = P\left(z > \frac{.3-.2}{.04} = 2.5\right) = .0062.$$

Harold should have studied.

RAPID REVIEW

1. A coin is known to be unbalanced in such a way that heads only comes up 0.4 of the time.

   (a) What is the probability the first head appears on the 4th toss?
   (b) How many tosses would it take, on average, to flip two heads?

   Answer:

   (a) $P(\text{first head appears on 4th toss}) = .4(1-.4)^{4-1} = .4(.6)^3 = .0864$
   (b) Avg. wait to flip two heads = 2 (average wait to flip one head)

   $$= 2\left(\frac{1}{.4}\right) = 5.$$

2. The coin of problem #1 is flipped 50 times. Let X be the number of heads. What is;

   (a) the probability of *exactly* 20 heads?
   (b) the probability of at *least* 20 heads?

   Answer:

   (a) $P(X = 20) = \binom{50}{20}(.4)^{20}(.6)^{30} = .115$ [or *binompdf(50, .4,20)*]

   (b) $P(X \geq 20) = \binom{50}{20}(.4)^{20}(.6)^{30} + \binom{50}{21}(.4)^{21}(.6)^{29}$

   $+ \cdots + \binom{50}{50}(.4)^{50}(.6)^{0}$ = [or 1-*binomcdf(50, .4, 19)* = .554]

3. A random variable X has $B(300, .2)$. Describe the sampling distribution of $\hat{p}$.

   Answer: Because $300(.2) = 60 \geq 10$ and $300(.8) = 240 \geq 10$, $\hat{p}$ has approximately a normal distribution with $\mu_{\hat{p}} = .2$ and $\sigma_{\hat{p}} = \sqrt{\frac{.2(1-.2)}{300}}$ = .023

4. A distribution is know to be highly skewed to the left with mean 25 and standard deviation 4. Samples of size 10 are drawn from this population and the mean of each sample is calculated. Describe the sampling distribution of $\bar{x}$.

   Answer: $\mu_{\bar{x}} = 25, \sigma_{\bar{x}} = \frac{4}{\sqrt{10}} = 1.26.$

   Since the samples are small, the shape of the sampling distribution would probably show some left-skewness but would be more mound-shaped than the original population.

5. What is the probability that a sample of size 35 drawn from a population with mean 65 and standard deviation 6 will have a mean less than 64?

   Answer: The sample size is large enough that we can use large-sample procedures. Hence,

   $$P(\bar{x} < 64) = P\left(z < \frac{64-65}{6/\sqrt{35}} = -.99\right) = .1611.$$

   [Calculator solution: *normalcdf*$\left(-100, 64, 65, 6/\sqrt{35}\right)$]

PRACTICE PROBLEMS

Multiple Choice

1. A binomial event has $n = 60$ trials. The probability of success on each trial is .4. Let X be the count of successes of the event during the 60 trials. What are μ_x and σ_x

   a. 24, 3.79
   b. 24, 14.4
   c. 4.90, 3.79
   d. 4.90, 14.4
   e. 2.4, 3.79

2. Consider repeated trials of a binomial random variable. Suppose the probability of the first success occurring on the second trial is .25, what is the probability of success on the first trial?

   a. ¼
   b. 1
   c. ½
   d. ⅛
   e. 3/16

3. To use a normal approximation to the binomial, which of the following does *not* have to be true?

   a. $np \geq 5,\ n(1-p) \geq 5$
   b. The individual trials must be independent.
   c. The sample size in the problem must be too large to permit doing the problem on a calculator.
   d. For the binomial, the population size must be at least 10 times as large as the sample size.
   e. All of the above are true.

4. You form a distribution of the means of all samples of size 9 drawn from an infinite population that is skewed to the left (like the scores on an easy Stat quiz!). The population from which the samples are drawn has a mean of 50 and a standard deviation of 12. Which one of the following statements is true of this distribution?

   a. $\mu_{\bar{x}} = 50$, $\sigma_{\bar{x}} = 12$, the sampling distribution is skewed somewhat to the left.
   b. $\mu_{\bar{x}} = 50$, $\sigma_{\bar{x}} = 4$, the sampling distribution is skewed somewhat to the left.
   c. $\mu_{\bar{x}} = 50$, $\sigma_{\bar{x}} = 12$, the sampling distribution is approximately normal.
   d. $\mu_{\bar{x}} = 50$, $\sigma_{\bar{x}} = 4$, the sampling distribution is approximately normal.
   e. $\mu_{\bar{x}} = 50$, $\sigma_{\bar{x}} = 4$, the sample size is too small to make any statements about the shape of the sampling distribution.

5. A 12-sided die has faces numbered from 1–12. Assuming the die is fair (that is, each face is equally likely to appear each time), which of the following would give the exact probability of getting at least 10 3s out of 50 rolls?

   a. $\binom{50}{0}(.083)^0(.917)^{50} + \binom{50}{1}(.083)^1(.917)^{49} + \cdots + \binom{50}{9}(.083)^9(.917)^{41}$

   b. $\binom{50}{11}(.083)^{11}(.917)^{39} + \binom{50}{12}(.083)^{12}(.917)^{38} + \cdots + \binom{50}{50}(.083)^{50}(.917)^0$

   c. $1 - \left(\binom{50}{0}(.083)^0(.917)^{50} + \binom{50}{1}(.083)^1(.917)^{49} + \cdots + \binom{50}{10}(.083)^{10}(.917)^{40}\right)$

   d. $1 - \left(\binom{50}{0}(.083)^0(.917)^{50} + \binom{50}{1}(.083)^1(.917)^{49} + \cdots + \binom{50}{10}(.083)^9(.917)^{41}\right)$

   e. $\binom{50}{0}(.083)^0(.917)^{50} + \binom{50}{1}(.083)^1(.917)^{49} + \cdots + \binom{50}{10}(.083)^{10}(.917)^{40}$

Free Response

1. A factory manufacturing tennis balls determines that the probability that a single can of three balls will contain at least one defective ball is .025. What is the probability that a case of 48 cans will contain at least two cans with a defective ball?

2. A sampling distribution is highly skewed to the left. Describe the shape of the sampling distribution of $\bar{x}$ if the sample size is (a) 3 or (b) 30.

3. Suppose you had gobs of time on your hands and decided to flip a coin 1,000,000 times and note whether each flip was a head or a tail. Let X be the count of heads. What is the probability that there are at least 1000 more heads than tails? (*Note:* this is a binomial but your calculator will not be able to do the binomial computation because the numbers are too large for it).

4. In Section 7.4, we had an example in which we asked if it would change the proportion of girls in the population (assumed to be .5) if families continued to have children until they had a girl and then they stopped. That problem was to be done by simulation. How could you use what you know about the geometric distribution to answer this same question?

5. At a school better know for football than academics (a school its football team can be proud of), it is known that only 20% of the scholarship athletes graduate within 5 years. The school is able to give 55 scholarships for football. What are the expected mean and standard deviation of the number of graduates for a group of 55 scholarship athletes?

6. Consider a population consisting of the numbers 2,4,5, and 7. List all possible samples of size two from this population and compute the mean and standard deviation of the sampling distribution of $\bar{x}$. Compare this with the values obtained by relevant formulas for the sampling distribution of $\bar{x}$. Note that the sample size is large relative to the population—this may affect how you compute $\sigma_{\bar{x}}$ by formula.

7. Approximately 10% of the population of the United States is known to have blood type B. If this is correct, what is the probability that between 11% and 15% of a random sample of 50 adults will have type B blood?

8. Which of the following is/are true of the central limit theorem? (More than one might be true.)

I. $\mu_{\bar{x}} = \mu$

II. $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$

III. The sampling distribution of a sample mean will be approximately normally distributed for sufficiently large samples, regardless of the shape of the original population.

IV. The sampling distribution of a sample mean will be normally distributed if the population from which the samples are drawn is normal.

9. A brake inspection station reports that 15% of all cars tested have brakes in need of replacement pads. For a sample of 20 cars that come to the inspection station,

(a) What is the probability that exactly 3 have defective breaks?
(b) What is the mean and standard deviation of cars that need replacement pads?

10. A tire manufacturer claims that his tires will last 40,000 miles with a standard deviation of 5000 miles.

(a) Assuming that the claim is true, describe the sampling distribution of the mean lifetime of a random sample of 160 tires. "Describe" means discuss center, spread, and shape.
(b) What is the probability that the mean life time of the sample of 160 tires will be less than 39,000 miles? Interpret the probability in terms of the truth of the manufacturer's claim.

11. The probability of winning a bet on red in roulette is .474. The binomial probability of winning money if you play 10 games is .31 and drops to .27 if you play 100 games. Use a normal approximation to the binomial to estimate your probability of coming out ahead (that is, winning more than ½ of your bets) if you play 1000 times. Justify being able to use a normal approximation for this situation.

12. Crabs off the coast of Northern California have a mean weight of 2 lbs. with a standard deviation of 5 oz. A large trap captures 35 crabs.

(a) Describe the sampling distribution for the average weight of a random sample of 35 crabs taken from this population.
(b) What would the mean weight of a sample of 35 crabs have to be in order to be in the top 10% of all such samples?

13. The probability that a person recovers from a particular type of cancer operation is .7. Suppose 8 people have the operation. What is the probability that

(a) exactly 5 recover?
(b) they all recover?
(c) at least one of them recovers?

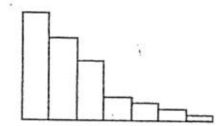
14. A certain type of light bulb is advertised to have an average life of 1200 hours. If, in fact, light bulbs of this type only average 1185 hours with a standard deviation of 80 hours, what is the probability that a sample of 100 bulbs will have an average life of at least 1200 hours?

15. Your task is to explain to your friend Gretchen, who knows virtually nothing (and cares even less) about statistics, just what a sampling distribution of the mean is. Explain the idea of a sampling distribution in such a way that even Gretchen, if she pays attention, will understand.

16. Consider the distribution shown at the right:

Describe the sampling distribution of $\bar{x}$ for samples of size n if

(a) $n = 3$
(b) $n = 40$



17. After the Challenger disaster of 1986, it was discovered that the explosion was caused by defective O-rings. The probability that a single O-ring was defective and would fail (with catastrophic consequences) was .003 and there were 12 of them (6 outer and 6 inner). What was the probability that at least one of the O-rings would fail (as it actually did)?

18. Your favorite cereal has a little prize in each box. There are 5 such prizes. Each box is equally likely to contain any one of the prizes. So far, you have been able to collect 2 of the prizes. What is:

(a) the probability that you will get the third different prize on the next box you buy?
(b) the average number of boxes of cereal you will have to buy before getting the third prize?

19. We wish to approximate the binomial distribution $B(40, .8)$ with a normal curve $N(\mu,\sigma)$, Is this an appropriate approximation and, if so, what are μ and σ for the approximating normal curve?

20. Opinion polls in 2002 showed that about 70% of the population had a favorable opinion of President Bush. That same year, a simple random sample of 600 adults living in the San Francisco Bay Area showed found only 65% that had a favorable opinion of President Bush. What is the probability of getting a rating of 65% or less in a random sample of this size if the true proportion in the population was .70?

CUMULATIVE REVIEW PROBLEMS

1. Toss three fair coins and let X be the count of heads among the three coins. Construct the probability distribution for this experiment.

2. You are doing a survey for your school newspaper and want to select a sample of 25 seniors. You decide to do this by randomly selecting 5 students from each of the 5 senior-level classes, each of which contains 28 students. The school data clerk assures you that students have been randomly assigned, by computer, to each of the 5 classes. Is this sample

   (a) a random sample?
   (b) a simple random sample?

3. Data are collected in an experiment to measure a person's reaction time (in seconds) as a function of the number of milligrams of a new drug. The least-squares regression line for the data is *Reaction Time* = $-.2 + .8(mg)$. Interpret the slope of the regression line in the context of the situation.

4. If $P(A) = .5$, $P(B) = .3$, and $P(A \text{ or } B) = .65$, are events A and B *independent*?

5. Which of the following examples of *quantitative data* and which are examples of *qualitative data*?

   a. The height of an individual, measured in inches.
   b. The color of the shirts in my closet.
   c. The outcome of a flip of a coin described as "heads" or "tails."
   d. The value of the change in your pocket.
   e. Individuals, after they are weighed, are identified as thin, normal, or heavy.
   f. Your pulse rate.
   g. Your religion.