2016 Q3

(a) each flight is a "trial"
- $p = P(\text{upgrade}) = 0.10$ constant & independent
- multiple trials
- finding probability related to when 1st success occurs

this is a Geometric model scenario w/ $p = 0.10$

let X = number of flights until 1st upgrade

$$\begin{array}{c|cccccc} X & 1^{st} & 2^{nd} & 3 & 4 & 5 & 6 \ldots \\ \hline & & & & & & \end{array}$$

$P(X>3) = 1 - P(X \leq 3)$
$= 1 - \text{geometcdf}(\underset{p}{0.10}, \underset{x}{3})$
$= 1 - .271 = \boxed{.729}$

or by formula $(q^{x-1})(p)^1$

$P(X>3) = 1 - P(X \leq 3)$
$= 1 - [P(X=1) + P(X=2) + P(X=3)]$
$= 1 - [(.1) + (.9)(.1) + (.9)^2(.1)]$
$= 1 - .271 = \boxed{.729}$

(AP may prefer the hand calculation, but they state calculator methods are ok too, as long as all values are labelled)

(b) now finding a probability of a number of upgrades in 20 flights, so switch to Binomial model:

Let Y = # of upgrades in 20 flights:

$$\begin{array}{c|cccccccc} Y & 0 & 1 & 2 & 3 & 4 & 5 & \ldots & 19 \quad 20 \\ \hline & & & & & & & & \end{array}$$

$P(Y=2) = \text{binompdf}(\underset{n}{20}, \underset{p}{.10}, \underset{x}{2}) = \boxed{.2852}$

or

$= {}_{20}C_2 (.1)^2 (.9)^{18} = 190(.1)^2(.9)^{18} = \boxed{.2852}$

(c) This is again a binomial scenario with $n = 104$, $p = .10$ so let W = # of upgrades in 104 flights:

$$\begin{array}{c|cccccccc} W & 0 & 1 & 2 & \ldots & 19 & 20, & 21 & \ldots\ 104 \\ \hline & & & & & & & & \end{array}$$

$P(W>20) = 1 - P(W \leq 20)$
$= 1 - \text{binomcdf}(\underset{n}{104}, \underset{p}{.10}, \underset{x}{20}) = 1 - .99863 = .00136$

the probability of more than 20 upgrades in 104 flights is less than 1%. so I would be surprised if Sam receives more than 20 upgrades.