AP Calc BC

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Name answers/solutions

9.1 Worksheet (Conics Review)

Period____

Classify each conic section.

1) $x^2 + y^2 - 6x - 8y + 24 = 0$ circle

2) $3x^2 - 5y^2 - 10y - 80 = 0$ hyperbola

3) $2y^2 + x - 12y + 20 = 0$ ellipse

4) $3x^2 + 4y^2 - 8y - 116 = 0$ ellipse

5) $x^2 + y^2 + 8y + 10 = 0$ circle

6) $-2x^2 + 20x + y - 47 = 0$ parabola

7) $4x^2 - 9y^2 + 54y - 117 = 0$ hyperbola

8) $9x^2 + 49y^2 + 294y = 0$ ellipse

9) $49x^2 + y^2 - 294x + 392 = 0$ ellipse

10) $x^2 + y^2 - 4x + 8y + 19 = 0$ circle

11) $9x^2 + 49y^2 + 98y - 392 = 0$ ellipse

12) $-x^2 + y^2 - 2x + 2y - 4 = 0$ hyperbola

Identify the center and radius of each.

13) $(x+8)^2 + (y-11)^2 = 25$

Center: $(-8, 11)$
radius $= 5$

14) $(x-5)^2 + (y-2)^2 = 25$

Center: $(5, 2)$
radius $= \sqrt{25} = 5$

15) $(x+8)^2 + (y+2)^2 = 67$

Center: $(-8, -2)$
radius $= \sqrt{67}$

16) $(x-13)^2 + (y-5)^2 = 13$

center: $(13, 5)$
radius $= \sqrt{13}$

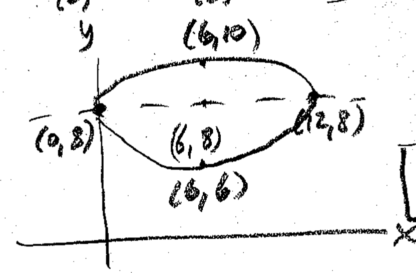
Identify the center, vertices, co-vertices, and foci of each.

17) $\dfrac{(x+1)^2}{4} + \dfrac{(y+5)^2}{81} = 1$

Center: $(-1, -5)$
vertices: $(-1, 4)\ (-1, -14)$
co-vertices: $(-3, -5)$ u $(1, -5)$
Foci: $(-1, -5+\sqrt{77})\ (-1, -5-\sqrt{77})$

18) $\dfrac{(x-6)^2}{36} + \dfrac{(y-8)^2}{4} = 1$

$(6)^2$ $(2)^2$



center: $(6, 8)$
vertices: $(12, 8)\ (0, 8)$
co-vertices: $(6, 10)\ (6, 6)$

$c^2 = a^2 - b^2$
$c^2 = 36 - 4 = 32$
$c = \sqrt{32}$

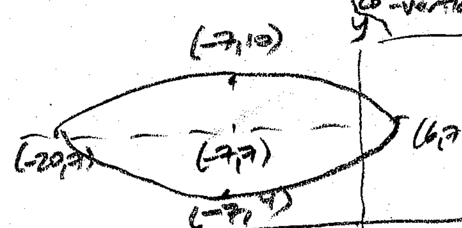
Foci: $(6-\sqrt{32}, 8)$
$(6+\sqrt{32}, 8)$

19) $\dfrac{(x+2)^2}{81} + \dfrac{(y-10)^2}{4} = 1$

Center: $(-2, 10)$
vertices: $(-11, 10)\ (7, 10)$
co-vertices: $(-2, 12)\ (-2, 8)$
Foci: $(-2+\sqrt{77}, 10)\ (-2-\sqrt{77}, 10)$

20) $\dfrac{(x+7)^2}{169} + \dfrac{(y-7)^2}{9} = 1$

center: $(-7, 7)$
vertices: $(-20, 7)\ (6, 7)$
co-vertices: $(-7, 10)\ (-7, 4)$



$c^2 = a^2 - b^2$
$c^2 = 169 - 9 = 160$
$c = \sqrt{160}$

Foci: $(-7+\sqrt{160}, 7)$
$(-7-\sqrt{160}, 7)$

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Identify the vertices, foci, and direction of opening of each.

21) $\frac{(y-6)^2}{100} - \frac{(x-6)^2}{100} = 1$

center: $(6,6)$

$\boxed{\text{vertices: } (6,16)\ (6,-4) \quad \text{foci: } (6, 6+\sqrt{200})\ (6, 6-\sqrt{200})}$

$\boxed{\text{direction: } \smile\ \frown}$

22) $\frac{(x+3)^2}{121} - \frac{(y+1)^2}{144} = 1$

$(11)^2 \quad (12)^2$

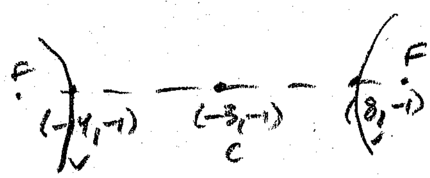
center: $(-3,-1)$

$\boxed{\text{vertices: } (-14,-1)\ (8,-1)}$

$c^2 = a^2 + b^2$

$c^2 = 121 + 144 = 265$

$c = \sqrt{265}$



$\boxed{\text{foci: } (-3+\sqrt{265}, -1)\ (-3-\sqrt{265}, -1)}$

23) $\frac{(x-1)^2}{121} - (y+4)^2 = 1$

center: $(1,-4)$

vertices: $(-10,-4)\ (12,-4)$

foci: $(1+\sqrt{122}, -4)\ (1-\sqrt{122}, -4)$

direction:) (

24) $\frac{(x+4)^2}{81} - \frac{(y+6)^2}{25} = 1$

$(9)^2 \quad (5)^2$

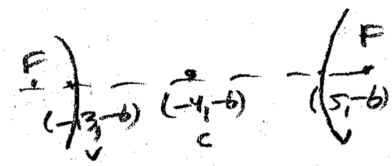
center: $(-4,-6)$

$\boxed{\text{vertices: } (-13,-6)\ (5,-6)}$

$c^2 = a^2 + b^2$

$c^2 = 81 + 25 = 106$

$c = \sqrt{106}$



$\boxed{\text{foci: } (-4+\sqrt{106}, -6)\ (-4-\sqrt{106}, -6)}$

Identify the vertex, intercepts on the axis parallel to the axis of symmetry, and intercepts on the axis perpendicular to the axis of symmetry of each.

25) $x = y^2 - 4y + 4$

$\boxed{\text{vertex: } (0,2) \quad (0,2) \quad (4,0)}$

26) $y = -x^2 + 9x - 20$

$y + 20 - \frac{81}{4} = -(x^2 - 9x + \frac{81}{4})$

$(y - \frac{1}{4}) = -(x - \frac{9}{2})^2$

$\boxed{\text{vertex: } (\frac{9}{2}, \frac{1}{4})}$

y int $(x=0)$

$y = -(0)^2 + 9(0) - 20$

$y = -20$ $\boxed{(0,-20)}$

x int $(y=0)$

$-x^2 + 9x - 20 = 0$

$x = \frac{-9 \pm \sqrt{81 - 4(-1)(-20)}}{2(-1)}$

non-real N/A

27) $x = -3y^2 - 30y - 76$

$\boxed{\text{vertex: } (-1,5)}$

$\boxed{(-76,0)}$

28) $x = 7y^2 - 56y + 105$

$x - 105 = 7(y^2 - 8y \quad)$

$x - 105 + 112 = 7(y^2 - 8y + 16)$

$(x + 7) = 7(y-4)^2$ $\boxed{\text{vertex: } (-7,4)}$

y int $(x=0)$

$7y^2 - 56y + 105 = 0$

$y = \frac{56 \pm \sqrt{(56)^2 - 4(7)(105)}}{2(7)} = \frac{56 \pm 14}{14} = \frac{19}{14} \ \& \ \frac{-9}{14}$

$\boxed{(0, \frac{19}{14})(0, \frac{-9}{14})}$

x int $(y=0)$

$x = 7(0)^2 - 56(0) + 105$

$x = 105$

$\boxed{(105,0)}$

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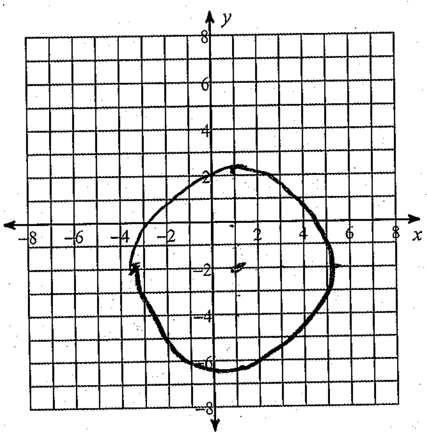
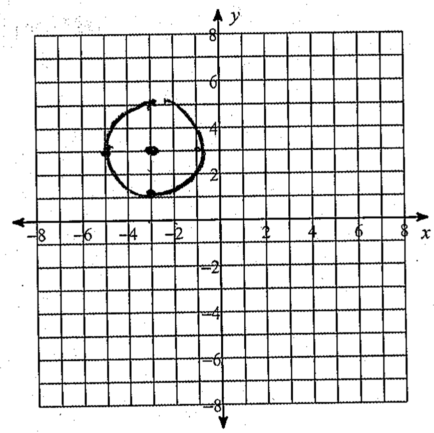
Graph each equation.

29) $(x-1)^2+(y+2)^2=17$

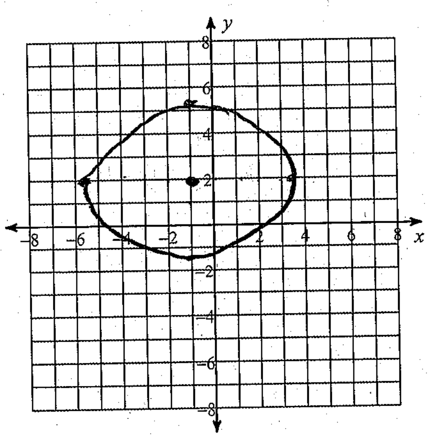


30) $(x+3)^2+(y-3)^2=4$

circle
center: $(-3,3)$
$r=\sqrt{4}=2$



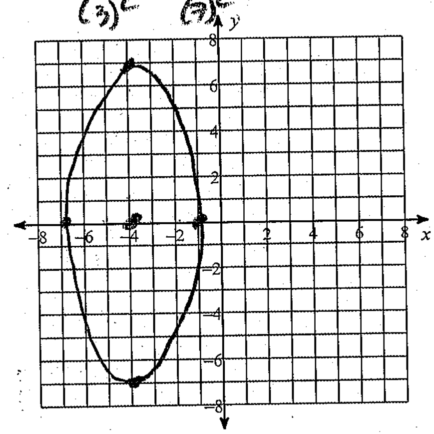
31) $\dfrac{(x+1)^2}{20}+\dfrac{(y-2)^2}{10}=1$



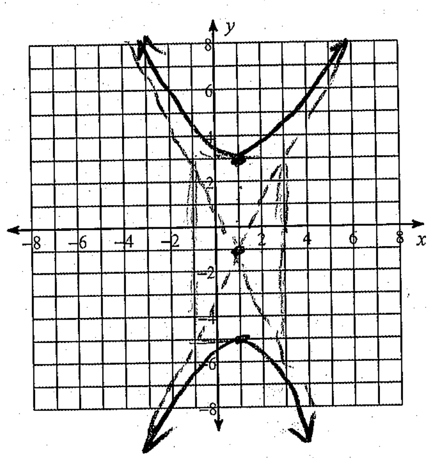
32) $\dfrac{(x+4)^2}{9}+\dfrac{y^2}{49}=1$

$(3)^2$ $(7)^2$

ellipse
center: $(-4,0)$
$a=\sqrt{49}=7$
$b=\sqrt{9}=3$



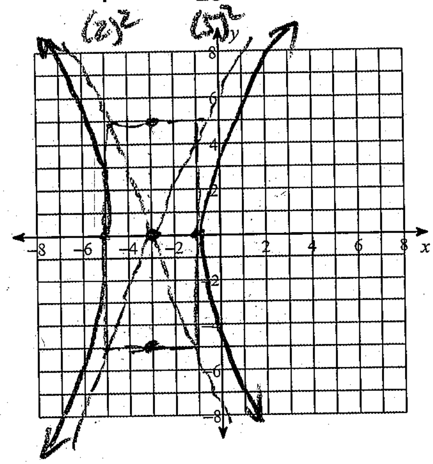
33) $\dfrac{(y+1)^2}{16}-\dfrac{(x-1)^2}{4}=1$



34) $\dfrac{(x+3)^2}{4}-\dfrac{y^2}{25}=1$

$(2)^2$ $(5)^2$

hyperbola
center: $(-3,0)$
$a=\sqrt{4}=2$
$b=\sqrt{25}=5$



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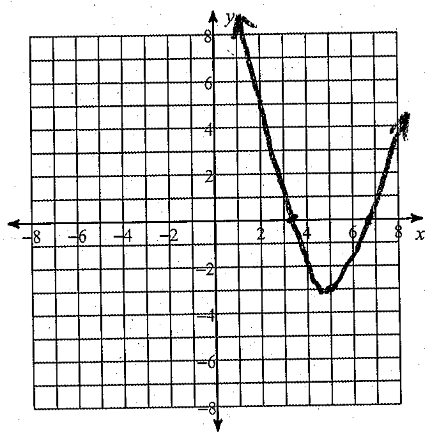
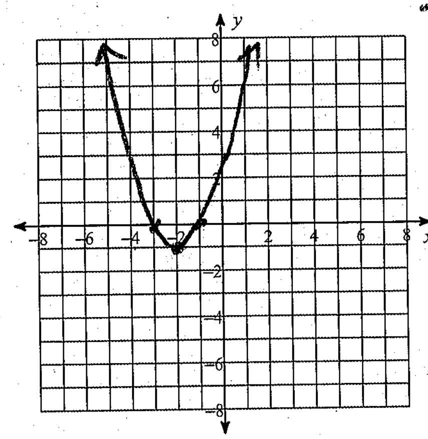
35) $y = x^2 - 10x + 22$

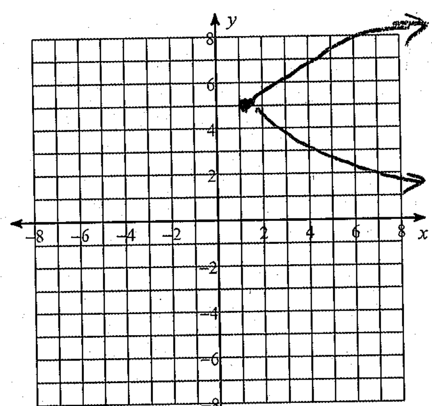


36) $y = x^2 + 4x + 3$



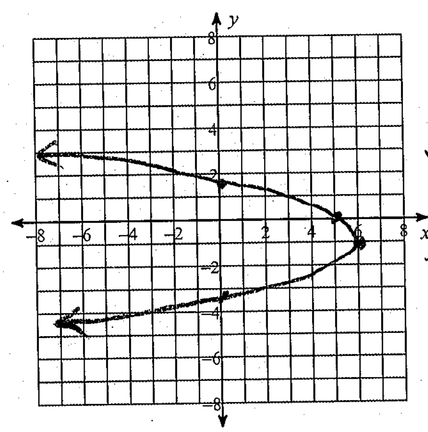
parabola
$y - 3 = (x^2 + 4x)$
$y - 3 + 4 = (x^2 + 4x + 4)$
$(y+1) = (x+2)^2$
vertex: $(-2, -1)$
xint ($y=0$)
$x^2 + 4x + 3 = 0$
$(x+1)(x+3) = 0$
$x = -1, x = -3$
$(-1, 0)$ $(-3, 0)$
yint ($x=0$) $y = 3$
$(0, 3)$

37) $x = y^2 - 10y + 26$



(difficult to graph — intercept is off the graph)

38) $x = -y^2 - 2y + 5$



parabola
$x - 5 - 1 = -(y^2 + 2y + 1)$
$(x - 6) = -(y+1)^2$
vertex: $(6, -1)$
xint ($y=0$) $x = 5$ $(5, 0)$
yint ($x=0$)
$-y^2 - 2y + 5 = 0$
$y^2 + 2y - 5 = 0$
$y = \frac{-2 \pm \sqrt{4 + (4)(5)}}{2(1)}$
$= \frac{-2 \pm \sqrt{24}}{2} \approx \frac{-2 \pm 4.9}{2}$
$\approx (0, 1.45)$ $(0, -3.45)$

Write the equation of each conic section in standard form.

39) $x^2 + y^2 - 8x + 6y + 24 = 0$

40) $x^2 + y^2 + 6x + 6y + 2 = 0$

41) $x^2 - 16y^2 - 2x + 128y - 271 = 0$

42) $-x^2 + 25y^2 - 150y + 200 = 0$

43) $x^2 + 16y^2 - 2x - 192y + 561 = 0$

44) $4x^2 + y^2 - 2y - 35 = 0$

45) $-x^2 + 6x + y - 10 = 0$

46) $y^2 + x - 5 = 0$

on separate page...

#39) $\boxed{(x-4)^2+(y+3)^2=1}$

(circle)

#40) $x^2+y^2+6x+6y+2=0$

$(x^2+6x)+(y^2+6y)=-2$

$(x^2+6x+\underline{9})+(y^2+6y+\underline{9})=-2+\underline{9}+\underline{9}$

$\boxed{(x+3)^2+(y+3)^2=16}$ (circle)

#41) $\boxed{\dfrac{(x-1)^2}{16}-\dfrac{(y-4)^2}{1}=1}$

(hyperbola)

#42) $-x^2+25y^2-150y+200=0$

$-x^2+(25y^2-150y)=-200$

$-(x-0)^2+25(y^2-6y+\underline{9})=-200+\underline{225}$

$\dfrac{-(x-0)^2}{25}+\dfrac{25(y-3)^2}{25}=\dfrac{25}{25}$

$\boxed{-\dfrac{(x-0)^2}{25}+\dfrac{(y-3)^2}{1}=1}$ (hyperbola)

#43) $\boxed{\dfrac{(x-1)^2}{16}+\dfrac{(y-6)^2}{1}=1}$

(ellipse)

#44) $4x^2+y^2-2y-35=0$

$4x^2+(y^2-2y+\underline{1})=35+\underline{1}$

$\dfrac{4(x-0)^2}{36}+\dfrac{(y-1)^2}{36}=\dfrac{36}{36}$

$\boxed{\dfrac{(x-0)^2}{9}+\dfrac{(y-1)^2}{36}=1}$ (ellipse)

#45) $\boxed{(y-1)=(x-3)^2}$

(parabola)

#46) $y^2+x-5=0$

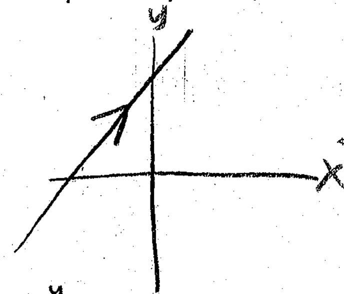
$x-5=-y^2$

$\boxed{(x-5)=-(y-0)^2}$ (parabola)

9.2 Worksheet

Sketch the curve of the parametric equation by hand (use a table), write the corresponding rectangular equation.

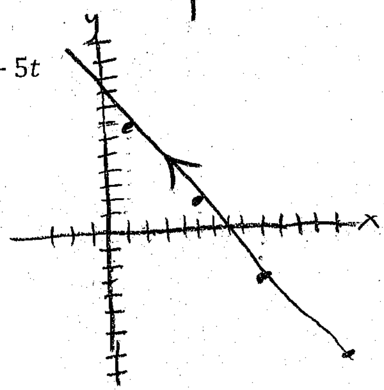
1. $x = 2t - 3, y = 3t + 1$



$\boxed{y = \frac{3}{2}x + \frac{11}{2}}$

2. $x = 5 - 4t, y = 2 + 5t$

t	(x, y)
-2	(13, -8)
-1	(9, -3)
0	(5, 2)
1	(1, 7)
2	(-3, 12)



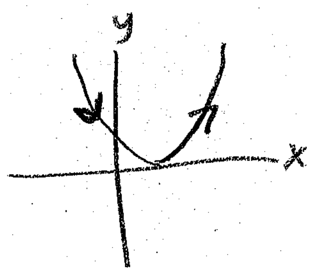
$x = 5 - 4t$ $\quad y = 2 + 5t$

$4t = 5 - x$ $\quad y = 2 + 5\left(\frac{5-x}{4}\right)$

$t = \frac{5-x}{4}$ $\quad y = 2 + \frac{25}{4} - \frac{5}{4}x$

$\boxed{y = -\frac{5}{4}x + \frac{33}{4}}$

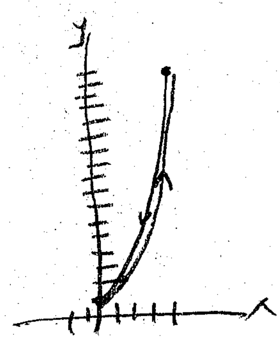
3. $x = t + 1, y = t^2$



$\boxed{(y-0) = (x-1)^2}$

4. $x = t^2, y = t^4 + 1$

t	(x, y)
-2	(4, 17)
-1	(1, 2)
0	(0, 1)
1	(1, 2)
2	(4, 17)

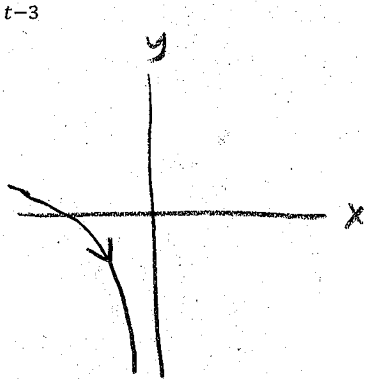


$x = t^2$ $\quad y = t^4 + 1$

$t^2 = x \rightarrow$ $\quad y = (t^2)^2 + 1$

$\boxed{y = (x)^2 + 1}$

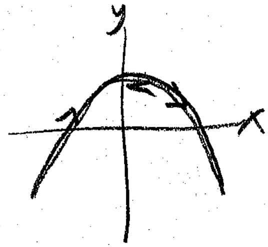
5. $x = t - 3, y = \frac{t}{t-3}$



$\boxed{y = \frac{x+3}{x}}$

Use a graphing calculator to graph the parametric equation. Eliminate the parameter and write in rectangular form.

6. $x = 6\sin(\theta), y = 4\cos(2\theta)$



$x = 6\sin\theta$

$\sin\theta = \frac{x}{6}$

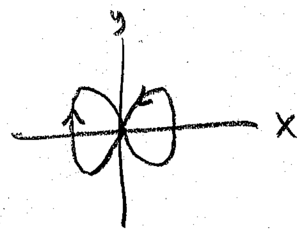
$\theta = \arcsin\left(\frac{x}{6}\right)$

$y = 4\cos(2\theta)$

$\boxed{y = 4\cos\left(2\arcsin\left(\frac{x}{6}\right)\right)}$

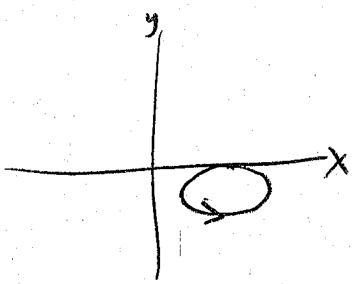
7. $x = \cos(\theta), y = 2\sin(2\theta)$

algebraically, $y = 2\sin(2\arccos(x))$ but graph doesn't show complete curve.
Instead use Pythagorean trig identity to obtain: (also need $\sin(2\theta) = 2\sin\theta\cos\theta$)



$$\boxed{\begin{gathered}(x)^2 + \left(\frac{y}{4x}\right)^2 = 1 \\ \text{or} \\ y = \pm\sqrt{16x^2(1-x^2)}\end{gathered}}$$

8. $x = 4 + 2\cos(\theta), y = -1 + \sin(\theta)$



$x = 4 + 2\cos\theta$

$2\cos\theta = x - 4$

$\cos\theta = \frac{x-4}{2}$

$y = -1 + \sin\theta$

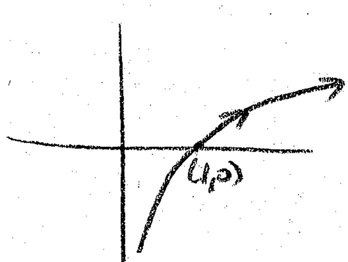
$\sin\theta = y + 1$

now: $\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{x-4}{2}\right)^2 + (y+1)^2 = 1$

$\boxed{\frac{(x-4)^2}{4} + \frac{(y+1)^2}{1} = 1}$ (ellipse)

9. $x = t^3, y = 3\ln(t)$



$\boxed{y = 3\ln\left(\sqrt[3]{x}\right)}$

9.3 Worksheet

Find dy/dx

1. $x = t^2, y = 7 - 6t$

$\boxed{-\frac{3}{t}}$

2. $x = t^{1/3}$

$x = \sqrt[3]{t}, y = 4 - t$

$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{-1}{\frac{1}{3}t^{-2/3}} = \boxed{-3t^{2/3}}$

3. $x = \sin^2(\theta), y = \cos^2(\theta)$

$\boxed{-1}$

4. $x = 2e^{\theta}, y = e^{-\frac{\theta}{2}}$

$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{e^{\left(-\frac{1}{2}\theta\right)}\left(-\frac{1}{2}\right)}{2e^{\theta}} = \boxed{-\frac{1}{4}e^{-3/2\,\theta}}$

Find $\frac{dy}{dx}$ and $\frac{d^2y}{(dx)^2}$, and find the slope and concavity (if possible) at the given value of the parameter.

5. $x = 4t, y = 3t - 2$ Parameter: $t = 3$

$\boxed{\frac{dy}{dx} = \frac{3}{4}}$

$\boxed{\frac{d^2y}{dx^2} = 0}$

6. $x = t^{1/2}$

$x = \sqrt{t}, y = 3t - 1$ Parameter: $t = 1$

$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{3}{\frac{1}{2}t^{-1/2}} = 6t^{1/2} = 6\sqrt{t}\Big|_{t=1} = 6\sqrt{1} = \boxed{6}$

$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{\left(\frac{dx}{dt}\right)} = \frac{6\left(\frac{1}{2}t^{-1/2}\right)}{1/2\,t^{-1/2}} = 6\Big|_{t=1} = \boxed{6}$

7. $x = 4\cos(\theta), y = 4\sin(\theta)$ Parameter: $\theta = \frac{\pi}{4}$

$$\boxed{\frac{dy}{dx} = -1}$$

$$\boxed{\frac{d^2y}{dx^2} = \frac{1}{\sqrt{2}}}$$

8. $x = \cos(\theta), y = 3\sin(\theta)$ Parameter: $\theta = 0$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{3\cos\theta}{-\sin\theta} = -3\cot\theta\Big|_{\theta=0} = \frac{3\cos(0)}{-\sin(0)} = \frac{3(1)}{0} \boxed{\text{undefined}}$$ (vertical tangent)

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{d\theta}\left[\frac{dy}{dx}\right]}{\left(\frac{dx}{d\theta}\right)} = \frac{3\csc^2\theta}{-\sin\theta} = -3\csc^3\theta\Big|_{\theta=0} = \frac{-3}{(\sin 0)^3} = \frac{-3}{0} \boxed{\text{undefined}}$$

9. $x = \sqrt{t}, y = \sqrt{t-1}$ Parameter: $t = 2$

$$\boxed{\frac{dy}{dx} = \sqrt{2}}$$

$$\boxed{\frac{d^2y}{dx^2} = -1}$$

10. $x = \sqrt{t+1}, y = \sqrt{t-1}$ Parameter: $t = 2$

$x = (t+1)^{1/2}$ $y = (t-1)^{1/2}$

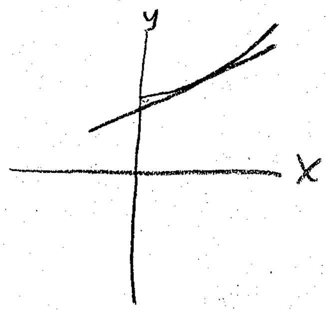
$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\frac{1}{2}(t-1)^{-1/2}(1)}{\frac{1}{2}(t+1)^{-1/2}(1)} = \frac{(t+1)^{1/2}}{(t-1)^{1/2}} = \frac{\sqrt{t+1}}{\sqrt{t-1}}\Bigg|_{t=2} = \frac{\sqrt{3}}{\sqrt{1}} = \boxed{\sqrt{3}}$$

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{\left(\frac{dx}{dt}\right)} = \frac{\left[\frac{(t-1)^{1/2}\left(\frac{1}{2}(t+1)^{-1/2}(1)\right) + (t+1)^{1/2}\left(\frac{1}{2}(t-1)^{-1/2}(1)\right)}{\left[(t-1)^{1/2}\right]^2}\right]}{\left(\frac{1}{2\sqrt{t+1}}\right)} = \frac{\left(\frac{\sqrt{t-1}}{2\sqrt{t+1}} + \frac{\sqrt{t+1}}{2\sqrt{t-1}}\right)}{\left(\frac{1}{2\sqrt{t+1}}\right)}\Bigg|_{t=2}$$

$$= \frac{\left(\frac{1}{2\sqrt{3}} + \frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2\sqrt{3}}\right)} = \frac{\frac{1}{2\sqrt{3}} + \frac{3}{2\sqrt{3}}}{\frac{1}{2\sqrt{3}}} = \frac{1+3}{1} = \boxed{4}$$

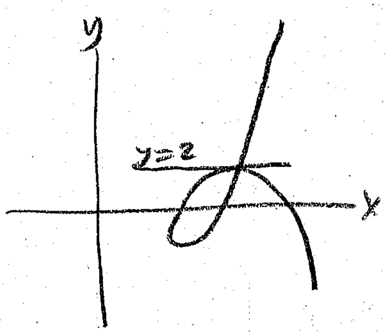
Graph the curve on your calculator. Use your calculator to find $\frac{dy}{dx}, \frac{dy}{dt}, \frac{dx}{dt}$ at the given parameter. Find a tangent line equation to the Cartesian Coordinate at the given parameter. Use your graphing calculator to verify that the line is tangent. *This is mostly just figure out how to use your calculator, right??*

11. $x = 6t, y = t^2 + 4$ Parameter: $t = 1$



$\boxed{y = \frac{1}{3}x + 3}$

12. $t^2 - t + 2, y = t^3 - 3t$ Parameter: $t = -1$



2nd trace;

$\frac{dy}{dx} = 0$

$\frac{dy}{dt} = 0$

$\frac{dx}{dt} = -3$

$y(-1) = (-1)^3 - 3(-1) = -1 + 3 = 2$

$x(-1) = (-1)^2 - (-1) + 2 = 1 + 1 + 2 = 4$

$(y-2) = 0(x-4)$

$y - 2 = 0$

$\boxed{y = 2}$

Find the equations of the tangent lines at the point where the curve crosses itself.

13. $x = 2\sin(2t), y = 3\sin(t)$

$$\boxed{\begin{array}{l} y = \frac{3}{4}x \\ y = -\frac{3}{4}x \end{array}}$$

14. $x = t^2 - t, y = t^3 - 3t - 1$

Intersection occurs when $t = -1$ & $t = 2$:

$t = -1$: $\boxed{(y-1) = \frac{1}{3}(x-2)}$

$t = 2$: $\boxed{(y-1) = 3(x-2)}$

Find all points (if any) of horizontal and vertical tangency to the portion of the curve shown. Use your calculator to confirm your results.

15. $x = 4 - t, y = t^2$

$\boxed{\text{no points}}$ of vertical tangency

horizontal tangent at $\boxed{(4,0)}$

16. $x = t+1, y = t^2+3t$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{2t+3}{1}$$

vertical tangent when denom $=0$ $\boxed{(\text{NONE})}$

horiz. tangent when numerator $=0$ $\quad 2t+3=0$

$t = -\frac{3}{2}$

$x(-\frac{3}{2}) = (-\frac{3}{2})+1 = -\frac{1}{2}$

$y(-\frac{3}{2}) = (-\frac{3}{2})^2+3(-\frac{3}{2}) = \frac{9}{4} - \frac{9}{2} = \frac{9}{4} - \frac{18}{4} = -\frac{9}{4}$

horiz. tangent at $\boxed{(-\frac{1}{2}, -\frac{9}{4})}$

15. $x = 3\cos(\theta), y = 3\sin(\theta)$

vertical tangents at $\boxed{(3,0) \text{ and } (-3,0)}$

horiz. tangents at $\boxed{(0,-3) \text{ and } (0,3)}$

16. $x = \cos(\theta), y = 2\sin(2\theta)$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{2\cos(2\theta)2}{-\sin\theta}$$

vertical tangent when denom $=0$ $(\sin\theta = 0)$

$\theta = -\pi, 0, \pi \ldots$

$x(0) = \cos 0 = 1, \quad y(0) = 2\sin 0 = 0$ $\quad \boxed{(1,0)}$

$x(\pi) = \cos\pi = -1, \quad y(\pi) = 2\sin\pi = 0$ $\quad \boxed{(-1,0)}$

horiz tangent when numerator $=0$ $(\cos(2\theta) = 0)$

$2\theta = -\frac{3\pi}{2}, -\frac{\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2} \ldots$

$\theta = -\frac{3\pi}{4}, -\frac{\pi}{4}, \frac{\pi}{4}, \frac{3\pi}{4} \ldots$

$x(-\frac{3\pi}{4}) = \cos(-\frac{3\pi}{4}) = -\frac{\sqrt{2}}{2}, \quad y(-\frac{3\pi}{4}) = 2\sin(-\frac{3\pi}{2}) = 2$ $\quad (-\frac{\sqrt{2}}{2}, 2)$

$x(-\frac{\pi}{4}) = \cos(-\frac{\pi}{4}) = \frac{\sqrt{2}}{2}, \quad y(-\frac{\pi}{4}) = 2\sin(-\frac{\pi}{2}) = -2$ $\quad (\frac{\sqrt{2}}{2}, -2)$

$x(\frac{\pi}{4}) = \cos(\frac{\pi}{4}) = \frac{\sqrt{2}}{2}, \quad y(\frac{\pi}{4}) = 2\sin(\frac{\pi}{2}) = 2$ $\quad (\frac{\sqrt{2}}{2}, 2)$

$x(\frac{3\pi}{4}) = \cos(\frac{3\pi}{4}) = -\frac{\sqrt{2}}{2}, \quad y(\frac{3\pi}{4}) = 2\sin(\frac{3\pi}{2}) = -2$ $\quad (-\frac{\sqrt{2}}{2}, -2)$

Determine the open t-intervals on which the curve is concave down or concave up.

17. $x = 2t + \ln(t), y = 2t - \ln(t)$

$$\boxed{\begin{array}{l}\text{Concave up: } t>0 \\ \text{concave down: nowhere}\end{array}}$$

18. $x = t^2, y = \ln(t)$

$$\frac{dy}{dx} = \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\left(\frac{1}{t}\right)}{2t} = \frac{1}{2t^2} = \frac{1}{2}t^{-2}$$

inflection pts when $f'' = 0$ or DNE

$$\frac{d^2y}{dx^2} = \frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{\left(\frac{dx}{dt}\right)} = \frac{-t^{-3}}{2t} = \frac{-1}{2t^4}$$

$\frac{d^2y}{dx^2} = 0$ N/A

$\frac{d^2y}{dx^2}$ DNE : $2t^4 = 0$, $t = 0$

t: $-\infty$ … 0 … ∞

(-1): $\frac{d^2y}{dx^2} \frac{(-)}{(+)}$ $-$

(1): $\frac{d^2y}{dx^2} \frac{(-)}{(+)}$ $-$

not in domain

would be concave up nowhere

concave down $t<0$ & $t>0$

but… $y = \ln(t)$

domain t: $t > 0$

so

$$\boxed{\begin{array}{l}\text{concave up: nowhere} \\ \text{concave down: } t>0\end{array}}$$

Find the arc length of the curve on the given interval.

19. $x = 3t + 5, y = 7 - 2t$ over $-1 \leq t \leq 3$

arclength $= \boxed{4\sqrt{13}}$

20. $x = 6t^2, y = 2t^3$ over $1 \leq t \leq 4$

arclength $= \int_1^4 \sqrt{(12t)^2 + (6t^2)^2}\, dt = \int_1^4 \sqrt{144t^2 + 36t^4}\, dt$ (math 9)

$\approx \boxed{156.525}$

21. $x = e^{-t}\cos(t), y = e^{-t}\sin(t)$ over $0 \leq t \leq \frac{\pi}{2}$

arclength $= \int_0^{\pi/2} \sqrt{2e^{-2t}}\, dt$ (math 9) $\approx \boxed{1.1202}$

22. $x = \arcsin(t), y = \ln(\sqrt{1 - t^2})$ over $0 \leq t \leq \frac{1}{2}$

$\frac{dx}{dt} = \frac{1}{\sqrt{1-t^2}}$

$\left(\frac{dx}{dt}\right)^2 = \frac{1}{1-t^2}$

$\frac{dy}{dt} = \frac{1}{\sqrt{1-t^2}} \cdot \frac{1}{2}(1-t^2)^{-1/2}(-2t)$

$= \frac{-t}{1-t^2}$

$\left(\frac{dy}{dt}\right)^2 = \frac{t^2}{(1-t^2)^2}$

arclength $= \int_0^{1/2} \sqrt{\frac{1}{1-t^2} + \frac{t^2}{(1-t^2)^2}}\, dt$

$= \int_0^{1/2} \sqrt{\frac{(1-t^2) + t^2}{(1-t^2)^2}}\, dt$

(math 9)

$\approx \boxed{0.435}$

Multiple Choice:

If $x = 2t^4$ and $y = (4t+1)^2$, then $\frac{dy}{dx} =$

a) $\frac{1}{t^2} + \frac{1}{4t^3}$

b) $\frac{4}{t^2} + \frac{1}{t^3}$

c) $\frac{t^3}{4t+1}$

d) $\frac{t^2}{4} + t^3$

$\boxed{B}$

Multiple Choice:

The length of the curve given by $x = \sin(3t)$ and $y = \cos(2t)$ from $t = 0$ to $t = \pi$ is represented by

a) $\int_0^{\pi} \sqrt{9\sin^2(3t) + 4\cos^2(2t)}\, dt$

b) $\int_0^{\pi} \sqrt{9\cos^2(3t) + 4\sin^2(2t)}\, dt$

c) $\int_0^{\pi} \sqrt{3\,sin(3t) + 2\,cos(2t)}\, dt$

d) $\int_0^{\pi} \sqrt{3\,cos(3t) - 2\,sin(2t)}\, dt$

$\boxed{B}$

Free Response:

At time $t \geq 0$, the position of the particle moving along a curve in the xy-plane is $(x(t), y(t))$, where

$\frac{dx}{dt} = 2t - 5\cos(t)$ and $\frac{dy}{dt} = -\sin(t)$.

At time $t = 4$, the particle is at the point $(-1,3)$.

a. Write an equation for the tangent line to the path of the particle at time $t = 4$.

b. Find the time t when the tangent line to the path of the particle is vertical. Is the direction of the motion of the particle up or down at that moment? Explain your reasoning.

c. Find the y-coordinate of the position of the particle at time $t = 0$.

d. Find the total distance traveled by the particle on the interval $0 \leq t \leq 4$.

(a) $\boxed{(y-3) = \left[\frac{-\sin(4)}{8-5\cos(4)}\right](x+1), \quad (y-3) = 0.06716(x+1)}$

(b) $\boxed{t = 1.1105105, \text{ down b/c } \frac{dy}{dt} < 0.}$

(c) $\boxed{y(0) = 4 - \cos(4) \approx 4.6536}$

(d) total dist. traveled = arclength $\approx \boxed{26.656}$

Calculus 2
Unit 9 Part 1 REVIEW

Identify the conic, put the equation in standard form, and sketch:

#1. $x^2 - 6x - 8y - 7 = 0$

#2. $y^2 + 16x + 2y - 63 = 0$

#3. $3x^2 + 3y^2 + 12x + 18y + 12 = 0$

#4. $2x^2 + 2y^2 - 16x + 4y + 24 = 0$

#5. $9x^2 + 4y^2 + 18x - 16y - 11 = 0$

#6. $2x^2 + 50y^2 - 20x + 300y + 450 = 0$

#7. $4x^2 - 9y^2 + 16x + 54y - 101 = 0$

#8. $9x^2 - 25y^2 - 54x - 50y + 281 = 0$

Convert the equation to rectangular form and sketch the curve (include direction arrows):

#9. $x = 2t, \quad y = t^2 + 3$

#10. $x = 3\cos t, \quad y = 5\sin t$

#11. $x = 2 - 3\cos\theta, \quad y = -3 + 3\sin\theta$

Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the curve given by the parametric equations set, then find the slope and concavity at the given parameter value:

#12. $x = 4t^3 + t^2 - 2, \quad y = t^2 - 1 \quad$ at $t = 2$

#13. $x = \ln(t), \quad y = \frac{1}{t^2} \quad$ at $t = 1$

Find the arc length of the curve on the given interval:

#14. $x = 3\cos^2 t, \quad y = e^{2t} \quad 1 \le t \le 4$

#15. $x = \ln(t^2), \quad y = t^3 + 2 \quad 1 \le t \le 2$

Find an equation of the tangent line to the curve at the given value of the parameter:

#16. $x = 2\cos(t), \quad y = 3\sin(t) \quad at\ t = \frac{\pi}{6}$

#17. $x = t^3 + 4t^2, \quad y = 3t^{4/3} \quad at\ t = \frac{1}{8}$

Find a set of parametric equations which represents the given curve:

#18. A circle centered at (3, -4) with radius = 3, clockwise direction.

#19. An ellipse centered at the origin with major axis in y direction, a = 3, minor axis in x direction b = 1, and counter-clockwise direction.

Unit 9 Part I Review – SOLUTIONS

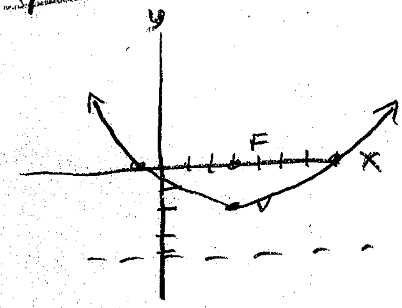
(1) $x^2 - 6x - 8y - 7 = 0$

$x^2 - 6x + \underline{9} = 8y + 7 + \underline{9}$

$(x-3)^2 = 8y + 16$

$\boxed{(x-3)^2 = 8(y+2)}$

$\boxed{\text{parabola}}$ $4p = 8,\ p = 2$



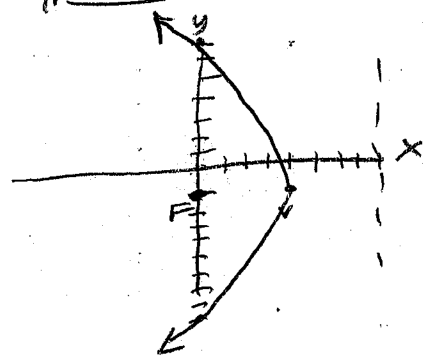
(2) $y^2 + 16x + 2y - 63 = 0$

$y^2 + 2y + \underline{1} = -16x + 63 + \underline{1}$

$(y+1)^2 = -16x + 64$

$\boxed{(y+1)^2 = -16(x-4)}$

$\boxed{\text{parabola}}$ $4p = -16,\ p = -4$



(3) $3x^2 + 3y^2 + 12x + 18y + 12 = 0$

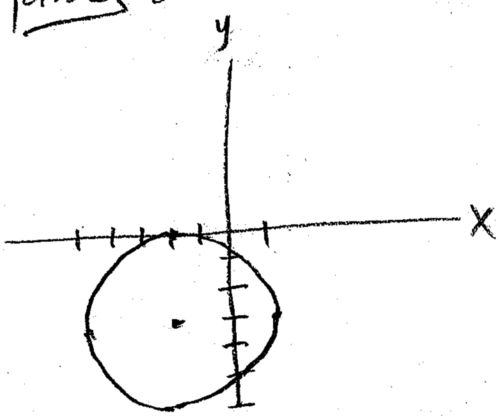
$3x^2 + 12x \quad + 3y^2 + 18y = -12$

$3(x^2 + 4x + \underline{4}) + 3(y^2 + 6y + \underline{9}) = -12 + \underline{12} + \underline{27}$

$3(x+2)^2 + 3(y+3)^2 = 27$

$\boxed{(x+2)^2 + (y+3)^2 = 9}$

$\boxed{\text{circle}}$ $(r = 3)$



(4) $2x^2 + 2y^2 - 16x + 4y + 24 = 0$

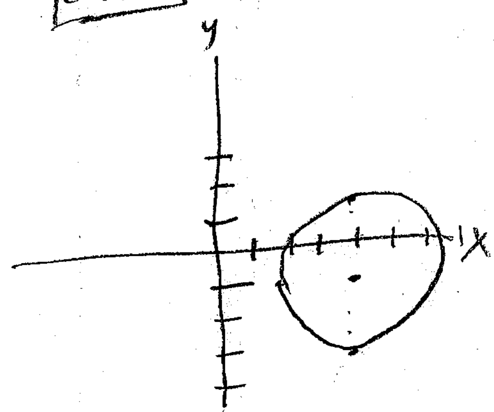
$2x^2 - 16x \quad + 2y^2 + 4y = -24$

$2(x^2 - 8x + \underline{16}) + 2(y^2 + 2y + 1) = -24 + \underline{32} + \underline{2}$

$2(x-4)^2 + 2(y+1)^2 = 10$

$\boxed{(x-4)^2 + (y+1)^2 = 5}$

$\boxed{\text{circle}}$ $(r = \sqrt{5})$



(5) $9x^2+4y^2+18x-16y-11=0$

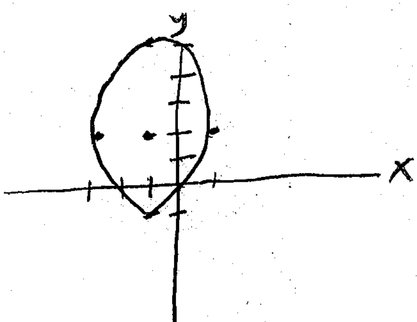
$9x^2+18x \quad +4y^2-16y \quad =11$

$9(x^2+2x+1)+4(y^2-4y+4)=11+9+16$

$9(x+1)^2+4(y-2)^2=36$

$$\boxed{\frac{(x+1)^2}{4}+\frac{(y-2)^2}{9}=1}$$

ellipse



(6) $2x^2+50y^2-20x+300y+450=0$

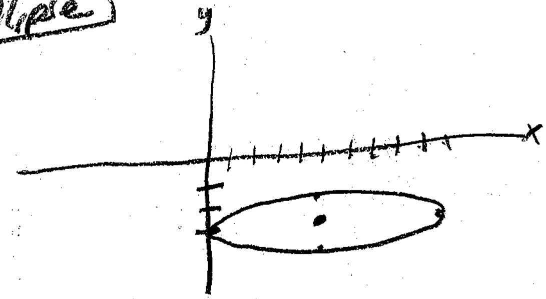
$2x^2-20x \quad +50y^2+300y \quad =-450$

$2(x^2-10x+25)+50(y^2+6y+9)=-450+50+450$

$2(x-5)^2+50(y+3)^2=50$

$$\boxed{\frac{(x-5)^2}{25}+\frac{(y+3)^2}{1}=1}$$

ellipse



(7) $4x^2-9y^2+16x+54y-101=0$

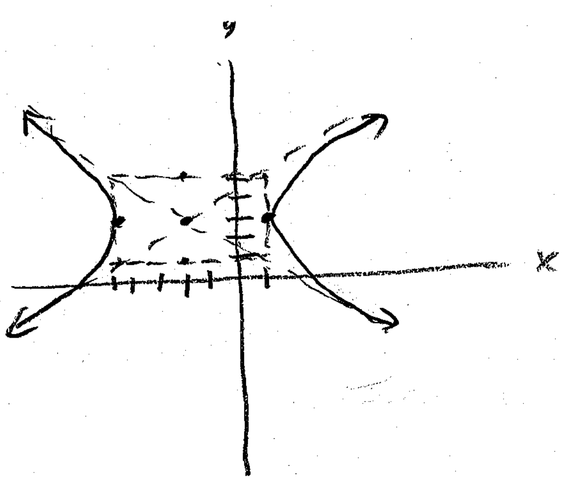
$4x^2+16x \quad -9y^2+54y \quad =101$

$4(x^2+4x+4)-9(y^2-6y+9)=101+16-81$

$4(x+2)^2-9(y-3)^2=36$

$$\boxed{\frac{(x+2)^2}{9}-\frac{(y-3)^2}{4}=1}$$

hyperbola



(8) $9x^2-25y^2-54x-50y+281=0$

$9x^2-54x \quad -25y^2-50y \quad =-281$

$9(x^2-6x+9)-25(y^2+2y+1)=-281+81-$

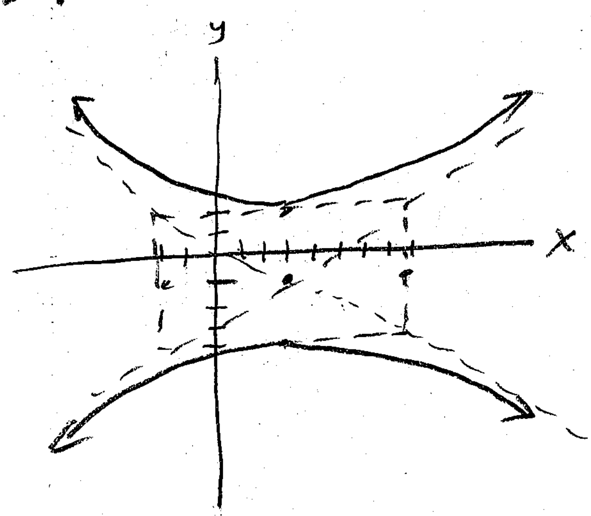
$9(x-3)^2-25(y+1)^2=-225$

$-9(x-3)^2+25(y+1)^2=225$

$25(y+1)^2-9(x-3)^2=225$

$$\boxed{\frac{(y+1)^2}{9}-\frac{(x-3)^2}{25}=1}$$

hyperbola



(9) $X=2t,\ y=t^2+3$

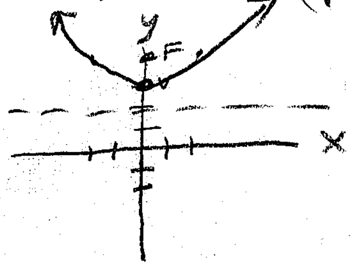
$t=\frac{x}{2} \nearrow y=\left(\frac{x}{2}\right)^2+3$

$\boxed{y=\frac{1}{4}x^2+3}$

$4y=x^2+12$

$(x-0)^2=4y-12$

$(x-0)^2=4(y-3)$ $\begin{pmatrix}4p=4\\ p=1\end{pmatrix}$



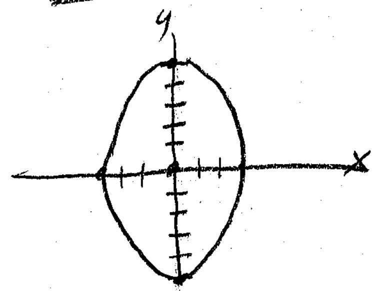
(10) $x=3\cos t,\ y=5\sin t$

$\cos t=\frac{x}{3}$ $\sin t=\frac{y}{5}$

$\cos^2 t=\frac{x^2}{9}$ $\sin^2 t=\frac{y^2}{25}$

$\cos^2 t+\sin^2 t=1$

$\boxed{\frac{x^2}{9}+\frac{y^2}{25}=1}$



(11) $x=2-3\cos\theta,\ y=-3+3\sin\theta$

$x-2=-3\cos\theta$ $y+3=3\sin\theta$

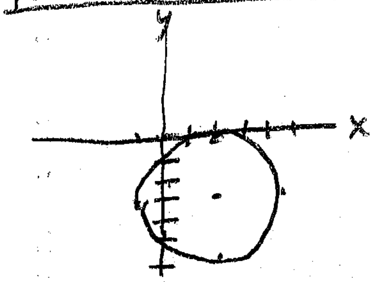
$\cos\theta=\frac{x-2}{-3}$ $\sin\theta=\frac{y+3}{3}$

$\cos^2\theta=\frac{(x-2)^2}{9}$ $\sin^2\theta=\frac{(y+3)^2}{9}$

$\cos^2 t+\sin^2 t=1$

$\frac{(x-2)^2}{9}+\frac{(y+3)^2}{9}=1$

$\boxed{(x-2)^2+(y+3)^2=9}$



(12) $x=4t^3+t^2-2$

$y=t^2-1$ (@ $t=2$)

$\frac{dy}{dx}=\frac{(dy/dt)}{(dx/dt)}=\boxed{\frac{2t}{12t^2+t}}$

slope $=\left.\frac{dy}{dx}\right|_{t=2}=\frac{2(2)}{12(2)^2+2}=\boxed{\frac{1}{13}}$

$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{\left(\frac{dx}{dt}\right)}=\frac{\frac{(12t^2+t)(2)-(2t)(24t+1)}{(12t^2+t)^2}}{12t^2+t}$

$=\frac{24t^2+\cancel{2t}-48t^2-\cancel{2t}}{(12t^2+t)^3}=\boxed{\frac{-24t^2}{(12t^2+t)^3}}$

concavity $=\left.\frac{d^2y}{dx^2}\right|_{t=2}=\frac{-24(2)^2}{(12(2)^2+4)^3}=\boxed{\frac{-96}{140608}}$

(13) $x=\ln(t)$ (@ $t=1$)

$y=\frac{1}{t^2}=t^{-2}$

$\frac{dy}{dx}=\frac{(dy/dt)}{(dx/dt)}=\frac{-2t^{-3}}{\frac{1}{t}}$

$=\boxed{\frac{-2}{t^2}}=-2t^{-2}$

slope $=\left.\frac{dy}{dx}\right|_{t=1}=\frac{4t^{-3}}{\frac{1}{t}}=\frac{4t^{-3}}{t^{-1}}=\boxed{\frac{4}{t^2}}$

$\frac{d^2y}{dx^2}=\frac{\frac{d}{dt}\left[\frac{dy}{dx}\right]}{\left(\frac{dx}{dt}\right)}=\frac{-2}{t^2}=\frac{-2}{(1)^2}=\boxed{-2}$

concavity $=\left.\frac{d^2y}{dx^2}\right|_{t=1}=\frac{4}{(1)^2}=\boxed{4}$

(14) $x = 3\cos^2 t \quad \frac{dx}{dt} = 6\cos t(-\sin t)$

$y = e^{2t} \quad \frac{dy}{dt} = 2e^{2t}$

$1 \le t \le 4$

$\text{arclength} = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

$$\boxed{\int_1^4 \sqrt{(6\cos t(-\sin t))^2 + (2e^{2t})^2}\,dt}$$

$\approx \boxed{2973.6316}$ (mathematica)

(15) $x = \ln(t^2) \quad \frac{dx}{dt} = \frac{1}{t^2}(2t) = \frac{2}{t}$

$y = t^3 + 2 \quad \frac{dy}{dt} = 3t^2 \quad 1 \le t \le 2$

$\text{arclength} = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt$

$$\boxed{\int_1^2 \sqrt{\left(\frac{2}{t}\right)^2 + (3t^2)^2}\,dt}$$

$\approx \boxed{7.18715}$

(16) $x = 2\cos(t) \quad \frac{dx}{dt} = -2\sin t$

$y = 3\sin(t) \quad \frac{dy}{dt} = 3\cos t$

@ $t = \pi/6$

$x = 2\cos\left(\frac{\pi}{6}\right) = 2\frac{\sqrt{3}}{2} = \sqrt{3}$ point: $\left(\sqrt{3}, \frac{3}{2}\right)$

$y = 3\sin\left(\frac{\pi}{6}\right) = 3\left(\frac{1}{2}\right) = \frac{3}{2}$

$m = \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{3\cos t}{-2\sin t}$

at $t = \frac{\pi}{6}$: $m = \frac{3\cos(\pi/6)}{-2\sin(\pi/6)} = \frac{3\frac{\sqrt{3}}{2}}{-2(1/2)}$

$m = \frac{3\sqrt{3}}{-2}$

$$\boxed{\text{tangent line: } \left(y - \frac{3}{2}\right) = -\frac{3\sqrt{3}}{2}\left(x - \sqrt{3}\right)}$$

(17) $x = t^3 + 4t^2 \quad \frac{dx}{dt} = 3t^2 + 8t$

$y = 3t^{4/3} \quad \frac{dy}{dt} = 4t^{1/3}$

@ $t = \frac{1}{8}$

$x = \left(\frac{1}{8}\right)^3 + 4\left(\frac{1}{8}\right)^2 = \frac{33}{512}$ point: $\left(\frac{33}{512}, \frac{3}{16}\right)$

$y = 3\left(\frac{1}{8}\right)^{4/3} = \frac{3}{16}$

$m = \frac{dy}{dx} = \frac{(dy/dt)}{(dx/dt)} = \frac{4t^{1/3}}{3t^2 + 8t}$

at $t = \frac{1}{8}$: $m = \frac{4\sqrt[3]{\frac{1}{8}}}{3\left(\frac{1}{8}\right)^2 + 8\left(\frac{1}{8}\right)} = \frac{128}{67}$

$$\boxed{\text{tangent line: } \left(y - \frac{3}{16}\right) = \frac{128}{67}\left(x - \frac{33}{512}\right)}$$

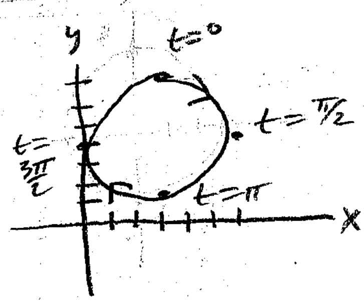
(18) circle center (3,4), clockwise

$(x-3) = 3\sin t \quad (y-4) = 3\cos t$

$$\boxed{x = 3 + 3\sin t, \quad y = 4 + 3\cos t}$$

check:

t	(x,y)
0	(3,7)
$\frac{\pi}{2}$	(6,4)
π	(3,1)
$\frac{3\pi}{2}$	(0,4)



(19) ellipse center (0,0) major y (a=3) clockwise
minor x (b=1)

$$\boxed{x = (1)\cos t, \quad y = 3\sin t}$$

check:

t	(x,y)
0	(1,0)
$\frac{\pi}{2}$	(0,3)
π	(-1,0)
$\frac{3\pi}{2}$	(0,-3)