AP Calculus BC

9.6 Worksheet

Name: answers/solutions

Period: ____________

Show that **u** and **v** are equivalent.

1. **u** : (3,2), (5,6)

   v = (1,4), (3,8)

   $\vec{u} = \langle 2,4 \rangle$ ✓

   $\vec{v} = \langle 2,4 \rangle$

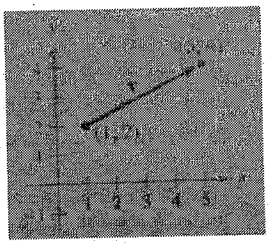
2. **u** = (-4,0), (1,8)

   v = (2,-1), (7,7)

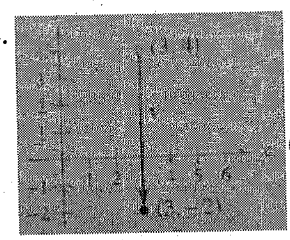
   $\vec{u} = \langle 1-(-4), 8-0 \rangle = \langle 5,8 \rangle$ ✓

   $\vec{v} = \langle 7-2, 7-(-1) \rangle = \langle 5,8 \rangle$

Find the component form of the vector **v** and sketch the vector with its initial point at the origin.

3. 

   $\boxed{\vec{v} = \langle 4,2 \rangle}$

4. 

   $\vec{v} = \langle -2-4, 3-3 \rangle$

   $\boxed{\vec{v} = \langle -6,0 \rangle}$

Find the magnitude of the vector **v**.

5. **v** = <7, 0>

   $\boxed{|\vec{v}| = 7}$

6. **v** = <-3, 0>

   $|\vec{v}| = \sqrt{(-3)^2+(0)^2} = \boxed{3}$

7. **v** = <4, 3>

   $\boxed{|\vec{v}| = 5}$

8. **v** = <12, -5>

   $|\vec{v}| = \sqrt{12^2+5^2} = \boxed{13}$

9. **v** = <6, -5>

   $\boxed{\vec{v} = \sqrt{61}}$

10. **v** = <-10, 3>

    $|\vec{v}| = \sqrt{10^2+3^2} = \boxed{\sqrt{109}}$

Perform the following operations on the vectors: 2/3**u**, 3**v**, **v** – **u**, 2**u** + 5**v**.

11. **u** = <4, 9>

    v = <2, -5>

    $2/3\vec{u} = \langle \frac{8}{3}, \frac{18}{3} \rangle$

    $3\vec{v} = \langle 6, -15 \rangle$

    $\vec{v}-\vec{u} = \langle -2, -14 \rangle$

    $2\vec{u}+5\vec{v} = \langle 18, -7 \rangle$

12. **u** = <-3, -8>

    v = <8, 25>

    $2/3\vec{u} = \frac{2}{3}\langle -3,-8 \rangle = \boxed{\langle -2, \frac{-16}{3} \rangle}$

    $3\vec{v} = 3\langle 8,25 \rangle = \boxed{\langle 24,75 \rangle}$

    $\vec{v}-\vec{u} = \langle 8,25 \rangle - \langle -3,-8 \rangle = \boxed{\langle 11,33 \rangle}$

    $2\vec{u}+5\vec{v} = 2\langle -3,-8 \rangle + 5\langle 8,25 \rangle$

    $= \langle -6,-16 \rangle + \langle 40,125 \rangle$

    $= \boxed{\langle 34,109 \rangle}$

Find the following: $\|\boldsymbol{u}\|, \|\boldsymbol{v}\|, \|\boldsymbol{u}+\boldsymbol{v}\|, \left\|\frac{u}{\|u\|}\right\|, \left\|\frac{v}{\|v\|}\right\|, \left\|\frac{u+v}{\|u+v\|}\right\|$

13. **u** = <1, -1>

v = <-1, 2>

$$\boxed{\begin{aligned} |\vec{u}| &= \sqrt{2} \\ |\vec{v}| &= \sqrt{5} \\ |\vec{u}+\vec{v}| &= 1 \\ \left|\frac{\vec{u}}{|\vec{u}|}\right| &= 1 \\ \left|\frac{\vec{v}}{|\vec{v}|}\right| &= 1 \\ \left|\frac{\vec{u}+\vec{v}}{|\vec{u}+\vec{v}|}\right| &= 1 \end{aligned}}$$

14. **u** = <0, 1> $|\vec{u}| = \sqrt{0^2+1^2} = \boxed{1}$

v = <3, -3> $|\vec{v}| = \sqrt{3^2+3^2} = \boxed{\sqrt{18}}$

$|\vec{u}+\vec{v}| = |\langle 0,1\rangle + \langle 3,-3\rangle| = |\langle 3,-2\rangle| = \sqrt{3^2+2^2} = \boxed{\sqrt{13}}$

$\left|\frac{\vec{u}}{|\vec{u}|}\right| = \left|\frac{\langle 0,1\rangle}{1}\right| = |\langle 0,1\rangle| = \sqrt{0^2+1^2} = \sqrt{1} = \boxed{1}$

$\left|\frac{\vec{v}}{|\vec{v}|}\right| = \left|\frac{\langle 3,-3\rangle}{\sqrt{18}}\right| = \left|\left\langle \frac{3}{\sqrt{18}}, \frac{-3}{\sqrt{18}}\right\rangle\right|$

$= \sqrt{\left(\frac{3}{\sqrt{18}}\right)^2 + \left(\frac{-3}{\sqrt{18}}\right)^2} = \sqrt{\frac{9}{18}+\frac{9}{18}} = \sqrt{1} = \boxed{1}$

$\left|\frac{\vec{u}+\vec{v}}{|\vec{u}+\vec{v}|}\right| = \left|\frac{\langle 3,-2\rangle}{\sqrt{13}}\right| = \left|\left\langle \frac{3}{\sqrt{13}}, \frac{-2}{\sqrt{13}}\right\rangle\right|$

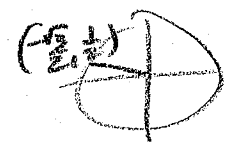
$= \sqrt{\left(\frac{3}{\sqrt{13}}\right)^2 + \left(\frac{-2}{\sqrt{13}}\right)^2} = \sqrt{\frac{9}{13}+\frac{4}{13}} = \sqrt{1} = \boxed{1}$

Find the vector **v**, given its magnitude and direction.

15. $\|v\| = 3, \theta = 45°$

$$\boxed{\vec{v} = \left\langle \frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right\rangle}$$

16. $\|v\| = 2, \theta = 150°$



$\vec{v} = \langle r\cos\theta, r\sin\theta\rangle$

$= \langle 2\cos 150°, 2\sin 150°\rangle$

$= \langle 2\left(-\frac{\sqrt{3}}{2}\right), 2\left(\frac{1}{2}\right)\rangle$

$$\boxed{\vec{v} = \langle -\sqrt{3}, 1\rangle}$$

AP Calculus BC

9.7 Worksheet

Name: ____________________

Period: __________

Find the domain of the vector-valued function.

1. $r(t) = \langle \frac{1}{t+1}, \frac{t}{2} \rangle$

$\boxed{D: (-\infty,-1)\cup(-1,\infty)}$

2. $r(t) = \langle \sqrt{4-t^2}, t^2 \rangle$

$4-t^2 \geq 0$

$t^2 = 0,\ t = \pm 2$

$-\infty$ × ✓ ✓ ✓ × ∞

(-3) -2 (0) 2 (3)

$\boxed{D: (-\infty,\infty)}$

3. $r(t) = \langle \ln(t), -e^t \rangle$

$\boxed{D: (0,\infty)}$

4. $r(t) = \langle \sin(t), \cos(t) \rangle$

$\boxed{D: (-\infty,\infty)}$

Evaluate (if possible) the vector-valued function at each given time t.

5. $r(t) = \langle \frac{1}{2}t^2, -t+1 \rangle$

a. r(1) = $\boxed{\langle \frac{1}{2}, 0 \rangle}$

b. r(0) = $\langle \frac{1}{2}(0)^2, -(0)+1 \rangle$

$= \boxed{\langle 0, 1 \rangle}$

c. r(s + 1) = $\boxed{\langle \frac{1}{2}(s+1)^2, -s \rangle}$

d. r(2 + Δt) – r(2) = $\langle \frac{1}{2}(2+\Delta t)^2, -(2+\Delta t)+1 \rangle$

$- \langle \frac{1}{2}(2)^2, -(2)+1 \rangle$

$= \langle \frac{1}{2}(2+\Delta t)^2 - 2, -2-\Delta t+1+1 \rangle$

$= \boxed{\langle \frac{1}{2}(2+\Delta t)^2 - 2, -\Delta t \rangle}$

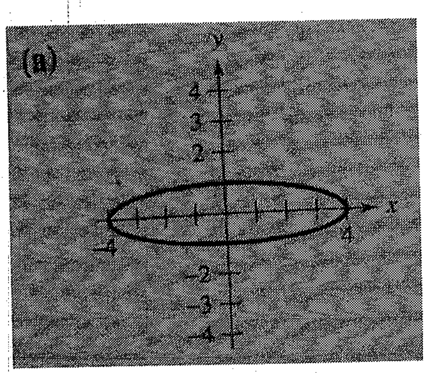
6. Match the equation with its graph. (Note: use calculator parametric mode to graph)

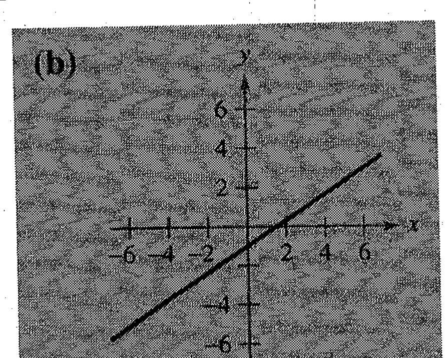
$r(t) = \langle 3t, 2t-1 \rangle$ __b__

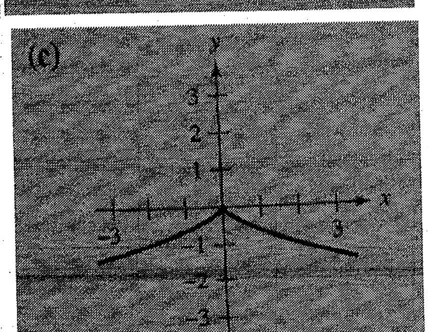
$r(t) = \langle 2t^3, -t^2 \rangle$ __c__

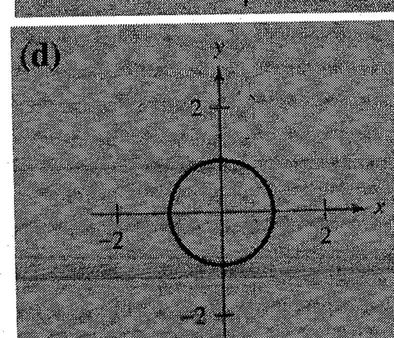
$r(t) = \langle \cos(t), \sin(t) \rangle$ __d__

$r(t) = \langle 4\cos(t), \sin(t) \rangle$ __a__



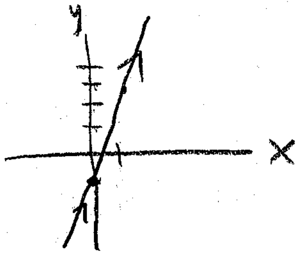






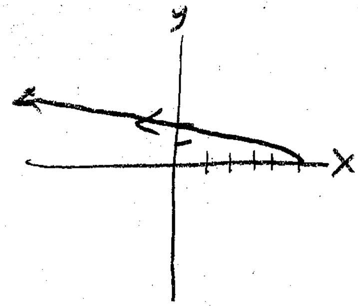
Sketch the plane curve represented by the vector-valued function and give the orientation of the curve.

7. $r(t) = \langle \frac{t}{4}, t - 1 \rangle$

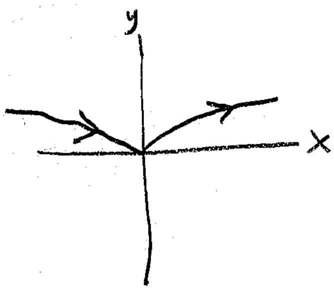


8. $r(t) = \langle 5 - t, \sqrt{t} \rangle$

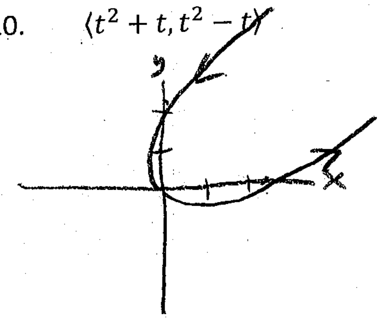
Use calculator in parametric mode to graph



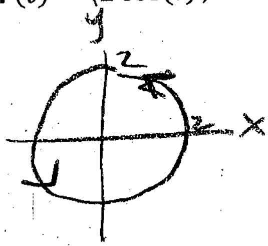
9. $r(t) = \langle t^3, t^2 \rangle$



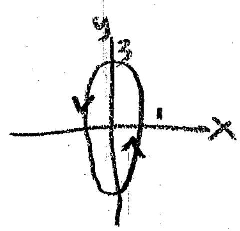
10. $\langle t^2 + t, t^2 - t \rangle$



11. $r(t) = \langle 2\cos(t), 2\sin(t) \rangle$

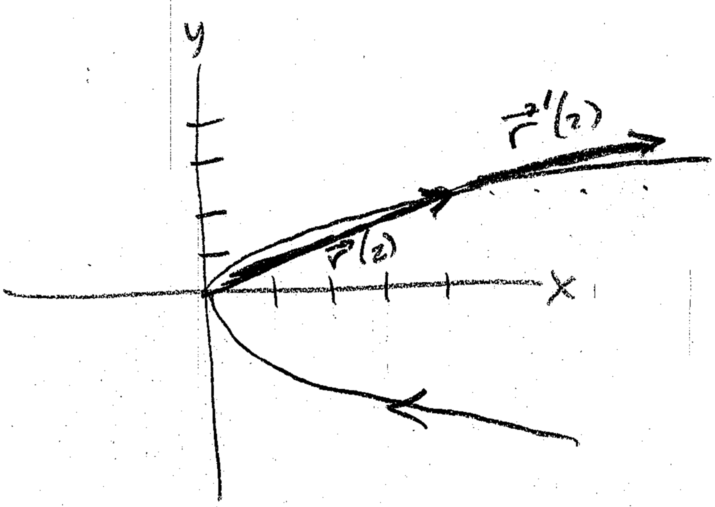


12. $\langle \cos(t), 3\sin(t) \rangle$



Find **r**′(t), **r**(t₀) and **r**′(t₀). Then sketch the plane curve represented by **r**(t) and sketch the vectors **r**(t₀) and **r**′(t₀). Position the vectors such that the initial point of **r**(t₀) is at the origin, and the initial point of **r**′(t₀) is at the terminal point of **r**′(t₀).

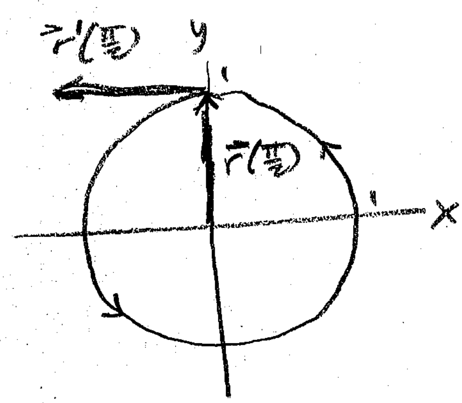
13. $r(t) = \langle t^2, t \rangle, t_0 = 2$



14. $r(t) = \langle \cos(t), \sin(t) \rangle, t_0 = \frac{\pi}{2}$

$\vec r'(t) = \langle -\sin t, \cos t \rangle$

$\vec r(\pi/2) = \langle 0, 1 \rangle$ $\vec r'(\frac{\pi}{2}) = \langle -1, 0 \rangle$



AP Calculus BC

Name: ____________________

9.8 Worksheet

Period: __________

r(t) represents the path of an object moving on a plane.

(a) Find the velocity vector, speed, and acceleration vector of the object.

(b) Evaluate the velocity vector and acceleration vector of the object at the given value of t.

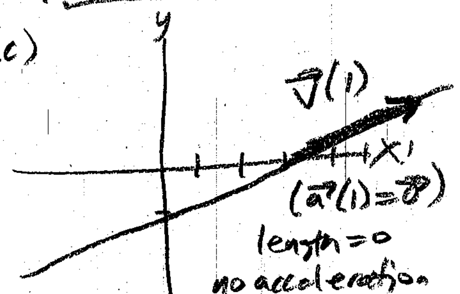
(c) Sketch the graph of the path, and sketch the velocity and acceleration vectors at the given value of t.

1. $\mathbf{r}(t) = \langle 3t, t-1 \rangle, t = 1$

(a) $\boxed{\vec{v}(t) = \langle 3, 1 \rangle \quad \vec{a}(t) = \langle 0, 0 \rangle}$

(b) $\boxed{\vec{v}(1) = \langle 3, 1 \rangle \quad \vec{a}(1) = \langle 0, 0 \rangle}$

(c)



($\vec{a}(1) = \vec{0}$)
length = 0
no acceleration
(speed is constant)

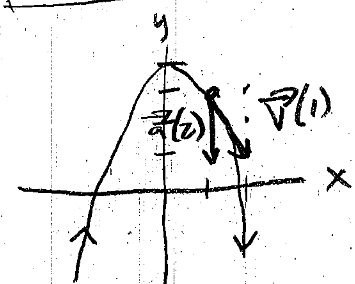
2. $\mathbf{r}(t) = \langle t, -t^2 + 4 \rangle, t = 1$

(a) $\boxed{\vec{v}(t) = \langle 1, -2t \rangle \quad \vec{a}(t) = \langle 0, -2 \rangle}$

(b) at $t = 1$:

$\boxed{\vec{v}(1) = \langle 1, -2(1) \rangle = \langle 1, -2 \rangle \quad \vec{a}(1) = \langle 0, -2 \rangle}$

(c)

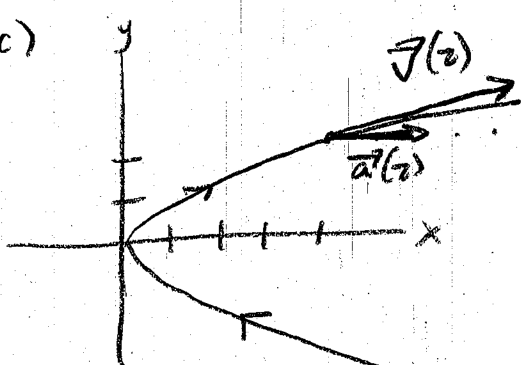


3. $\mathbf{r}(t) = \langle t^2, t \rangle, t = 2$

(a) $\boxed{\vec{v}(t) = \langle 2t, 1 \rangle \quad \vec{a}(t) = \langle 2, 0 \rangle}$

(b) $\boxed{\vec{v}(2) = \langle 4, 1 \rangle \quad \vec{a}(2) = \langle 2, 0 \rangle}$

(c)



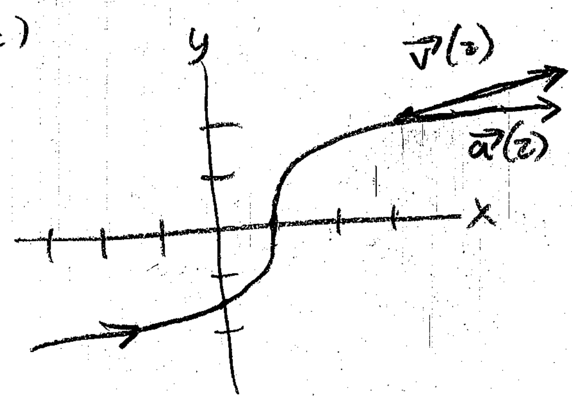
4. $\mathbf{r}(t) = \langle \frac{1}{4}t^3 + 1, t \rangle, t = 2$

(a) $\boxed{\vec{v}(t) = \langle \frac{3}{4}t^2, 1 \rangle \quad \vec{a}(t) = \langle \frac{3}{2}t, 0 \rangle}$

(b) at $t = 2$

$\boxed{\vec{v}(2) = \langle \frac{3}{4}(2)^2, 1 \rangle = \langle 3, 1 \rangle \quad \vec{a}(2) = \langle \frac{3}{2}(2), 0 \rangle = \langle 3, 0 \rangle}$

(c)

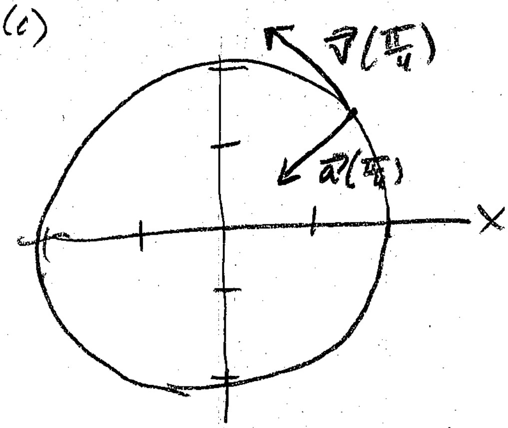


5. $r(t) = \langle 2\cos(t), 2\sin(t)\rangle, t = \frac{\pi}{4}$

(a) $\boxed{\begin{array}{l}\vec v(t) = \langle -2\sin t, 2\cos t\rangle \\ \vec a(t) = \langle -2\cos t, -2\sin t\rangle\end{array}}$

(b) $\boxed{\begin{array}{l}\vec v(\frac{\pi}{4}) = \langle -\sqrt2, \sqrt2\rangle \\ \vec a(\frac{\pi}{4}) = \langle -\sqrt2, -\sqrt2\rangle\end{array}}$

(c)



6. $r(t) = \langle t - \sin(t), 1 - \cos(t)\rangle, t = \pi$

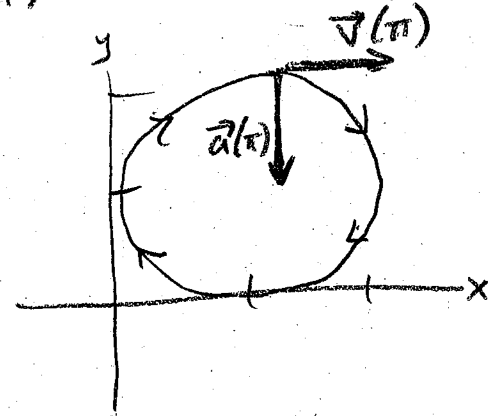
(a) $\boxed{\begin{array}{l}\vec v(t) = \langle 1-\cos t, \sin t\rangle \\ \vec a(t) = \langle \sin t, \cos t\rangle\end{array}}$

(b) at $t=\pi$:

$\vec v(\pi) = \langle 1-\cos\pi, \sin\pi\rangle = \langle 2,0\rangle$

$\vec a(\pi) = \langle \sin\pi, \cos\pi\rangle = \langle 0,-1\rangle$

(c)



The velocity vector **v**(t) and the position of a particle at time t = 0 are given.

(a) Find the position of the particle at time t = 3.

(b) Find the total distance travelled on the interval 0 ≤ t ≤ 3.

(c) Find the position vector of the particle.

7. $v(t) = \langle 3,1\rangle, (4,5)$

(a) $\boxed{\vec r(3) = \langle 13, 8\rangle}$

(b) $\boxed{\text{distance} = 3\sqrt{10} = 9.487}$

(c) $\boxed{\vec r(t) = \langle 3t+4, t+5\rangle}$

8. $v(t) = \langle 4,10\rangle, (3,1)$

(a) $\int_0^3 \vec v(t)dt = \vec r(3) - \vec r(0)$

$\langle \int_0^3 4dt, \int_0^3 10dt\rangle = \vec r(3) - \langle 3,1\rangle$

$\langle [4t]_0^3, [10t]_0^3\rangle = \vec r(3) - \langle 3,1\rangle$

$\langle 12-0, 10-0\rangle = \vec r(3) - \langle 3,1\rangle$

$\vec r(3) = \langle 12,10\rangle + \langle 3,1\rangle = \boxed{\langle 15,11\rangle}$

(b) distance $= \int_0^3 |\vec v| dt \qquad |\vec v| = \sqrt{4^2+10^2} = \sqrt{116}$

distance $= \int_0^3 \sqrt{116}\,dt = \boxed{3\sqrt{116} = 32.311}$

(c) $\vec r(t) = \int \vec v(t)dt = \langle \int 4dt, \int 10dt\rangle$

$= \langle 4t, 10t\rangle + \vec c$

$\langle 3,1\rangle = \langle 4(0), 10(0)\rangle + \vec c, \quad \vec c = \langle 3,1\rangle$

$\boxed{\vec r(t) = \langle 4t+3, 10t+1\rangle}$

9. $v(t) = \langle 3t^2, 2t\rangle, (1,2)$

(a) $\boxed{\vec{r}(3) = \langle 28, 11\rangle}$

(b) $\boxed{\text{distance} = 28.728}$

(c) $\boxed{\vec{r}(t) = \langle t^3+1, t^2+2\rangle}$

10. $v(t) = \langle 8t-1, 6t^2+1\rangle, (4,0)$

(a) $\int_0^3 \vec{v}(t)dt = \vec{r}(3) - \vec{r}(0)$

$\langle \int_0^3 (8t-1)dt, \int_0^3 (6t^2+1)dt\rangle = \vec{r}(3) - \langle 4,0\rangle$

$\langle [4t^2-t]_0^3, [2t^3+t]_0^3\rangle = \vec{r}(3) - \langle 4,0\rangle$

$\langle 33-0, 57-0\rangle = \vec{r}(3) - \langle 4,0\rangle$

$\vec{r}(3) = \langle 33, 57\rangle + \langle 4,0\rangle = \boxed{\langle 37, 57\rangle}$

(b) $|\vec{v}| = \sqrt{(8t-1)^2+(6t^2+1)^2}$

distance $= \int_0^3 |\vec{v}|dt = \int_0^3 \sqrt{(8t-1)^2+(6t^2+1)^2}\,dt = \boxed{66.427}$

(c) $\vec{r}(t) = \int \vec{v}(t)dt = \langle \int 8t-1dt, \int (6t^2+1)dt\rangle$

$= \langle 4t^2-t, 2t^3+t\rangle + \vec{C}$, $\vec{C} = \langle 4,0\rangle$

$\langle 4,0\rangle = \langle 4(0)^2-(0), 2(0)^3+(0)\rangle + \vec{C}$,

$\boxed{\vec{r}(t) = \langle 4t^2-t+4, 2t^3+t\rangle}$

Use the given information to find the velocity and position vectors. Then find the position at time t = 2.

11. $a(t) = \langle 2,3\rangle, v(0) = \langle 0,4\rangle, r(0) = \langle 0,0\rangle$

$\boxed{\vec{v}(t) = \langle 2t, 3t+4\rangle}$

$\boxed{\vec{r}(t) = \langle t^2, \frac{3}{2}t^2+4t\rangle}$

$\boxed{\vec{r}(2) = \langle 4, 14\rangle}$

12. $a(t) = \langle t,t\rangle, v(0) = \langle 3,1\rangle, r(0) = \langle 1,5\rangle$

$\vec{v}(t) = \int \vec{a}(t)dt = \langle \int tdt, \int tdt\rangle = \langle \frac{1}{2}t^2, \frac{1}{2}t^2\rangle + \vec{C}$

$\langle 3,1\rangle = \langle \frac{1}{2}(0)^2, \frac{1}{2}(0)^2\rangle + \vec{C}$, $\vec{C} = \langle 3,1\rangle$

$\boxed{\vec{v}(t) = \langle \frac{1}{2}t^2+3, \frac{1}{2}t^2+1\rangle}$

$\vec{r}(t) = \int \vec{v}(t)dt = \langle \int(\frac{1}{2}t^2+3)dt, \int(\frac{1}{2}t^2+1)dt\rangle$

$= \langle \frac{1}{6}t^3+3t, \frac{1}{6}t^3+t\rangle + \vec{C}$, $\vec{C} = \langle 1,5\rangle$

$\langle 1,5\rangle = \langle \frac{1}{6}(0)^3+3(0), \frac{1}{6}(0)^3+(0)\rangle + \vec{C}$,

$\boxed{\vec{r}(t) = \langle \frac{1}{6}t^3+3t+1, \frac{1}{6}t^3+t+5\rangle}$

$\vec{r}(2) = \langle \frac{1}{6}(2)^3+3(2)+1, \frac{1}{6}(2)^3+(2)+5\rangle = \boxed{\langle \frac{25}{3}, \frac{25}{3}\rangle}$

13. $a(t) = \langle 4t, t^2\rangle, v(0) = \langle 5,0\rangle, r(0) = \langle 4,2\rangle$

$\boxed{\vec{v}(t) = \langle 4t+5, \frac{1}{3}t^3\rangle}$

$\boxed{\vec{r}(t) = \langle 2t^2+5t+4, \frac{1}{12}t^4+2\rangle}$

$\boxed{\vec{r}(2) = \langle 22, \frac{10}{3}\rangle}$

14. $a(t) = \langle t, \sin(t)\rangle, v(0) = \langle 0,-1\rangle, r(0) = \langle 0,0\rangle$

$\vec{v}(t) = \int \vec{a}(t)dt = \langle \int tdt, \int \sin tdt\rangle = \langle \frac{1}{2}t^2, -\cos t\rangle + \vec{C}$

$\langle 0,-1\rangle = \langle \frac{1}{2}(0)^2, \underset{(-1)}{-\cos(0)}\rangle + \vec{C}$, $\vec{C} = \langle 0,-1\rangle - \langle 0,-1\rangle$

$\vec{C} = \langle 0,0\rangle$

$\boxed{\vec{v}(t) = \langle \frac{1}{2}t^2, -\cos t\rangle}$

$\vec{r}(t) = \int \vec{v}(t)dt = \langle \int \frac{1}{2}t^2dt, \int -\cos tdt\rangle$

$= \langle \frac{1}{6}t^3, -\sin t\rangle + \vec{C}$, $\vec{C} = \langle 0,0\rangle$

$\langle 0,0\rangle = \langle \frac{1}{6}(0)^3, \underset{(0)}{-\sin(0)}\rangle + \vec{C}$,

$\boxed{\vec{r}(t) = \langle \frac{1}{6}t^3, -\sin t\rangle}$

$\vec{r}(2) = \langle \frac{1}{6}(2)^3, -\sin 2\rangle = \boxed{\langle \frac{4}{3}, -\sin(2)\rangle = \langle 1.333, -0.909\rangle}$