Calculus 2 Unit 9 Part 3 (Vectors) Test Review Name_

Period

#1. A sailboat starts sailing from a point on the shore of a large, calm lake. A person on the boat wishes to sail to an island in the middle of the lake which is 5 miles to the South and 3 miles to the West of the starting point of the sailboat on the shore. There is a steady 8 mile per hour wind blowing in a direction from Southeast directly towards the Northwest.

(a) In what direction should the person steer the sailboat so that they end up at the island?

(Report the sailboat's velocity vector in the form <x-component, y-component> where x is in the East direction and y is in the North direction)

(b) What is the required speed of the sailboat?

#2. An object is moving in the x-y plane with a velocity vector, for $-\pi \le t \le \pi$, is given by $\overrightarrow{v}(t) = \langle 3\cos(2t) + 2, 4\sin(2t) + 3 \rangle$.

The position of the object at time $t = \frac{\pi}{4}$ seconds is $\left(\frac{5}{2} + \frac{\pi}{2}, 3 + \frac{3\pi}{4}\right)$.

(a) Find the position of the object at time $t = \frac{\pi}{3}$ seconds.

(b) Find the acceleration vector of the object at any time t.

(c) For what time(s), t > 0, is the tangent line to the object's path horizontal?

(d) For what time(s), t > 0, is the tangent line to the object's path vertical?

(e) For what time(s), t > 0, does the tangent line to the object's path have a slope of -2?

(f) Set up (but do not evaluate) an integral for the total distance traveled by the object for $-1 \le t \le 2$.

#3. (a) Sketch the plane curve for the position vector $\vec{r}(t) = \langle 8\sin t, 3\cos t \rangle$.

(b) Find the velocity vector, $\vec{v}(t)$.

(c) To your sketch in part a, add: $\overrightarrow{r}\left(\frac{2\pi}{3}\right)$, $\overrightarrow{v}\left(\frac{2\pi}{3}\right)$, $\overrightarrow{a}\left(\frac{2\pi}{3}\right)$.

#4. Given vectors $\overrightarrow{a} = \langle -3, 6 \rangle$, $\overrightarrow{b} = \langle 2, -3 \rangle$, $\overrightarrow{c} = \langle 1, 5 \rangle$ (a) Find $2\overrightarrow{a} - 3\overrightarrow{b} + 5\overrightarrow{c}$ (b) Find magnitude $\|\overrightarrow{a}\|$

#5. Given position vector $\overrightarrow{r}(t) = \langle 5t^2 - t, e^t + \sin(2t) \rangle$

(a) Find $\vec{r}(-2)$ (b) Find $\vec{r}(2m-3n)$