AP Calculus BC

Name: answers / Solutions

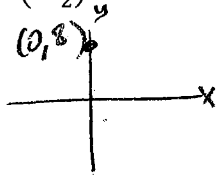
9.4 Worksheet

Period: ____________

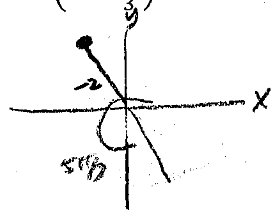
Plot the (r, θ) polar coordinate and find the corresponding rectangular (x, y) coordinate.

1. $\left(8, \frac{\pi}{2}\right)$

$\boxed{(0, 8)}$



2. $\left(-2, \frac{5\pi}{3}\right)$

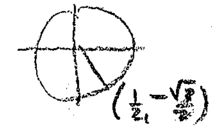


$x = (-2)\cos(5\pi/3)$

$= (-2)\left(\frac{1}{2}\right) = -1$

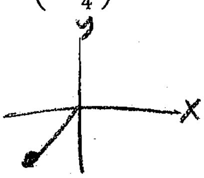
$y = (-2)\sin\left(\frac{5\pi}{3}\right)$

$= (-2)\left(-\frac{\sqrt{3}}{2}\right) = \sqrt{3}$



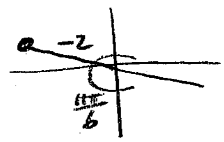
$\boxed{(-1, \sqrt{3})}$

3. $\left(7, \frac{5\pi}{4}\right)$



$\boxed{\left(-\frac{7\sqrt{2}}{2}, -\frac{7\sqrt{2}}{2}\right)}$

4. $\left(-2, \frac{11\pi}{6}\right)$

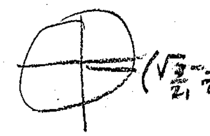


$x = (-2)\cos\left(\frac{11\pi}{6}\right)$

$= (-2)\left(\frac{\sqrt{3}}{2}\right) = -\sqrt{3}$

$y = (-2)\sin\left(\frac{11\pi}{6}\right)$

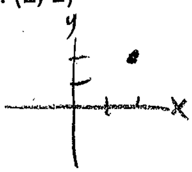
$= (-2)\left(-\frac{1}{2}\right) = 1$



$\boxed{(-\sqrt{3}, 1)}$

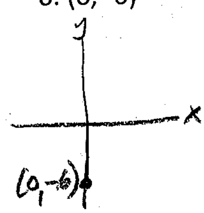
The rectangular (x, y) coordinate is given. Plot the coordinate, then find two sets of polar coordinates in $0 \leq \theta \leq 2\pi$.

5. (2, 2)



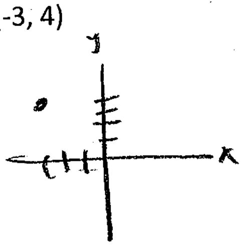
$\left(\sqrt{8}, \frac{\pi}{4}\right)$
$\left(\sqrt{8}, -\frac{7\pi}{4}\right)$
$\left(-\sqrt{8}, \frac{5\pi}{4}\right)$
⋮ etc.

6. (0, -6)



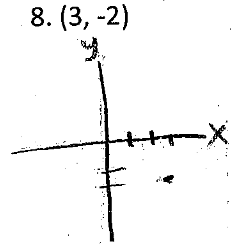
$\left(6, \frac{3\pi}{2}\right)$
$\left(6, -\frac{\pi}{2}\right)$
$\left(-6, \frac{\pi}{2}\right)$
⋮ etc.

7. (-3, 4)



$(5, 0.9273)$
$(5, -5.356)$
⋮

8. (3, -2)



$r = \sqrt{3^2 + 2^2} = \sqrt{13}$

$\theta = \tan^{-1}\left(-\frac{2}{3}\right) = -0.588$

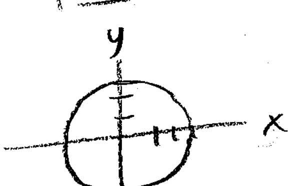
or $-0.588 + 2\pi = 5.695$

$\left(\sqrt{13}, -0.588\right)$
$\left(\sqrt{13}, 5.695\right)$

Covert the rectangular equation to polar form and sketch its graph.

9. $x^2 + y^2 = 9$

$\boxed{r = 3}$



10. $x^2 - y^2 = 9$

$(r\cos\theta)^2 - (r\sin\theta)^2 = 9$

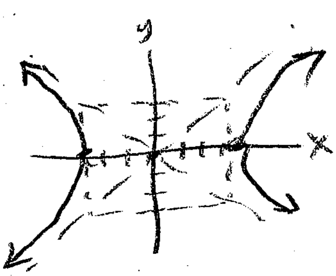
$r^2(\cos^2\theta - \sin^2\theta) = 9$

$\boxed{r^2 = \frac{9}{\cos^2\theta - \sin^2\theta}}$

or

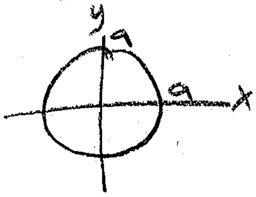
$\boxed{r = \frac{\pm 3}{\sqrt{\cos^2\theta - \sin^2\theta}}}$

$\frac{x^2}{9} - \frac{y^2}{9} = 1$



11. $x^2 + y^2 = a^2$

$\boxed{r = a}$



12. $x^2 - y^2 = a^2$ $\qquad \frac{x^2}{a} - \frac{y^2}{a} = 1$

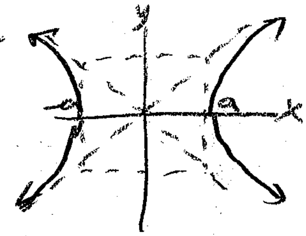
$(r\cos\theta)^2 - (r\sin\theta)^2 = a^2$

$r^2(\cos^2\theta - \sin^2\theta) = a^2$

$\boxed{r^2 = \frac{a^2}{\cos^2\theta - \sin^2\theta}}$

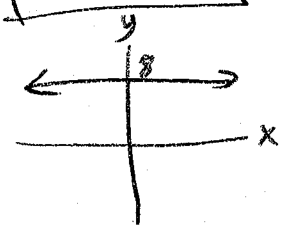
or

$\boxed{r = \frac{\pm a}{\sqrt{\cos^2\theta - \sin^2\theta}}}$



13. $y = 8$

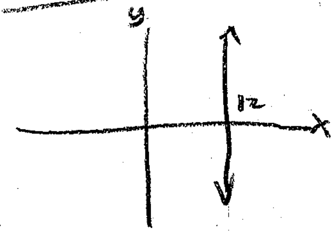
$\boxed{r = 8\csc\theta}$



14. $x = 12$

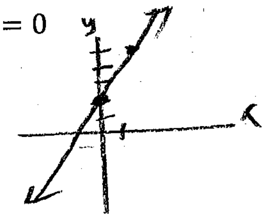
$r\cos\theta = 12$

$\boxed{r = \frac{12}{\cos\theta} = 12\sec\theta}$



15. $3x - y + 2 = 0$

$r = \frac{-2}{3\cos\theta - \sin\theta}$



16. $xy = 4$ $\qquad y = \frac{4}{x}$

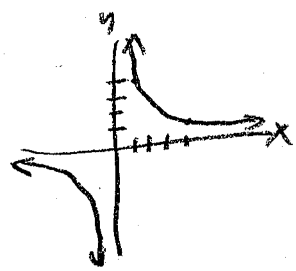
$(r\cos\theta)(r\sin\theta) = 4$

$r^2\cos\theta\sin\theta = 4$

$\boxed{r^2 = \frac{4}{\cos\theta\sin\theta}}$

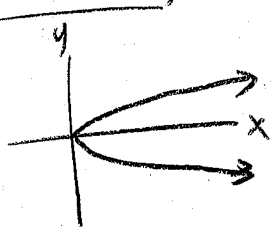
or

$\boxed{r = \frac{\pm 2}{\sqrt{\cos\theta\sin\theta}}}$



17. $y^2 = 9x$

$\boxed{\begin{array}{l} r = 9\cot\theta \\ (\text{or } r = 0) \end{array}}$



18. $(x^2 + y^2)^2 - 9(x^2 - y^2) = 0$

$(r^2)^2 - 9((r\cos\theta)^2 - (r\sin\theta)^2) = 0$

$r^4 - 9r^2(\cos^2\theta - \sin^2\theta) = 0$

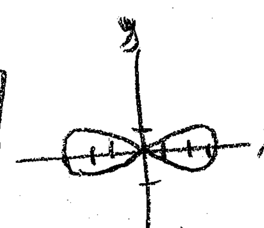
$r^2(r^2 - 9(\cos^2\theta - \sin^2\theta)) = 0$

$r = 0$ (the origin) or $r^2 - 9(\cos^2\theta - \sin^2\theta) = 0$

$r^2 = 9(\cos^2\theta - \sin^2\theta)$

$\boxed{r = \pm 3\sqrt{\cos^2\theta - \sin^2\theta}}$

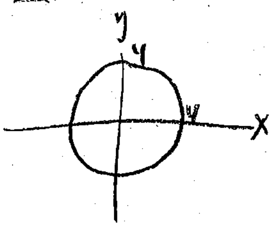
graph in calc. polar mode



Covert the polar equation to ~~polar~~ rectangular form and sketch its graph.

19. $r = 4$

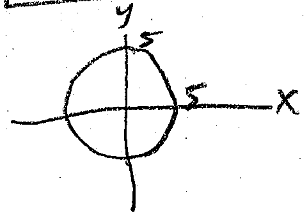
$$\boxed{x^2+y^2=16}$$



20. $r = -5$

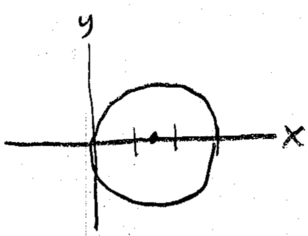
$$r^2=25$$

$$\boxed{x^2+y^2=25}$$



21. $r = 3\sin(\theta)$

$$\boxed{x^2+\left(y-\tfrac{3}{2}\right)^2=\tfrac{9}{4}}$$



22. $r = 5\cos(\theta)$

$$r^2=5r\cos\theta$$

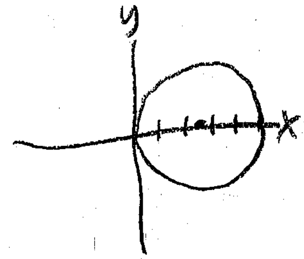
$$x^2+y^2=5x$$

$$x^2-5x+y^2=0$$

$$x^2-5x+\tfrac{25}{4}+(y-0)^2=\tfrac{25}{4}$$

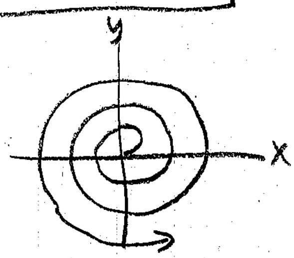
$$\boxed{\left(x-\tfrac{5}{2}\right)^2+(y-0)^2=\tfrac{25}{4}}$$

circle, center $(5/2, 0)$
radius $= 5/2$



23. $r = \theta$

$$\boxed{\tan\left(\sqrt{x^2+y^2}\right)=\frac{y}{x}}$$

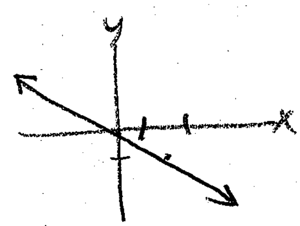


24. $\theta = \dfrac{5\pi}{6}$

$$\tan\theta=\tan\left(\tfrac{5\pi}{6}\right)=\frac{\sin(5\pi/6)}{\cos(5\pi/6)}=\frac{(1/2)}{(-\sqrt{3}/2)}=-\frac{1}{\sqrt{3}}$$

$$\frac{y}{x}=-\frac{1}{\sqrt{3}}$$

$$\boxed{y=-\frac{1}{\sqrt{3}}x}$$



Find the points of vertical and horizontal tangency (if any) to the polar curve.

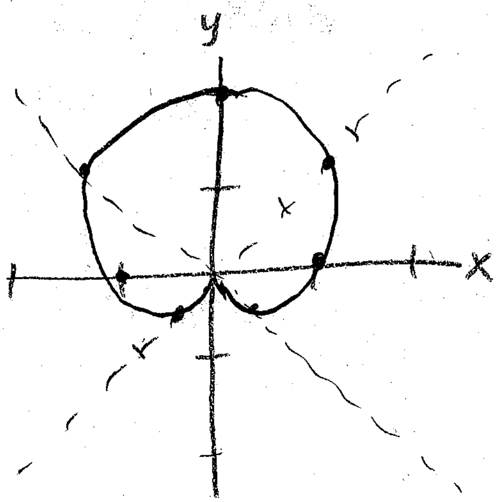
25. $r = 1 - \sin(\theta)$

		r, θ
vertical tangents	at $\theta=\frac{7\pi}{6}$	$\left(\frac{3}{2}, \frac{7\pi}{6}\right)$
	and $\theta=\frac{11\pi}{6}$	$\left(\frac{3}{2}, \frac{11\pi}{6}\right)$
horizontal tangents	at $\theta=\frac{\pi}{6}$	$\left(\frac{1}{2}, \frac{\pi}{6}\right)$
	and $\theta=\frac{3\pi}{2}$	$\left(2, \frac{3\pi}{2}\right)$
	and $\theta=\frac{5\pi}{6}$	$\left(\frac{1}{2}, \frac{5\pi}{6}\right)$

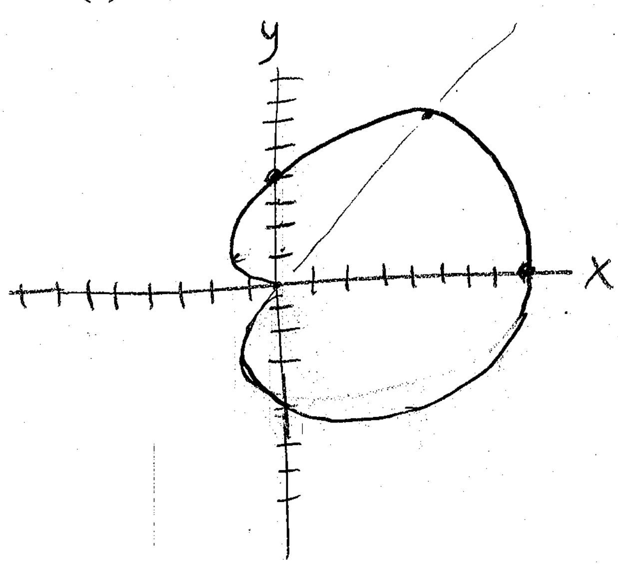
Sketch a graph of the polar equation by hand.

26. $r = 1 + \sin(\theta)$

θ	$r = 1+\sin\theta$
0	$1+0 = 1$
$\frac{\pi}{4}$	$1+\frac{\sqrt{2}}{2} = 1.707$
$\frac{\pi}{2}$	$1+1 = 2$
$\frac{3\pi}{4}$	$1+\frac{\sqrt{2}}{2} = 1.707$
π	$1+0 = 1$
$\frac{5\pi}{4}$	$1-\frac{\sqrt{2}}{2} = 0.293$
$\frac{3\pi}{2}$	$1-0 = 1$
$\frac{7\pi}{4}$	$1-\frac{\sqrt{2}}{2} = 0.293$
2π	$1+0 = 1$



27. $r = 4(1 + \cos(\theta))$



AP Calculus BC

9.5 Worksheet

Name: ______________________

Period: ____________

Write an integral that represents the area of the ~~shaded figure~~. entire figure

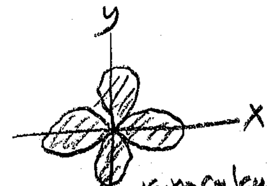
1. $r = 4\sin(\theta)$

$$\boxed{A = \frac{1}{2}\int_0^{\pi} (4\sin\theta)^2\, d\theta}$$

2. $r = \cos(2\theta)$

$$A = \frac{1}{2}\int_{\alpha}^{\beta} r^2\, d\theta$$

$$\boxed{A = \frac{1}{2}\int_0^{2\pi} (\cos(2\theta))^2\, d\theta}$$



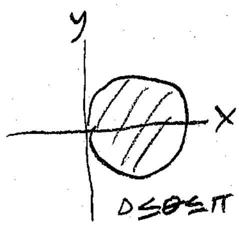
using calculator trace: entire figure required: $0 \le \theta \le 2\pi$

Find the area of the region.

3. Interior of $r = 6\sin(\theta)$

$\boxed{28.274}$

4. Interior of $r = 3\cos(\theta)$



$0 \le \theta \le \pi$

$$A = \frac{1}{2}\int_0^{\pi} (3\cos\theta)^2\, d\theta$$

(mathg)

$= \boxed{7.0686}$

5. One petal of $r = \sin(2\theta)$

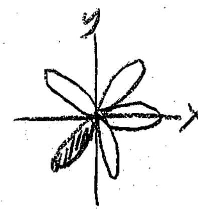
$\boxed{0.393}$

6. One petal of $r = \cos(5\theta)$

r=0 at start/end of 1 petal.

$\cos(5\theta) = 0$, $5\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\theta = \frac{\pi}{10}, \frac{3\pi}{10}$

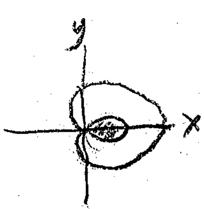


$$A = \frac{1}{2}\int_{\pi/10}^{3\pi/10} (\cos(5\theta))^2\, d\theta = \boxed{0.157}$$

(mathg)

Graph the polar equation on a calculator, then find the area analytically.

7. Inner loop of $r = 1 + 2\cos(\theta)$



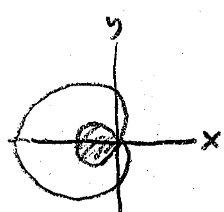
$\boxed{0.5435}$

8. Inner loop of $r = 2 - 4\cos(\theta)$

when $r=0$: $2 - 4\cos\theta = 0$, $\cos\theta = \frac{1}{2}$

$\theta = \pi/3$ to $5\pi/3$ ← calc. graph check this is large loop

$\theta = -\pi/3$ to $\pi/3$ (small loop)



$$A = \frac{1}{2}\int_{-\pi/3}^{\pi/3} (2 - 4\cos\theta)^2\, d\theta = \boxed{2.174}$$

Find the points of intersection of the graphs of the equations.

9. $r = 1 + \cos(\theta)$

$r = 1 - \cos(\theta)$

$\boxed{\theta = \pi/2 \quad r = 1}$ $\boxed{\theta = \frac{3\pi}{2} \quad r = 1}$

10. $r = 3(1 + \sin(\theta))$

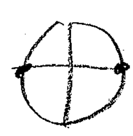
$r = 3(1 - \sin(\theta))$

$3(1+\sin\theta) = 3(1-\sin\theta)$

$1 + \sin\theta = 1 - \sin\theta$

$\sin\theta = -\sin\theta$

$2\sin\theta = 0$, $\sin\theta = 0$



$\boxed{\theta = 0 \quad r = 3}$ $\boxed{\theta = \pi \quad r = 3}$

11. $r = 4 - 5\sin(\theta)$

$r = 3\sin(\theta)$

$\boxed{\theta = \frac{\pi}{6} \quad r = 3/2}$ $\boxed{\theta = \frac{5\pi}{6} \quad r = 3/2}$

12. $r = 3 + \sin(\theta)$ $\quad 3+\sin\theta = 2\csc\theta = \frac{2}{\sin\theta}$

$r = 2\csc(\theta)$ $\quad \sin^2\theta + 3\sin\theta - 2 = 0$

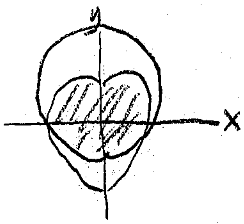
(not factorable – find zeros w/calc. graph)

plug into either to find r

$\boxed{\theta = 0.5963 \quad r = 3.5616}$ $\boxed{\theta = 2.5453 \quad r = 3.5616}$

Use a calculator to graph, then find the area analytically.

13. Common interior of $r = 3 - 2\sin(\theta)$ and $r = -3 + 2\sin(\theta)$.

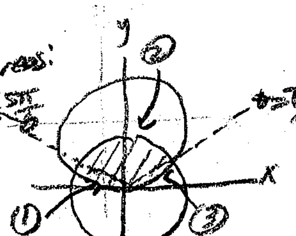


$\boxed{10.5575}$

Note: solve for intersections when $y = r\sin\theta = 0$ ← use either r rather than setting rs equal.
(Unusual problem where intersections don't occur 'coincident' (θ is not the same on both curves at an intersection point)

14. Common interior of $r = 4\sin(\theta)$ and $r = 2$.

3 areas: $\theta = \frac{5\pi}{6}$



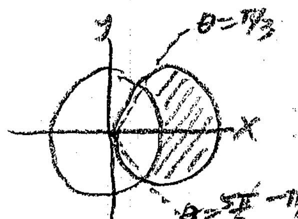
$4\sin\theta = 2$
$\sin\theta = \frac{1}{2}$
$\theta = \frac{\pi}{6}, \frac{5\pi}{6}$

$$A = \frac{1}{2}\int_0^{\pi/6} (4\sin\theta)^2 d\theta + \frac{1}{2}\int_{\pi/6}^{5\pi/6} (2)^2 d\theta + \frac{1}{2}\int_{5\pi/6}^{\pi} (4\sin\theta)^2 d\theta$$

$A = .3623442948 + 4.188790205 + .3623442948$

$A = \boxed{4.913}$

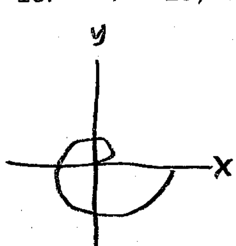
15. Inside $r = 2\cos(\theta)$ and outside $r = 1$.



$A = \boxed{1.913}$

Use your graphing calculator to graph and find the arc length.

16. $r = 2\theta,\ 0 \le \theta \le 2\pi$

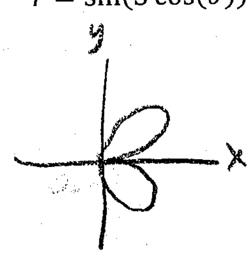


$\frac{dr}{d\theta} = 2$

$$\text{arclength} = \int_\alpha^\beta \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$= \int_0^{2\pi} \sqrt{(2\theta)^2 + (2)^2}\, d\theta = \boxed{42.5126}$$

17. $r = \sin(3\cos(\theta)),\ \ 0 \le \theta \le \pi$



arclength $= \boxed{4.388}$

Unit 9 Part 2 Review — SOLUTIONS

(1) $9x^2+9y^2=18y$

$9(x^2+y^2)=18y$

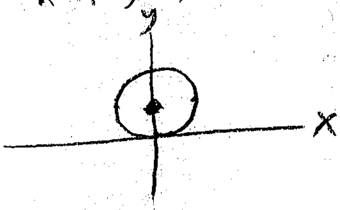
$\boxed{9r^2=18r\sin\theta}$

$9x^2+9y^2-18y=0$

$9x^2+9(y^2-2y+1)=0+9$

$9x^2+9(y-1)^2=9$

$x^2+(y-1)^2=1$



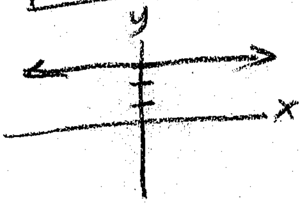
(2) $y=3$

$\boxed{r\sin\theta=3}$

or

$r=\frac{3}{\sin\theta}$

$\boxed{r=3\csc\theta}$



(3) $y=x^2$

$\boxed{r\sin\theta=(r\cos\theta)^2}$

also OKAY...

$r\sin\theta=r^2\cos^2\theta$

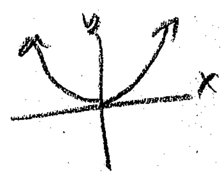
$r\sin\theta-r^2\cos^2\theta=0$

$r(\sin\theta-r\cos^2\theta)=0$

$r=0$ or $\sin\theta-r\cos^2\theta=0$

$r\cos^2\theta=\sin\theta$

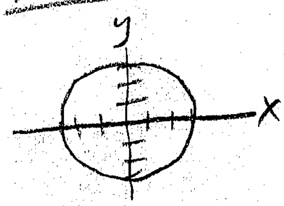
$\boxed{r=\frac{\sin\theta}{\cos^2\theta}}$



(4) $r=3$

$r^2=9$

$\boxed{x^2+y^2=9}$



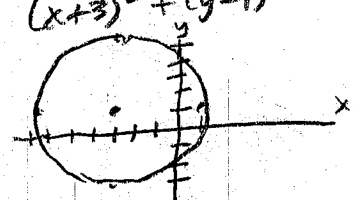
(5) $r+6\cos\theta-2\sin\theta=\frac{6}{r}$

$r^2+6r\cos\theta-2r\sin\theta=6$

$\boxed{x^2+y^2+6x-2y=6}$

$(x^2+6x+9)+(y^2-2y+1)=6+9+1$

$(x+3)^2+(y-1)^2=16$



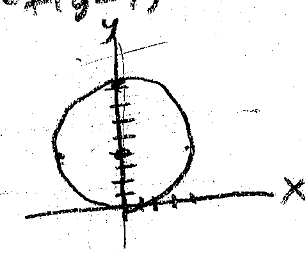
(6) $r=8\sin\theta$

$r^2=8r\sin\theta$

$x^2+y^2=8y$

$x^2+(y^2-8y+16)=0+16$

$x^2+(y-4)^2=16$



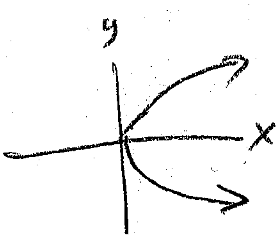
(7) $r=\cot\theta\csc\theta$

$r=\frac{\cos\theta}{\sin\theta}\cdot\frac{1}{\sin\theta}$

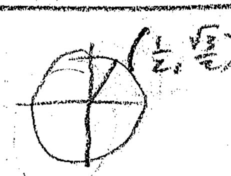
$r\sin\theta=\frac{\cos\theta}{\sin\theta}=\frac{1}{\tan\theta}$

$y=\frac{x}{y}$

$\boxed{y^2=x}$



(8) $\theta=\frac{\pi}{3}$ $\left(\tan\theta=\frac{y}{x}\right)$

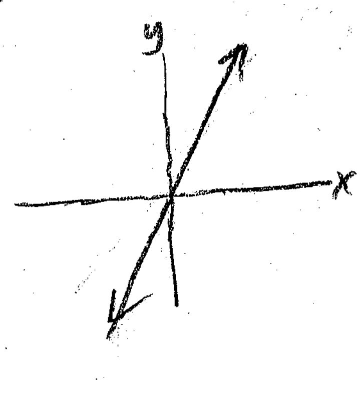


$\tan\theta=\tan\left(\frac{\pi}{3}\right)$

$\frac{y}{x}=\frac{\sin(\pi/3)}{\cos(\pi/3)}\quad\frac{\sqrt{3}/2}{1/2}=\sqrt{3}$

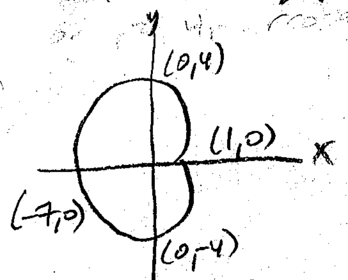
$\frac{y}{x}=\sqrt{3}$

$\boxed{y=\sqrt{3}x}$



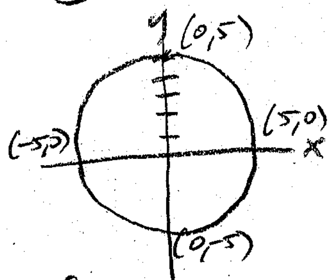
(9) $r = 4 - 3\cos\theta$

calculator to graph



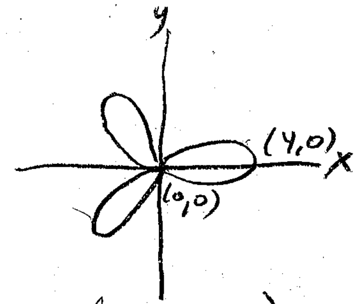
$(0 \le \theta \le 2\pi)$

(10) $r = 5$



$(0 \le \theta \le 2\pi)$

(11) $r = 4\cos(3\theta)$



$(0 \le \theta \le \pi)$

$4\cos(3\theta) = 0$

$\cos(3\theta) = 0$

$\cos(\phi) = 0$

$\phi \quad \pi/2 \quad 3\pi/2$

$3\theta \quad \pi/6 \quad \frac{3\pi}{6}\ \frac{\pi}{2}$

(12) $r = 5 + 4\sin\theta$

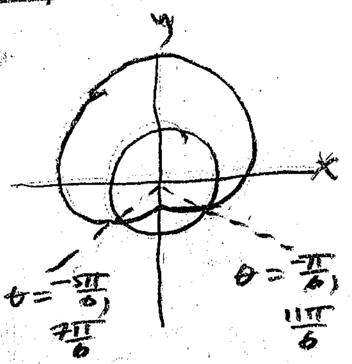
$r = 3$

$5 + 4\sin\theta = 3$

$4\sin\theta = -2$

$\sin\theta = -\frac{1}{2}$

$$\boxed{\begin{array}{l} \theta = -\frac{\pi}{6}\left(\frac{11\pi}{6}\right) \\ \theta = -\frac{5\pi}{6}\left(\frac{7\pi}{6}\right) \end{array}}$$



(13) $r = 4\cos\theta$

$r = 8\cos\theta$

$4\cos\theta = 8\cos\theta$

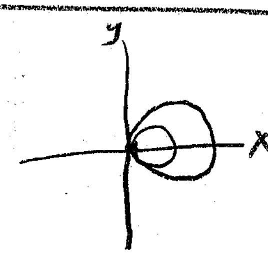
$4\cos\theta - 8\cos\theta = 0$

$(4-8)\cos\theta = 0$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

$\left(-\frac{\pi}{2}\right)\left(-\frac{3\pi}{2}\right)$



but at these angles $\cos\theta$, so $r = 4\cos\theta = 0$ ($r=0$) $\boxed{\text{origin only}}$

(14) $r = 5(1 - \cos\theta)$

$r = 5(1 + \cos\theta)$

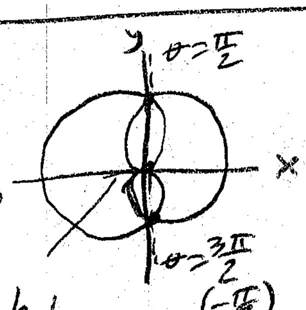
$5(1-\cos\theta) = 5(1+\cos\theta)$

$1 - \cos\theta = 1 + \cos\theta$

$-\cos\theta = \cos\theta$

$2\cos\theta = 0$

$\cos\theta = 0$



but also at ($r=0$) (which can occur at any angle)

$\boxed{\text{answer: } \theta = \frac{\pi}{2}, \frac{3\pi}{2} \text{ plus the origin}}$

(15) $r = -4\sin\theta$

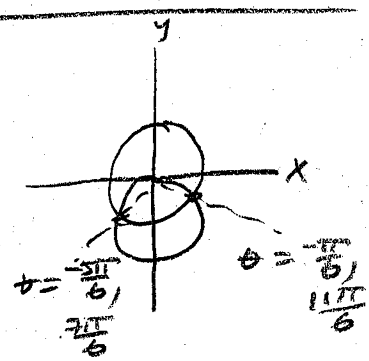
$r = 2$

$-4\sin\theta = 2$

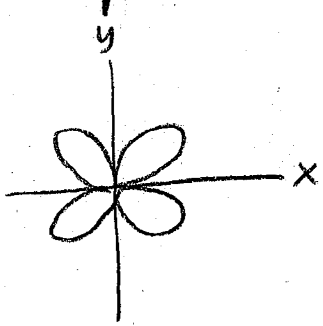
$\sin\theta = -\frac{1}{2}$

$\theta = -\frac{\pi}{6}\left(\frac{11\pi}{6}\right)$

$\theta = -\frac{5\pi}{6}\left(\frac{7\pi}{6}\right)$



(16) one petal of $r = 4\sin(2\theta)$



back to origin ($r=0$) when...

$4\sin(2\theta) = 0$

$\sin(\omega) = 0$

$\omega = 0, \pi$

$2\theta = 0$ or π

$2\theta = \pi$

$\theta = \frac{\pi}{2}$

So one petal for

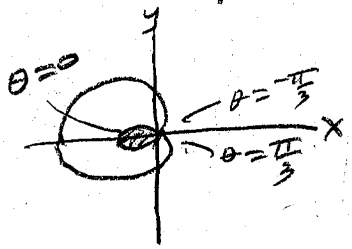
$0 \le \theta \le \frac{\pi}{2}$

$$\text{area} = \frac{1}{2}\int_\alpha^\beta r^2\,d\theta$$

$$\boxed{\text{area} = \frac{1}{2}\int_0^{\pi/2} (4\sin(2\theta))^2\,d\theta}$$

$\approx \boxed{6.28319}$

(17) inner loop of $r = 2 - 4\cos\theta$



back to origin ($r=0$) when...

$2 - 4\cos\theta = 0$

$-4\cos\theta = -2$

$\cos\theta = \frac{1}{2}$

$\theta = -\frac{\pi}{3}$ and $\frac{\pi}{3}$

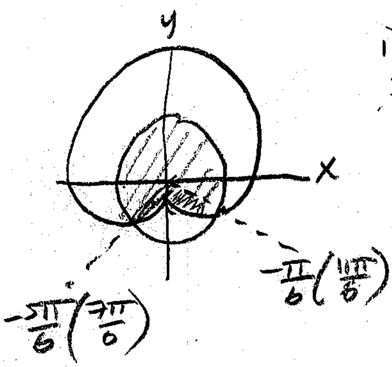
$\text{area} = \frac{1}{2}\int_{\alpha}^{\beta} r^2\,d\theta$

$$\boxed{\text{area} = \frac{1}{2}\int_{-\pi/3}^{\pi/3} (2 - 4\cos\theta)^2\,d\theta}$$

$\approx \boxed{2.17407}$

---

(18) area within both: $r = 5 + 4\sin\theta$
$r = 3$



intersections:

$5 + 4\sin\theta = 3$

$4\sin\theta = -2$

$\sin\theta = -\frac{1}{2}$

$\theta = -\frac{\pi}{6}\left(\frac{11\pi}{6}\right)$

$\theta = -\frac{5\pi}{6}\left(\frac{7\pi}{6}\right)$

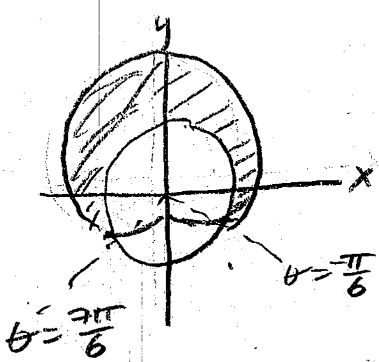
$$\boxed{\text{area} = \frac{1}{2}\int_{-5\pi/6}^{-\pi/6} (5 + 4\sin\theta)^2\,d\theta + \frac{1}{2}\int_{-\pi/6}^{7\pi/6} (3)^2\,d\theta}$$

$\approx 3.38060465 + 18.8495559$

$\boxed{22.2302}$

---

(19) area between curves: $r = 5 + 4\sin\theta$
$r = 3$



intersections:

$5 + 4\sin\theta = 3$

$4\sin\theta = -2$

$\sin\theta = -\frac{1}{2}$

$\theta = -\frac{\pi}{6}\left(\frac{11\pi}{6}\right)$

$\theta = -\frac{5\pi}{6}\left(\frac{7\pi}{6}\right)$

$\text{area} = \text{area}_{\text{outer}} - \text{area}_{\text{inner}}$

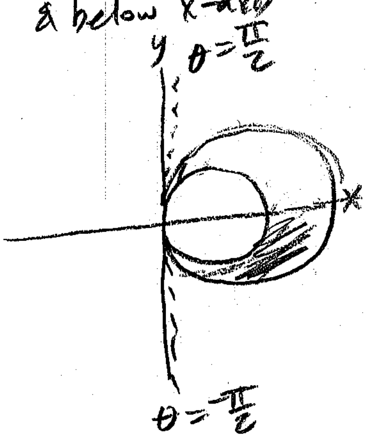
$$\boxed{\text{area} = \frac{1}{2}\int_{-\pi/6}^{7\pi/6} (5 + 4\sin\theta)^2\,d\theta - \frac{1}{2}\int_{-\pi/6}^{7\pi/6} (3)^2\,d\theta}$$

$\approx 100.29195 - 18.849556$

$\boxed{81.4424}$

---

(20) area between curves: $r = 4\cos\theta$
$r = 8\cos\theta$
& below x-axis



intersection: (origin)

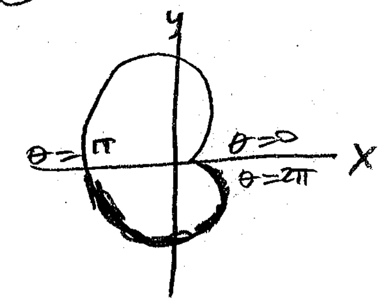
$\text{area} = \text{area}_{\text{outer}} - \text{area}_{\text{inner}}$

$$\boxed{\text{area} = \frac{1}{2}\int_{-\pi/2}^{0} (8\cos\theta)^2\,d\theta - \frac{1}{2}\int_{-\pi/2}^{0} (4\cos\theta)^2\,d\theta}$$

$= \frac{1}{2}\int_{-\pi/2}^{0} \left[(8\cos\theta)^2 - (4\cos\theta)^2\right]d\theta$

$= \boxed{18.8496}$

(21) arclength of $r=3-3\cos\theta$ below x-axis



$$\text{arclength} = \int_\alpha^\beta \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta \qquad \frac{dr}{d\theta} = -3(-\sin\theta) = 3\sin\theta$$

$$\boxed{\int_\pi^{2\pi} \sqrt{(3-3\cos\theta)^2+(3\sin\theta)^2}\,d\theta}$$

$$= \int_\pi^{2\pi} \sqrt{9-18\cos\theta+9\cos^2\theta+9\sin^2\theta}\,d\theta$$

$$= \int_\pi^{2\pi} \sqrt{9-18\cos\theta+9(\underbrace{\cos^2\theta+\sin^2\theta}_{1})}\,d\theta$$

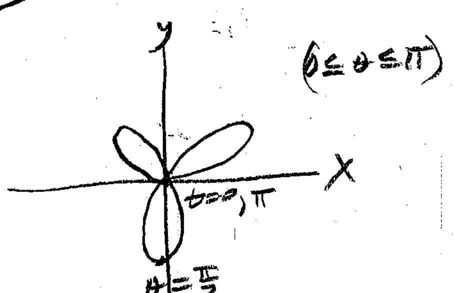
$$= \int_\pi^{2\pi} \sqrt{18-18\cos\theta}\,d\theta$$

this one requires math 9, but if this were simpler we may ask you to evaluate by hand.

$$= \boxed{12}$$

---

(22) one time around $r=4\sin(3\theta)$



$$\text{arclength} = \int_\alpha^\beta \sqrt{r^2+\left(\frac{dr}{d\theta}\right)^2}\,d\theta \qquad \frac{dr}{d\theta} = 4\cos(3\theta)\cdot 3 = 12\cos(3\theta)$$

$$= \boxed{\int_0^\pi \sqrt{(4\sin(3\theta))^2+(12\cos(3\theta))^2}\,d\theta}$$

$$= \int_0^\pi \sqrt{16\sin^2(3\theta)+144\cos^2(3\theta)}\,d\theta$$

math 9 $\approx \boxed{26.7298}$