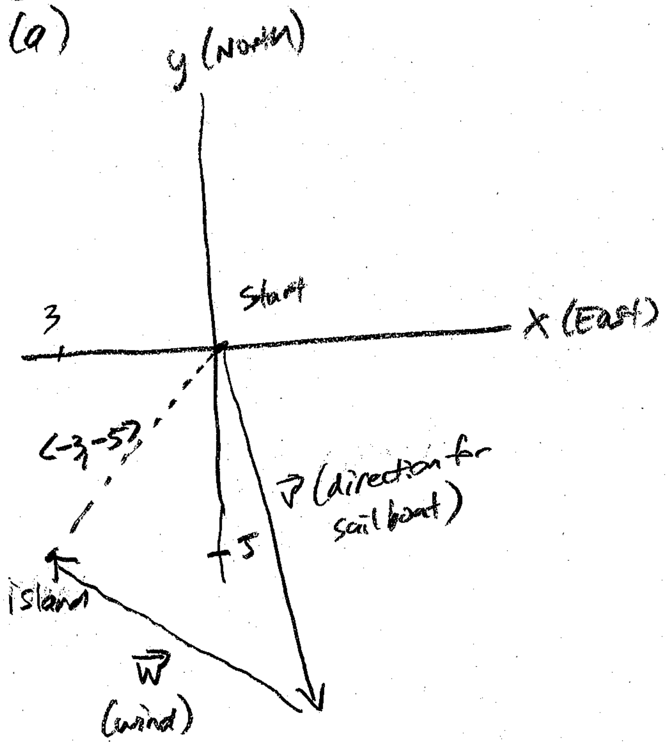
Unit 9 Part 3 (vectors) Test Review (SOLUTION)

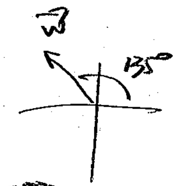
#1

(a)



$\vec{v} + \vec{w} = \langle -3, -5 \rangle$

$|\vec{w}| = 8$ mph



$\vec{w} = \langle 8\cos 135°, 8\sin 135° \rangle$

$\vec{w} = \langle 8(-\frac{\sqrt{2}}{2}), 8(\frac{\sqrt{2}}{2}) \rangle = \langle -4\sqrt{2}, 4\sqrt{2} \rangle$

so...

$\vec{v} + \langle -4\sqrt{2}, 4\sqrt{2} \rangle = \langle -3, -5 \rangle$

$\vec{v} = \langle -3, -5 \rangle - \langle -4\sqrt{2}, 4\sqrt{2} \rangle$

$\boxed{\vec{v} = \langle -3 + 4\sqrt{2}, -5 - 4\sqrt{2} \rangle \text{ mph}}$

$\approx \langle 2.657, -10.657 \rangle$ mph

(b) $\boxed{\text{speed} = |\vec{v}(t)| = \sqrt{(-3+4\sqrt{2})^2 + (-5-4\sqrt{2})^2} \text{ mph}}$

≈ 10.983 mph

#2 $\vec{v}(t) = \langle 3\cos(2t)+2,\ 4\sin(2t)+3\rangle$, $\vec{r}(\frac{\pi}{4}) = \langle \frac{5}{2}+\frac{\pi}{2},\ 3+\frac{3\pi}{4}\rangle$

(a) $\vec{r}(t) = \int \vec{v}(t)\,dt = \langle \int(3\cos(2t)+2)\,dt,\ \int(4\sin(2t)+3)\,dt\rangle$

$\vec{r}(t) = \langle 3\frac{\sin(2t)}{2} + 2t + C_1,\ 4(\frac{-\cos(2t)}{2} + 3t) + C_2\rangle$

$= \langle \frac{3}{2}\sin(2t)+2t,\ -2\cos(2t)+3t\rangle + \vec{C}$

at $t = \pi/4$

$\langle \frac{5}{2}+\frac{\pi}{2},\ 3+\frac{3\pi}{4}\rangle = \langle \frac{3}{2}\sin(2(\frac{\pi}{4})) + 2(\frac{\pi}{4}),\ -2\cos(2\frac{\pi}{4}) + 3(\frac{\pi}{4})\rangle + \vec{C}$

$\langle \frac{5}{2}+\frac{\pi}{2},\ 3+\frac{3\pi}{4}\rangle = \langle \frac{3}{2}\sin(\frac{\pi}{2}) + \frac{\pi}{2},\ -2\cos(\frac{\pi}{2}) + \frac{3\pi}{4}\rangle + \vec{C}$

$\langle \frac{5}{2}+\frac{\pi}{2},\ 3+\frac{3\pi}{4}\rangle = \langle \frac{3}{2} + \frac{\pi}{2},\ 0 + \frac{3\pi}{4}\rangle + \vec{C}$

so $\vec{C} = \langle \frac{5}{2}+\frac{\pi}{2},\ 3+\frac{3\pi}{4}\rangle - \langle \frac{3}{2}+\frac{\pi}{2},\ 0+\frac{3\pi}{4}\rangle$

$\vec{C} = \langle 1, 3\rangle$

and $\vec{r}(t) = \langle \frac{3}{2}\sin(2t)+2t,\ -2\cos(2t)+3t\rangle + \langle 1,3\rangle$

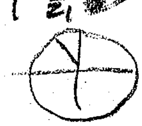
$\vec{r}(t) = \langle \frac{3}{2}\sin(2t) + 2t + 1,\ -2\cos(2t)+3t+3\rangle$

now find position at $t = \pi/3$ secs

$\vec{r}(\frac{\pi}{3}) = \langle \frac{3}{2}\sin(2(\frac{\pi}{3})) + 2(\frac{\pi}{3}) + 1,\ -2\cos(2(\frac{\pi}{3})) + 3(\frac{\pi}{3}) + 3\rangle$

$= \langle \frac{3}{2}\sin(\frac{2\pi}{3}) + \frac{2\pi}{3} + 1,\ -2\cos(\frac{2\pi}{3}) + \pi + 3\rangle$

$(-\frac{1}{2},$



$= \langle \frac{3}{2}(\frac{\sqrt{3}}{2}) + \frac{2\pi}{3} + 1,\ -2(-\frac{1}{2}) + \pi + 3\rangle$

$= \langle \frac{3\sqrt{3}}{4} + \frac{2\pi}{3} + 1,\ 1 + \pi + 3\rangle$

$= \langle \frac{3\sqrt{3}}{4} + \frac{2\pi}{3} + 1,\ 4 + \pi\rangle$

any of these answers are fine

$\vec{r}(\frac{\pi}{3}) \approx \langle 4.393,\ 7.142\rangle$

or can express as a coordinate point:

$\boxed{(\frac{3\sqrt{3}}{4} + \frac{2\pi}{3} + 1,\ 4+\pi)}$

#2 (continued)

(b) $\vec{a}(t) = \frac{d}{dt}[\vec{v}(t)] = \langle \frac{d}{dt}[3\cos(2t)+2], \frac{d}{dt}[4\sin(2t)+3] \rangle$

$\vec{a}(t) = \langle 3(-\sin(2t)\cdot 2 + 0), 4(\cos(2t)\cdot 2 + 0) \rangle$

$\boxed{\vec{a}(t) = \langle -6\sin(2t), 8\cos(2t) \rangle}$

(c) $m = \frac{(dy/dt)}{(dx/dt)} = \frac{4\sin(2t)+3}{3\cos(2t)+2}$ horiz tangent when $m = 0$, when $4\sin(2t)+3 = 0$

for $-\pi \le t \le \pi$, by calculator graph, this occurs when $\boxed{t = -1.147, \ t = -0.424, \ t = 1.995, \ t = 2.718 \text{ secs}}$

(d) vertical tangent when $3\cos(2t)+2 = 0$

for $-\pi \le t \le \pi$, by calculator graph, this occurs when $\boxed{t = -1.991, \ t = -1.150, \ t = 1.150, \ t = 1.991 \text{ secs}}$

(e) when $\frac{4\sin(2t)+3}{3\cos(2t)+2} = -2$

for $-\pi \le t \le \pi$, by calculator graph, this occurs when $\boxed{t = -1.398, \ t = 1.744 \text{ secs}}$

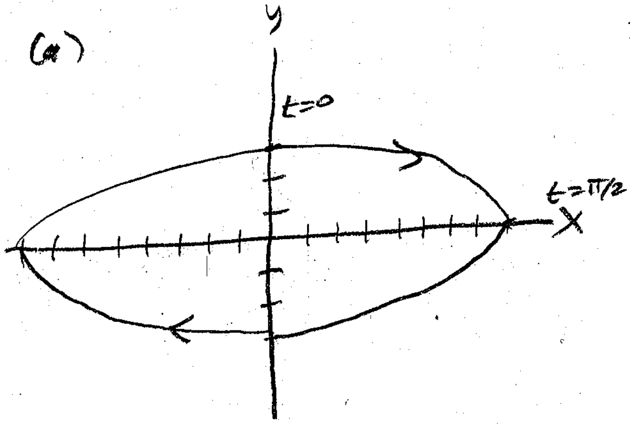
(f) total distance traveled $= \int_a^b |\vec{v}(t)| dt$

$$\boxed{\int_{-1}^{2} \sqrt{(3\cos(2t)+2)^2 + (4\sin(2t)+3)^2} \, dt}$$

#3 $\vec{r}(t) = \langle 8\sin t, 3\cos t \rangle$

t	(x,y)
0	$(0,3)$
$\frac{\pi}{2}$	$(8,0)$

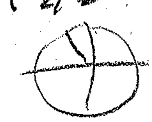
(a)



(b) $\vec{v}(t) = \langle \frac{d}{dt}[8\sin t], \frac{d}{dt}[3\cos t] \rangle$

$\boxed{\vec{v}(t) = \langle 8\cos t, -3\sin t \rangle}$

$\langle -\frac{1}{2}, \frac{\sqrt{3}}{2} \rangle$



(c) $\vec{r}\left(\frac{2\pi}{3}\right) = \langle 8\sin\left(\frac{2\pi}{3}\right), 3\cos\left(\frac{2\pi}{3}\right) \rangle = \langle 8\left(\frac{\sqrt{3}}{2}\right), 3\left(-\frac{1}{2}\right) \rangle$

$= \langle 6.9282, -1.5 \rangle$

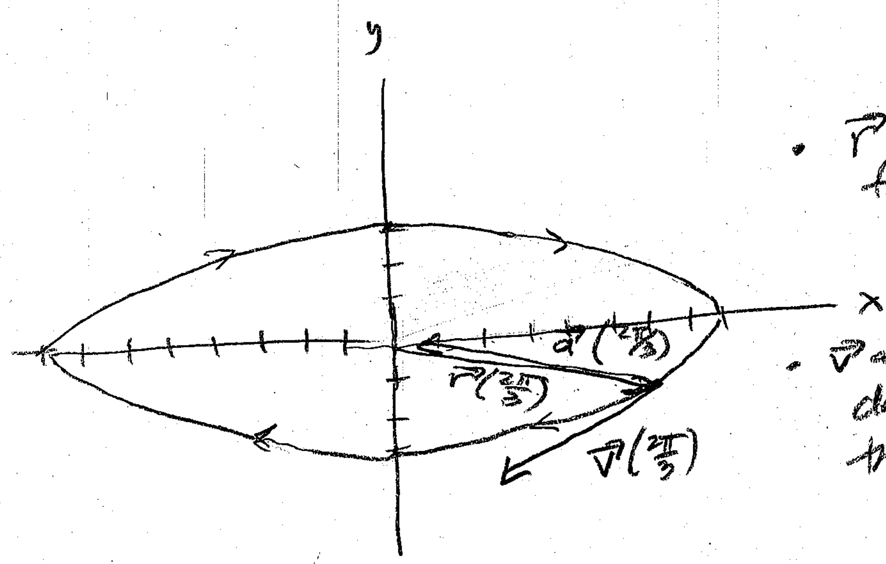
$\vec{v}\left(\frac{2\pi}{3}\right) = \langle 8\cos\left(\frac{2\pi}{3}\right), -3\sin\left(\frac{2\pi}{3}\right) \rangle = \langle 8\left(-\frac{1}{2}\right), -3\left(\frac{\sqrt{3}}{2}\right) \rangle$

$= \langle -4, -2.598 \rangle$

$\vec{a}(t) = \langle -8\sin t, -3\cos t \rangle$

$\vec{a}\left(\frac{2\pi}{3}\right) = \langle -8\sin\left(\frac{2\pi}{3}\right), -3\cos\left(\frac{2\pi}{3}\right) \rangle = \langle -8\left(\frac{\sqrt{3}}{2}\right), -3\left(-\frac{1}{2}\right) \rangle$

$= \langle -6.9282, 1.5 \rangle$



- $\vec{r}$ is always drawn from origin to the object
- $\vec{v}$ and $\vec{a}$ are always drawn starting from the object.

#4 $\vec{a} = \langle -3, 6 \rangle$ $\vec{b} = \langle 2, -3 \rangle$ $\vec{c} = \langle 1, 5 \rangle$

(a) $2\vec{a} - 3\vec{b} + 5\vec{c}$

$2\langle -3, 6 \rangle - 3\langle 2, -3 \rangle + 5\langle 1, 5 \rangle$

$\langle -6, 12 \rangle + \langle -6, 9 \rangle + \langle 5, 25 \rangle$

$\langle -6 - 6 + 5, 12 + 9 + 25 \rangle = \boxed{\langle -7, 46 \rangle}$

(b) $|\vec{a}| = \boxed{\sqrt{(-3)^2 + (6)^2}} = \sqrt{9 + 36} = \sqrt{45}$

#5 $\vec{r}(t) = \langle 5t^2 - t, e^t + \sin(2t) \rangle$

(a) $\vec{r}(-2) = \boxed{\langle 5(-2)^2 - (-2), e^{-2} + \sin(2(-2)) \rangle}$

$= \langle 22, e^{-2} + \sin(-4) \rangle$

(b) $\vec{r}(2m - 3n) = \boxed{\langle 5(2m - 3n)^2 - (2m - 3n), e^{(2m-3n)} + \sin(2(2m - 3n)) \rangle}$