AP Calculus BC

9.6 Worksheet

Name: full solutions

Period: ____________

Show that **u** and **v** are equivalent.

1. **u** : (3,2), (5,6)

   v = (1,4), (3,8)

$\vec{u} = \langle 5-3, 6-2 \rangle = \langle 2,4 \rangle$

$\vec{v} = \langle 3-2, 8-4 \rangle = \langle 2,4 \rangle$ ✓

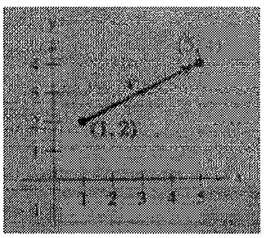
2. **u** = (-4,0), (1,8)

   v = (2,-1), (7,7)

$\vec{u} = \langle 1-(-4), 8-0 \rangle = \langle 5,8 \rangle$ ✓

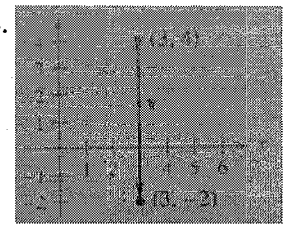
$\vec{v} = \langle 7-2, 7-(-1) \rangle = \langle 5,8 \rangle$

Find the component form of the vector **v** and sketch the vector with its initial point at the origin.

3. 

$\vec{v} = \langle 5-1, 4-2 \rangle$

$\boxed{\vec{v} = \langle 4, 2 \rangle}$

4. 

$\vec{v} = \langle -2-4, 3-3 \rangle$

$\boxed{\vec{v} = \langle -6, 0 \rangle}$

Find the magnitude of the vector **v**.

5. **v** = <7, 0>

$|\vec{v}| = \sqrt{7^2+0^2} = \boxed{7}$

6. **v** = <-3, 0>

$|\vec{v}| = \sqrt{(-3)^2+0^2} = \boxed{3}$

7. **v** = <4, 3>

$|\vec{v}| = \sqrt{4^2+3^2} = \boxed{5}$

8. **v** = <12, -5>

$|\vec{v}| = \sqrt{12^2+5^2} = \boxed{13}$

9. **v** = <6, -5>

$|\vec{v}| = \sqrt{6^2+(-5)^2} = \boxed{\sqrt{61}}$

10. **v** = <-10, 3>

$\sqrt{(-10)^2+(3)^2} = \boxed{\sqrt{109}}$

Perform the following operations on the vectors: 2/3**u**, 3**v**, **v** – **u**, 2**u** + 5**v**.

11. **u** = <4, 9>

    v = <2, -5>

$\frac{2}{3}\vec{u} = \frac{2}{3}\langle 4,9 \rangle = \boxed{\langle \frac{8}{3}, \frac{18}{3} \rangle}$

$3\vec{v} = 3\langle 2,-5 \rangle = \boxed{\langle 6,-15 \rangle}$

$\vec{v} - \vec{u} = \langle 2,-5 \rangle - \langle 4,9 \rangle = \boxed{\langle -2,-14 \rangle}$

$2\vec{u} + 5\vec{v} = 2\langle 4,9 \rangle + 5\langle 2,-5 \rangle$

$= \langle 8,18 \rangle + \langle 10,-25 \rangle$

$= \boxed{\langle 18, -7 \rangle}$

12. **u** = <-3, -8>

    v = <8, 25>

$\frac{2}{3}\vec{u} = \frac{2}{3}\langle -3,-8 \rangle = \boxed{\langle -2, \frac{-16}{3} \rangle}$

$3\vec{v} = 3\langle 8,25 \rangle = \boxed{\langle 24,75 \rangle}$

$\vec{v} - \vec{u} = \langle 8,25 \rangle - \langle -3,-8 \rangle = \boxed{\langle 11,33 \rangle}$

$2\vec{u} + 5\vec{v} = 2\langle -3,-8 \rangle + 5\langle 8,25 \rangle$

$= \langle -6,-16 \rangle + \langle 40, 125 \rangle$

$= \boxed{\langle 34, 109 \rangle}$

Find the following: $\|\boldsymbol{u}\|, \|\boldsymbol{v}\|, \|\boldsymbol{u}+\boldsymbol{v}\|, \left\|\frac{u}{\|u\|}\right\|, \left\|\frac{v}{\|v\|}\right\|, \left\|\frac{u+v}{\|u+v\|}\right\|$

13. **u** = <1, -1> $|\vec{u}| = \sqrt{1^2+1^2} = \boxed{\sqrt{2}}$

v = <-1, 2> $|\vec{v}| = \sqrt{1^2+2^2} = \boxed{\sqrt{5}}$

$|\vec{u}+\vec{v}| = |\langle 1,-1\rangle + \langle -1,2\rangle| = |\langle 0,1\rangle| = \sqrt{0^2+1^2} = \boxed{1}$

$\left|\frac{\vec{u}}{|\vec{u}|}\right| = \left|\frac{\langle 1,-1\rangle}{\sqrt{2}}\right| = \left|\langle \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\rangle\right| = \sqrt{(\frac{1}{\sqrt{2}})^2+(\frac{1}{\sqrt{2}})^2}$

$= \sqrt{\frac{1}{2}+\frac{1}{2}} = \sqrt{1} = \boxed{1}$

$\left|\frac{\vec{v}}{|\vec{v}|}\right| = \left|\frac{\langle -1,2\rangle}{\sqrt{5}}\right| = \left|\langle \frac{-1}{\sqrt{5}}, \frac{2}{\sqrt{5}}\rangle\right| = \sqrt{(\frac{-1}{\sqrt{5}})^2+(\frac{2}{\sqrt{5}})^2}$

$= \sqrt{\frac{1}{5}+\frac{4}{5}} = \sqrt{1} = \boxed{1}$

$\left|\frac{\vec{u}+\vec{v}}{|\vec{u}+\vec{v}|}\right| = \left|\frac{\langle 0,1\rangle}{1}\right| = |\langle 0,1\rangle| = \sqrt{0^2+1^2} = \sqrt{1} = \boxed{1}$

14. **u** = <0, 1> $|\vec{u}| = \sqrt{0^2+1^2} = \boxed{1}$

v = <3, -3> $|\vec{v}| = \sqrt{3^2+3^2} = \sqrt{18}$

$|\vec{u}+\vec{v}| = |\langle 0,1\rangle + \langle 3,-3\rangle| = |\langle 3,-2\rangle|$

$= \sqrt{3^2+2^2} = \sqrt{13}$

$\left|\frac{\vec{u}}{|\vec{u}|}\right| = \left|\frac{\langle 0,1\rangle}{1}\right| = |\langle 0,1\rangle| = \sqrt{0^2+1^2} = \sqrt{1} = \boxed{1}$

$\left|\frac{\vec{v}}{|\vec{v}|}\right| = \left|\frac{\langle 3,-3\rangle}{\sqrt{18}}\right| = \left|\langle \frac{3}{\sqrt{18}}, \frac{-3}{\sqrt{18}}\rangle\right|$

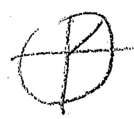
$= \sqrt{(\frac{3}{\sqrt{18}})^2+(\frac{-3}{\sqrt{18}})^2} = \sqrt{\frac{9}{18}+\frac{9}{18}} = \sqrt{1} = \boxed{1}$

$\left|\frac{\vec{u}+\vec{v}}{|\vec{u}+\vec{v}|}\right| = \left|\frac{\langle 3,-2\rangle}{\sqrt{13}}\right| = \left|\langle \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}}\rangle\right|$

$= \sqrt{(\frac{3}{\sqrt{13}})^2+(\frac{-2}{\sqrt{13}})^2} = \sqrt{\frac{9}{13}+\frac{4}{13}} = \sqrt{1} = \boxed{1}$

Find the vector **v**, given its magnitude and direction.

15. $\|v\| = 3, \theta = 45°$



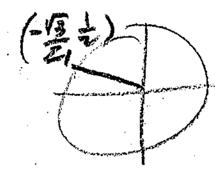
$\vec{v} = \langle r\cos\theta, r\sin\theta\rangle$

$\vec{v} = \langle 3\cos 45°, 3\sin 45°\rangle$

$\vec{v} = \langle 3(\frac{\sqrt{2}}{2}), 3(\frac{\sqrt{2}}{2})\rangle$

$\boxed{\vec{v} = \langle \frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\rangle}$

16. $\|v\| = 2, \theta = 150°$



$\vec{v} = \langle r\cos\theta, r\sin\theta\rangle$

$= \langle 2\cos 150°, 2\sin 150°\rangle$

$= \langle 2(-\frac{\sqrt{3}}{2}), 2(\frac{1}{2})\rangle$

$\boxed{\vec{v} = \langle -\sqrt{3}, 1\rangle}$

AP Calculus BC   Name: ____________________

9.7 Worksheet   Period: __________

Find the domain of the vector-valued function.

1. $r(t) = \langle \frac{1}{t+1}, \frac{t}{2} \rangle$

$\boxed{D: (-\infty,-1)\cup(-1,\infty)}$ $t \neq -1$

2. $r(t) = \langle \sqrt{4-t^2}, t^2 \rangle$

$4-t^2=0$

$t^2=4,\ t=\pm 2$

$-\infty$ ✗ ✓ ✗ ∞

(-3) -2 (0) 2 (3)

$\boxed{D: [-2,2] \text{ or } -2 \le t \le 2}$

3. $r(t) = \langle \ln(t), -e^t \rangle$

$\boxed{D: (0,\infty)}$

4. $r(t) = \langle \sin(t), \cos(t) \rangle$

$\boxed{D: (-\infty,\infty)}$

Evaluate (if possible) the vector-valued function at each given time t.

5. $r(t) = \langle \frac{1}{2}t^2, -t+1 \rangle$

a. r(1) = $\langle \frac{1}{2}(1)^2, -(1)+1 \rangle$

$= \boxed{\langle \frac{1}{2}, 0 \rangle}$

b. r(0) = $\langle \frac{1}{2}(0)^2, -(0)+1 \rangle$

$= \boxed{\langle 0, 1 \rangle}$

c. r(s + 1) = $\langle \frac{1}{2}(s+1)^2, -(s+1)+1 \rangle$

$= \boxed{\langle \frac{1}{2}(s+1)^2, -s \rangle}$

d. r(2 + Δt) – r(2) = $\langle \frac{1}{2}(2+\Delta t)^2, -(2+\Delta t)+1 \rangle - \langle \frac{1}{2}(2)^2, -(2)+1 \rangle$

$= \langle \frac{1}{2}(2+\Delta t)^2 - 2, -2-\Delta t+1+1 \rangle$

$= \boxed{\langle \frac{1}{2}(2+\Delta t)^2 - 2, -\Delta t \rangle}$

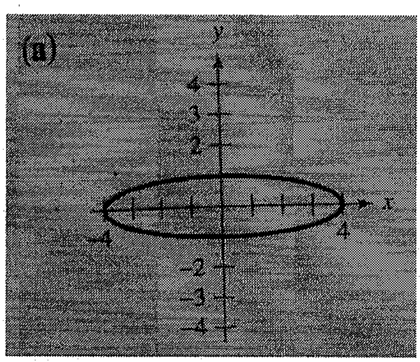
6. Match the equation with its graph. (use calculator parametric mode to graph)

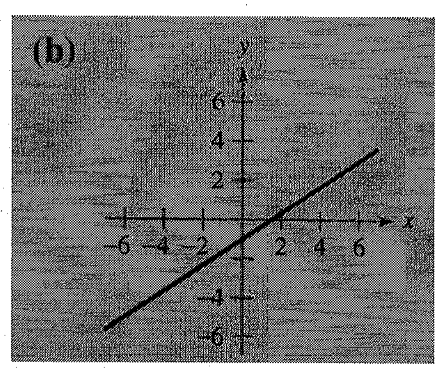
$r(t) = \langle 3t, 2t-1 \rangle$ __b__

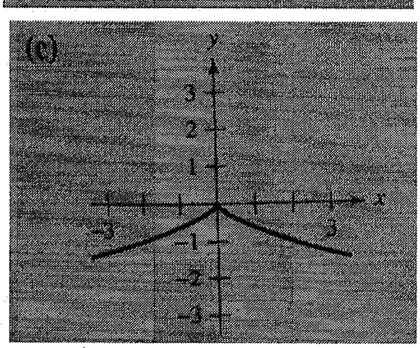
$r(t) = \langle 2t^3, -t^2 \rangle$ __c__

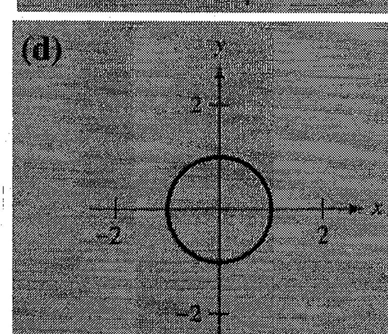
$r(t) = \langle \cos(t), \sin(t) \rangle$ __d__

$r(t) = \langle 4\cos(t), \sin(t) \rangle$ __a__



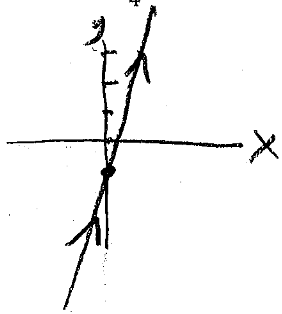




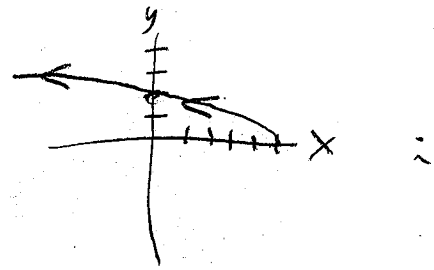


Sketch the plane curve represented by the vector-valued function and give the orientation of the curve.

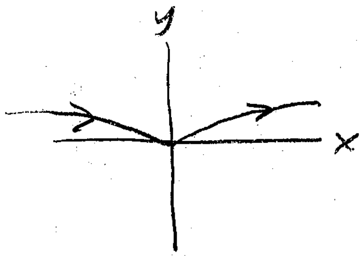
7. $\boldsymbol{r}(t) = \langle \frac{t}{4}, t-1 \rangle$



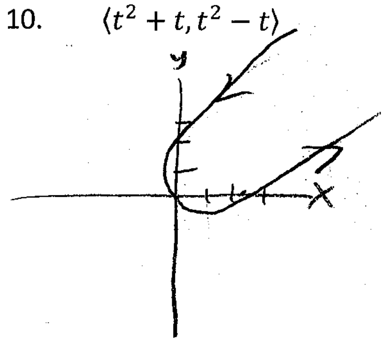
8. $\boldsymbol{r}(t) = \langle 5-t, \sqrt{t} \rangle$



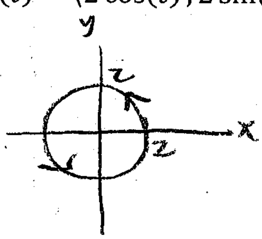
9. $\boldsymbol{r}(t) = \langle t^3, t^2 \rangle$



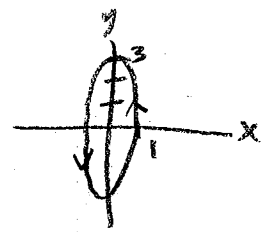
10. $\langle t^2 + t, t^2 - t \rangle$



11. $\boldsymbol{r}(t) = \langle 2\cos(t), 2\sin(t) \rangle$



12. $\langle \cos(t), 3\sin(t) \rangle$



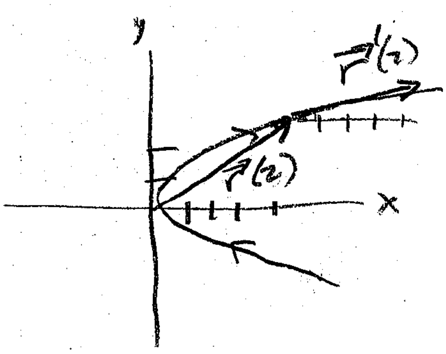
Find **r**′(t), **r**(t₀) and **r**′(t₀). Then sketch the plane curve represented by **r**(t) and sketch the vectors **r**(t₀) and **r**′(t₀). Position the vectors such that the initial point of **r**(t₀) is at the origin, and the initial point of **r**′(t₀) is at the terminal point of **r**′(t₀).

13. $\boldsymbol{r}(t) = \langle t^2, t \rangle, t_0 = 2$

$\vec{r}'(t) = \langle 2t, 1 \rangle$

$\vec{r}(2) = \langle 2^2, 2 \rangle = \langle 4, 2 \rangle$

$\vec{r}'(2) = \langle 2(2), 1 \rangle = \langle 4, 1 \rangle$

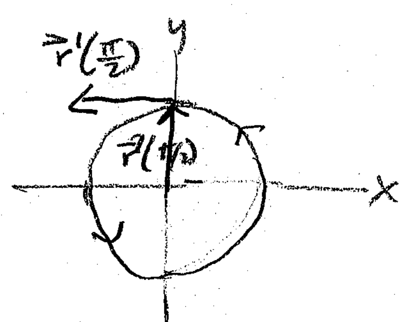


14. $\boldsymbol{r}(t) = \langle \cos(t), \sin(t) \rangle, t_0 = \frac{\pi}{2}$

$\vec{r}'(t) = \langle -\sin t, \cos t \rangle$

$\vec{r}(\pi/2) = \langle 0, 1 \rangle$

$\vec{r}'(\pi/2) = \langle -1, 0 \rangle$



AP Calculus BC

9.8 Worksheet

Name: ______________________

Period: __________

r(t) represents the path of an object moving on a plane.

(a) Find the velocity vector, speed, and acceleration vector of the object.

(b) Evaluate the velocity vector and acceleration vector of the object at the given value of t.

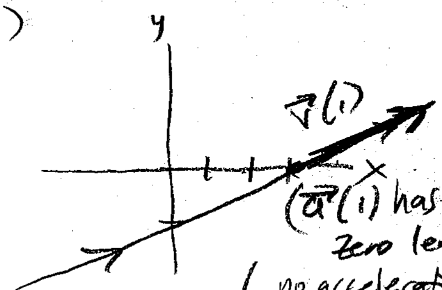
(c) Sketch the graph of the path, and sketch the velocity and acceleration vectors at the given value of t.

1. $\boldsymbol{r}(t) = \langle 3t, t-1 \rangle, t = 1$

(a) $\boxed{\vec{v}(t) = \langle 3, 1 \rangle \quad \vec{a}(t) = \langle 0, 0 \rangle}$

(b) at $t=1$: $\boxed{\vec{v}(1) = \langle 3, 1 \rangle \quad \vec{a}(1) = \langle 0, 0 \rangle}$

(c)



($\vec{a}(1)$ has zero length)

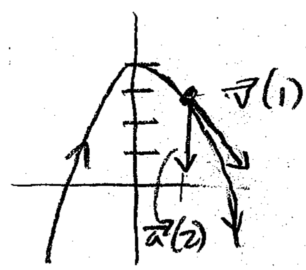
(no acceleration, speed is constant)

2. $\boldsymbol{r}(t) = \langle t, -t^2 + 4 \rangle, t = 1$

(a) $\boxed{\vec{v}(t) = \langle 1, -2t \rangle \quad \vec{a}(t) = \langle 0, -2 \rangle}$

(b) at $t=1$: $\boxed{\vec{v}(1) = \langle 1, -2(1) \rangle = \langle 1, -2 \rangle}$ $\boxed{\vec{a}(1) = \langle 0, -2 \rangle}$

(c)

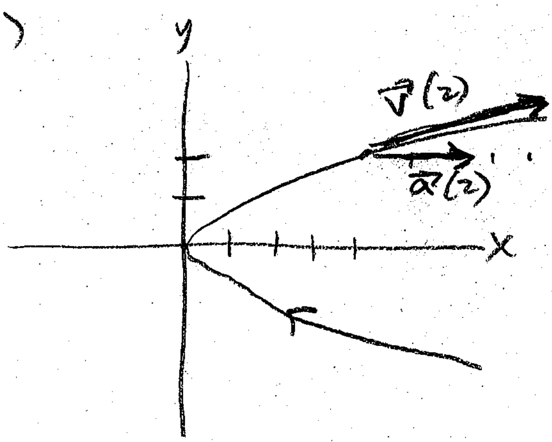


3. $\boldsymbol{r}(t) = \langle t^2, t \rangle, t = 2$

(a) $\boxed{\vec{v}(t) = \langle 2t, 1 \rangle \quad \vec{a}(t) = \langle 2, 0 \rangle}$

(b) $\boxed{\vec{v}(2) = \langle 2(2), 1 \rangle = \langle 4, 1 \rangle \quad \vec{a}(2) = \langle 2, 0 \rangle}$

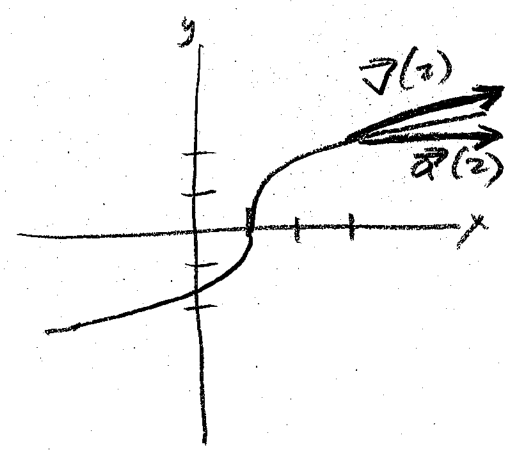
(c)



4. $\boldsymbol{r}(t) = \langle \frac{1}{4}t^3 + 1, t \rangle, t = 2$

(a) $\boxed{\vec{v}(t) = \langle \frac{3}{4}t^2, 1 \rangle \quad \vec{a}(t) = \langle \frac{3}{2}t, 0 \rangle}$

(b) at $t=2$: $\boxed{\vec{v}(2) = \langle \frac{3}{4}(2)^2, 1 \rangle = \langle 3, 1 \rangle \quad \vec{a}(2) = \langle \frac{3}{2}(2), 0 \rangle = \langle 3, 0 \rangle}$



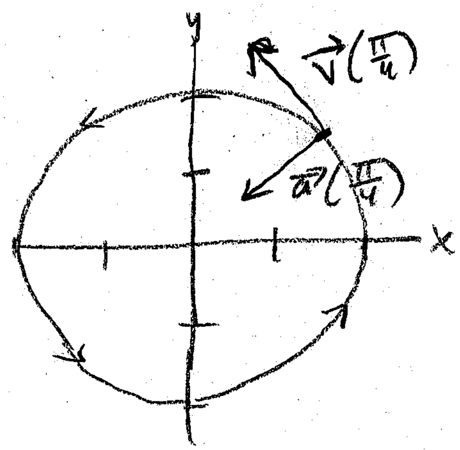
5. $r(t) = \langle 2\cos(t), 2\sin(t)\rangle, t = \frac{\pi}{4}$

(a) $\boxed{\vec{v}(t) = \langle -2\sin t, 2\cos t\rangle \quad \vec{a}(t) = \langle -2\cos t, -2\sin t\rangle}$

(b) at $t = \pi/4$: $\sqrt{2}/2$ $\sqrt{2}/2$

$\boxed{\vec{v}(\tfrac{\pi}{4}) = \langle -2\sin\tfrac{\pi}{4}, 2\cos\tfrac{\pi}{4}\rangle = \langle -\sqrt{2}, \sqrt{2}\rangle \quad \vec{a}(\pi/4) = \langle -2\cos\pi/4, -2\sin\pi/4\rangle = \langle -\sqrt{2}, -\sqrt{2}\rangle}$

(c)



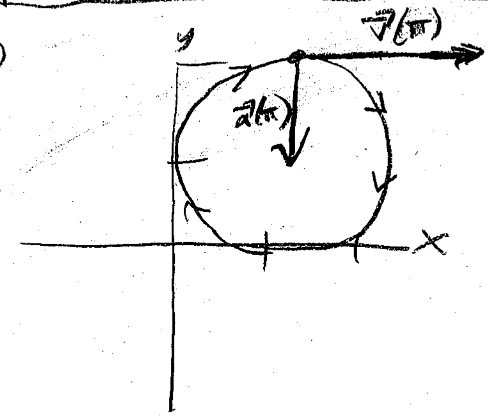
6. $r(t) = \langle t - \sin(t), 1 - \cos(t)\rangle, t = \pi$

(a) $\boxed{\vec{v}(t) = \langle 1 - \cos t, \sin t\rangle \quad \vec{a}(t) = \langle \sin t, \cos t\rangle}$

(b) at $t = \pi$:

$\boxed{\vec{v}(\pi) = \langle 1 - \cos\pi, \sin\pi\rangle = \langle 2, 0\rangle \quad \vec{a}(\pi) = \langle \sin\pi, \cos\pi\rangle = \langle 0, -1\rangle}$

(c)



The velocity vector **v**(t) and the position of a particle at time t = 0 are given.

(a) Find the position of the particle at time t = 3.

(b) Find the total distance travelled on the interval 0 ≤ t ≤ 3.

(c) Find the position vector of the particle.

7. $v(t) = \langle 3,1\rangle, (4,5)$

(a) $\int_0^3 \vec{v}(t)\,dt = \vec{r}(3) - \vec{r}(0)$

$\langle \int_0^3 3\,dt, \int_0^3 1\,dt\rangle = \vec{r}(3) - \langle 4,5\rangle$

$\langle [3t]_0^3, [t]_0^3\rangle = \vec{r}(3) - \langle 4,5\rangle$

$\langle 9-0, 3-0\rangle = \langle 9,3\rangle = \vec{r}(3) - \langle 4,5\rangle$

$\vec{r}(3) = \langle 9,3\rangle + \langle 4,5\rangle = \boxed{\langle 13,8\rangle}$

(b) $|\vec{v}| = \sqrt{3^2+1^2} = \sqrt{10}$

distance $= \int_0^3 \sqrt{10}\,dt = \boxed{3\sqrt{10} = 9.487}$

(c) $\vec{r}(t) = \langle \int 3\,dt, \int 1\,dt\rangle$

$= \langle 3t, t\rangle + \vec{c}$

$\langle 4,5\rangle = \langle 3(0), 0\rangle + \vec{c} \Rightarrow \vec{c} = \langle 4,5\rangle$

$\boxed{\vec{r}(t) = \langle 3t+4, t+5\rangle}$

8. $v(t) = \langle 4,10\rangle, (3,1)$

(a) $\int_0^3 \vec{v}(t)\,dt = \vec{r}(3) - \vec{r}(0)$

$\langle \int_0^3 4\,dt, \int_0^3 10\,dt\rangle = \vec{r}(3) - \langle 3,1\rangle$

$\langle [4t]_0^3, [10t]_0^3\rangle = \vec{r}(3) - \langle 3,1\rangle$

$\langle 12-0, 10-0\rangle = \vec{r}(3) - \langle 3,1\rangle$

$\vec{r}(3) = \langle 12,10\rangle + \langle 3,1\rangle = \boxed{\langle 15,11\rangle}$

(b) $|\vec{v}| = \sqrt{4^2+10^2} = \sqrt{116}$

distance $= \int_0^3 \sqrt{116}\,dt = \boxed{3\sqrt{116} = 32.311}$

(c) $\vec{r}(t) = \langle \int 4\,dt, \int 10\,dt\rangle$

$= \langle 4t, 10t\rangle + \vec{c}$

$\langle 3,1\rangle = \langle 4(0), 10(0)\rangle + C \rightarrow C = \langle 3,1\rangle$

$\boxed{\vec{r}(t) = \langle 4t+3, 10t+1\rangle}$

9. $v(t) = \langle 3t^2, 2t \rangle, (1,2)$

(a) $\int_0^3 \vec{v}(t)dt = \vec{r}(3) - \vec{r}(0)$

$\langle \int_0^3 3t^2 dt, \int_0^3 2t dt \rangle = \vec{r}(3) - \langle 1,2 \rangle$

$\langle [t^3]_0^3, [t^2]_0^3 \rangle = \vec{r}(3) - \langle 1,2 \rangle$

$\langle 27-0, 9-0 \rangle = \vec{r}(3) - \langle 1,2 \rangle$

$\vec{r}(3) = \langle 27, 9 \rangle + \langle 1,2 \rangle = \boxed{\langle 28, 11 \rangle}$

(b) $|\vec{v}| = \sqrt{(3t^2)+(2t)^2}$

distance $= \int_0^3 \sqrt{(3t^2)+(2t)^2}\,dt = \boxed{28.728}$

(c) $\vec{r}(t) = \int \vec{v}(t)dt = \langle \int 3t^2 dt, \int 2t dt \rangle$

$= \langle t^3, t^2 \rangle + \vec{c}$

$\langle 1,2 \rangle = \langle 0^3, 0^2 \rangle + \vec{c}, \quad \vec{c} = \langle 1,2 \rangle$

$\boxed{\vec{r}(t) = \langle t^3+1, t^2+2 \rangle}$

10. $v(t) = \langle 8t - 1, 6t^2 + 1 \rangle, (4,0)$

(a) $\int_0^3 \vec{v}(t)dt = \vec{r}(3) - \vec{r}(0)$

$\langle \int_0^3 (8t-1)dt, \int_0^3 (6t^2+1)dt \rangle = \vec{r}(3) - \langle 4,0 \rangle$

$\langle [4t^2-t]_0^3, [2t^3+t]_0^3 \rangle = \vec{r}(3) - \langle 4,0 \rangle$

$\langle 33-0, 57-0 \rangle = \vec{r}(3) - \langle 4,0 \rangle$

$\vec{r}(3) = \langle 33, 57 \rangle + \langle 4,0 \rangle = \boxed{\langle 37, 57 \rangle}$

(b) $|\vec{v}| = \sqrt{(8t-1)^2+(6t^2+1)^2}$

distance $= \int_0^3 \sqrt{(8t-1)^2+(6t^2+1)^2}\,dt = \boxed{66.429}$

(c) $\vec{r}(t) = \int \vec{v}dt = \langle \int (8t-1)dt, \int (6t^2+1)dt \rangle$

$= \langle 4t^2-t, 2t^3+t \rangle + \vec{c}$

$\langle 4,0 \rangle = \langle 4(0)^2-0, 2(0)^3+0 \rangle + \vec{c}, \quad \vec{c} = \langle 4,0 \rangle$

$\boxed{\vec{r}(t) = \langle 4t^2-t+4, 2t^3+t \rangle}$

Use the given information to find the velocity and position vectors. Then find the position at time t = 2.

11. $a(t) = \langle 2,3 \rangle, v(0) = \langle 0,4 \rangle, r(0) = \langle 0,0 \rangle$

$\vec{v}(t) = \int \vec{a}(t)dt = \langle \int 2dt, \int 3dt \rangle = \langle 2t, 3t \rangle + \vec{c}$

$\langle 0,4 \rangle = \langle 2(0), 3(0) \rangle + \vec{c}, \quad \vec{c} = \langle 0,4 \rangle$

$\boxed{\vec{v}(t) = \langle 2t, 3t+4 \rangle}$

$\vec{r}(t) = \langle \int 2t dt, \int (3t+4)dt \rangle = \langle t^2, \frac{3}{2}t^2+4t \rangle + \vec{c}$

$\langle 0,0 \rangle = \langle (0)^2, \frac{3}{2}(0)^2+4(0) \rangle + \vec{c}, \quad \vec{c} = \langle 0,0 \rangle$

$\boxed{\vec{r}(t) = \langle t^2, \frac{3}{2}t^2+4t \rangle}$

$\vec{r}(2) = \langle (2)^2, \frac{3}{2}(2)^2+4(2) \rangle = \boxed{\langle 4, 14 \rangle}$

12. $a(t) = \langle t, t \rangle, v(0) = \langle 3,1 \rangle, r(0) = \langle 1,5 \rangle$

$\vec{v}(t) = \int \vec{a}(t)dt = \langle \int t dt, \int t dt \rangle = \langle \frac{1}{2}t^2, \frac{1}{2}t^2 \rangle + \vec{c}$

$\langle 3,1 \rangle = \langle \frac{1}{2}(0)^2, \frac{1}{2}(0)^2 \rangle + \vec{c}, \quad \vec{c} = \langle 3,1 \rangle$

$\boxed{\vec{v}(t) = \langle \frac{1}{2}t^2+3, \frac{1}{2}t^2+1 \rangle}$

$\vec{r}(t) = \int \vec{v}(t)dt = \langle \int (\frac{1}{2}t^2+3)dt, \int (\frac{1}{2}t^2+1)dt \rangle$

$= \langle \frac{1}{6}t^3+3t, \frac{1}{6}t^3+t \rangle + \vec{c}$

$\langle 1,5 \rangle = \langle \frac{1}{6}(0)^3+3(0), \frac{1}{6}(0)^3+(0) \rangle + \vec{c}, \quad \vec{c} = \langle 1,5 \rangle$

$\boxed{\vec{r}(t) = \langle \frac{1}{6}t^3+3t+1, \frac{1}{6}t^3+t+5 \rangle}$

$\vec{r}(2) = \langle \frac{1}{6}(2)^3+3(2)+1, \frac{1}{6}(2)^3+(2)+5 \rangle = \boxed{\langle \frac{25}{3}, \frac{25}{3} \rangle}$

13. $a(t) = \langle 4t, t^2 \rangle, v(0) = \langle 5,0 \rangle, r(0) = \langle 4,2 \rangle$

$\vec{v}(t) = \langle \int 4dt, \int t^2 dt \rangle = \langle 4t, \frac{1}{3}t^3 \rangle + \vec{c}$

$\langle 5,0 \rangle = \langle 4(0), \frac{1}{3}(0)^3 \rangle + \vec{c}, \quad \vec{c} = \langle 5,0 \rangle$

$\boxed{\vec{v}(t) = \langle 4t+5, \frac{1}{3}t^3 \rangle}$

$\vec{r}(t) = \langle \int (4t+5)dt, \int \frac{1}{3}t^3 dt \rangle$

$= \langle 2t^2+5t, \frac{1}{12}t^4 \rangle + \vec{c}$

$\langle 4,2 \rangle = \langle 2(0)^2+5(0), \frac{1}{12}(0)^4 \rangle + \vec{c}, \quad \vec{c} = \langle 4,2 \rangle$

$\boxed{\vec{r}(t) = \langle 2t^2+5t+4, \frac{1}{12}t^4+2 \rangle}$

$\vec{r}(2) = \langle 2(2)^2+5(2)+4, \frac{1}{12}(2)^4+2 \rangle = \boxed{\langle 22, \frac{10}{3} \rangle}$

14. $a(t) = \langle t, \sin(t) \rangle, v(0) = \langle 0,-1 \rangle, r(0) = \langle 0,0 \rangle$

$\vec{v}(t) = \int \vec{a}(t)dt = \langle \int t dt, \int \sin t dt \rangle = \langle \frac{1}{2}t^2, -\cos t \rangle + \vec{c}$

$\langle 0,-1 \rangle = \langle \frac{1}{2}(0)^2, \underbrace{-\cos(0)}_{(-1)} \rangle + \vec{c}, \quad \vec{c} = \langle 0,-1 \rangle - \langle 0,-1 \rangle$

$\vec{c} = \langle 0,0 \rangle$

$\boxed{\vec{v}(t) = \langle \frac{1}{2}t^2, -\cos t \rangle}$

$\vec{r}(t) = \int \vec{v}(t)dt = \langle \int \frac{1}{2}t^2 dt, \int -\cos t dt \rangle$

$= \langle \frac{1}{6}t^3, -\sin t \rangle + \vec{c}$

$\langle 0,0 \rangle = \langle \frac{1}{6}(0)^3, \underbrace{-\sin(0)}_{(0)} \rangle + \vec{c}, \quad \vec{c} = \langle 0,0 \rangle$

$\boxed{\vec{r}(t) = \langle \frac{1}{6}t^3, -\sin t \rangle}$

$\vec{r}(2) = \langle \frac{1}{6}(2)^3, -\sin 2 \rangle = \boxed{\langle \frac{4}{3}, -\sin 2 \rangle = \langle 1.333, -0.909 \rangle}$