AP Calculus BC

Name: answers/solutions

7.7 Worksheet

Period: ____________

Use the table section "forms involving a+bu" to find the indefinite integral.

1. $\int \frac{x^2}{x+5}\,dx$

$$\boxed{-5x+\frac{1}{2}x^2+25\ln|5+x|+C}$$

2. $\int \frac{2}{x^2\,(4+3x)^2}\,dx = 2\int \frac{1}{x^2(4+3x)^2}\,dx$

table: $\int \frac{1}{u^2(a+bu)}\,du = -\frac{1}{a^2}\left[\frac{a+2bu}{u(a+bu)}+\frac{2b}{a}\ln\left|\frac{u}{a+bu}\right|\right]$

$u=x \quad a=4 \quad b=3$

$du=dx$

$$2\left(-\frac{1}{(4)^2}\left[\frac{4+2(3)x}{x(4+3x)}+\frac{2(3)}{4}\ln\left|\frac{x}{4+3x}\right|\right]\right)$$

$$-\frac{1}{8}\left[\frac{4+6x}{x(4+3x)}+\frac{3}{2}\ln\left|\frac{x}{4+3x}\right|\right]$$

$$\boxed{-\frac{1}{8}\left[\frac{2(2+3x)}{x(4+3x)}+\frac{3}{2}\ln\left|\frac{x}{4+3x}\right|\right]}$$

Use the table section "forms involving $\sqrt{a^2-u^2}$" to find the indefinite integral.

3. $\int \frac{1}{x^2\sqrt{1-x^2}}\,dx$

$$\boxed{\frac{-\sqrt{1-x^2}}{x}+C}$$

4. $\int \frac{\sqrt{64-x^4}}{x}\,dx$

table: $\int \frac{\sqrt{a^2-u^2}}{u^2}\,du = -\frac{1}{u}\sqrt{a^2-u^2}-\sin^{-1}\left(\frac{u}{a}\right)$

$u=x^2 \quad a=8$

$du=2x\,dx$

$x\,dx=\frac{1}{2}du$

$$\int \frac{\sqrt{(8)^2-(x^2)^2}}{x}\left(\frac{x}{x}\right)dx = \int \frac{\sqrt{(8)^2-(x^2)^2}}{x^2}\,x\,dx$$

$$\int \frac{\sqrt{a^2-u^2}}{u^2}\left(\frac{1}{2}du\right)$$

$$\frac{1}{2}\int \frac{\sqrt{a^2-u^2}}{u^2}\,du$$

$$\frac{1}{2}\left[-\frac{1}{(x^2)}\sqrt{(8)^2-(x^2)^2}-\sin^{-1}\left(\frac{x^2}{8}\right)\right]+C$$

$$\boxed{-\frac{1}{2x^2}\sqrt{64-x^4}-\frac{1}{2}\sin^{-1}\left(\frac{x^2}{8}\right)+C}$$

Use the table section "trigonometric forms" to find the indefinite integral.

5. $\int \cos^4(3x)\ dx$

$\boxed{\frac{1}{12}\cos^3(3x)\sin(3x) + \frac{3}{8}x + \frac{1}{8}\sin(3x)\cos(3x) + C}$

6. $\int \frac{\sin^4 \sqrt{x}}{\sqrt{x}}\ dx$

u-sub:

$u = \sqrt{x} = x^{1/2}$

$du = \frac{1}{2}x^{-1/2}dx = \frac{1}{2\sqrt{x}}dx$

$\frac{1}{\sqrt{x}}dx = 2du$

$2\int \sin^4(u)\,du$

table: $\int \sin^n(u)du = -\frac{1}{n}\sin^{n-1}(u)\cos(u) + \frac{n-1}{n}\int \sin^{n-2}(u)\,du$

$n=4$

$2(-\frac{1}{4}\sin^3(u)\cos(u) + \frac{3}{4}\underbrace{\int \sin^2(u)\,du)}_{\text{again w/ } n=2}$

$\sin^{2-2}(u) = (\sin(u))^0 = 1$

$2(-\frac{1}{4}\sin^3(u)\cos(u) + \frac{3}{4}[-\frac{1}{2}\sin(u)\cos(u) + \frac{1}{2}\int 1\,du]$

$-\frac{1}{2}\sin^3(u)\cos(u) - \frac{3}{8}\sin(u)\cos(u) + \frac{3}{8}u + C$

$\boxed{-\frac{1}{2}\sin^3(\sqrt{x})\cos(\sqrt{x}) - \frac{3}{8}\sin(\sqrt{x})\cos(\sqrt{x}) + \frac{3}{8}\sqrt{x} + C}$

7. $\int \frac{1}{\sqrt{x}(1-\cos\sqrt{x})}\ dx$

$\boxed{-2\cot(\sqrt{x}) - 2\csc(\sqrt{x}) + C}$

8. $\int \frac{1}{1+\cot(4x)}\ dx$

table: $\int \frac{1}{1\pm\cot(u)}du = \frac{1}{2}(u \mp \ln|\sin u \pm \cos u|)$

$u=4x,\ du = 4dx,\ dx = \frac{1}{4}du$

$\frac{1}{4}\int \frac{1}{1+\cot(u)}du$

$\frac{1}{4}(\frac{1}{2}(u - \ln|\sin u + \cos u|)) + C$

$\frac{1}{8}(4x) - \frac{1}{8}\ln|\sin(4x) + \cos(4x)| + C$

$\boxed{\frac{1}{2}x - \frac{1}{8}\ln|\sin(4x) + \cos(4x)| + C}$

Use the table section "exponential and logarithmic forms" to find the indefinite integral.

9. $\int \frac{1}{1+e^{2x}}\,dx$

$\boxed{x-\frac{1}{2}\ln(1+e^{2x})+c}$

10. $\int e^{-4x}\sin(3x)\,dx$

table: $\int e^{au}\sin(bu)\,du=\frac{e^{au}}{a^2+b^2}(a\sin(bu)-b\cos(bu))$

$a=-4,\ b=3,\ u=x,\ du=dx$

$\int e^{au}\sin(bu)\,du$

$\frac{e^{-4x}}{\underbrace{(-4)^2+(3)^2}_{5}}(-4\sin(3x)-3\cos(3x))+c$

$\boxed{-\frac{4}{5}e^{-4x}\sin(3x)-\frac{3}{5}e^{-4x}\cos(3x)+c}$

11. $\int x^7\ln x\,dx$

$\boxed{\frac{1}{8}x^8\ln(x)-\frac{1}{64}x^8+c}$

12. $\int(\ln x)^3\,dx$

table: $\int(\ln u)^n du=u(\ln u)^n-n\int(\ln u)^{n-1}du$

$u=x,\ du=dx,\ n=3$

$\int(\ln u)^n du$

$x(\ln x)^3-3\underbrace{\int(\ln u)^2 du}_{\text{again, w/ } n=2}$

$x(\ln x)^3-3(x(\ln x)^2-2\underbrace{\int\ln u\,du}_{\text{table: } \int\ln u\,du=u(-1+\ln u)})$

$x(\ln x)^3-3x(\ln x)^2+6(x+\ln x)+c$

$\boxed{x(\ln x)^3-3x(\ln x)^2+6x+6\ln x+c}$

Use the integral tables to find the indefinite integral.

13. $\int x^2\sqrt{2+9x^2}\,dx$

$$\boxed{\frac{1}{4}x^3+\frac{1}{36}x-\frac{1}{54}\ln\left|3x+\sqrt{9x^2+2}\right|+C}$$

14. $\int \frac{1}{x^2\sqrt{x^2-4}}\,dx$

table: $\int \frac{1}{u^2\sqrt{u^2-a^2}}\,du = \frac{\sqrt{u^2-a^2}}{a^2u}$

$u=x,\ du=dx,\ a=2$

$\int \frac{1}{u^2\sqrt{u^2-a^2}}\,du$

$$\boxed{\frac{\sqrt{x^2-4}}{4x}+C}$$

15. $\int \cot^4 x\,dx$

$$\boxed{-\frac{1}{3}\cot^3(x)+\cot(x)+x+C}$$

16. $\int \frac{x}{1-\sec^2 x}\,dx$

$\frac{\sin^2x}{\cos^2x}+\frac{\cos^2x}{\cos^2x}=\frac{1}{\cos^2x}$

$\tan^2x+1=\sec^2x$

$1-\sec^2x=-\tan^2x$

$\int \frac{x}{-\tan^2x}\,dx$

$-\int x\cot^2x\,dx$ by parts

$u=x$ $\quad dv=\cot^2x\,dx$

$\frac{du}{dx}=1$ $\quad \int dv=\int\cot^2x\,dx$

$du=dx$ $\quad$ table for this

table: $\int\cot^2u\,du=-u-\cot u$

$v=-x-\cot x$

$-\left[uv-\int v\,du\right]$

$-\left[(x)(-x-\cot x)-\int(-x-\cot x)\,dx\right]$

$x^2+x\cot x+\int x\,dx+\int\cot x\,dx$

$\int\frac{\cos x}{\sin x}\,dx$ $\quad$ used $u=\sin x$, $du=\cos x\,dx$

$\int\frac{1}{u}\,du$

$$\boxed{x^2+x\cot x+\frac{1}{2}x^2+\ln|\sin x|+C}$$

7.8 Worksheet

Decide whether the integral is improper or not. Explain your reasoning.

1. $\int_0^1 \frac{dx}{5x-3}$

Improper
(explanation)

2. $\int_1^5 \frac{dx}{2x-3}$

Zero at $2x-3=0$
$2x=3$
$x=3/2=1.5$ is in $[1,5]$

improper, due to vertical asymptote at $x=3/2$

3. $\int_0^1 \frac{2x-5}{x^2-5x+6}\,dx$

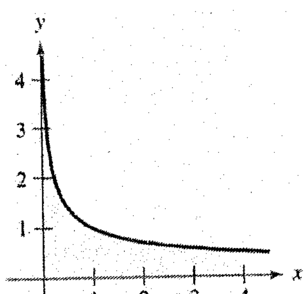
not improper

4. $\int_{-\infty}^{\infty} \frac{\sin x}{4+x^2}\,dx$

improper, due to infinite limits of integration

Explain why the integral is improper and determine if it converges or diverges.

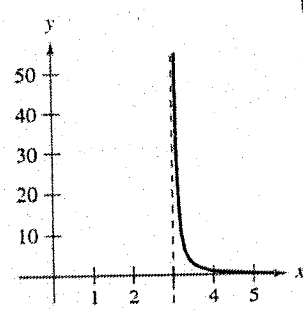
5. $\int_0^4 \frac{1}{\sqrt{x}}\,dx$



improper due to vertical asymptote at $x=0$

Converges

6. $\int_3^4 \frac{1}{(x-3)^{3/2}}\,dx$



Improper due to vertical asymptote at $x=3$

usub: $u=x-3$, $du=dx$

$\int (x-3)^{-3/2}dx$

$\int u^{-3/2}du = -2u^{-1/2} = -\frac{2}{\sqrt{x-3}}$

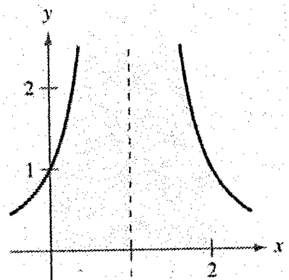
$\lim_{b\to 3^+} \int_b^4 (x-3)^{-3/2}\,dx$

$\lim_{b\to 3^+} \left[\frac{-2}{\sqrt{x-3}}\right]_b^4 = \frac{-2}{\sqrt{4-3}} - \lim_{b\to 3^+} \frac{-2}{\sqrt{b-3}}$ (3.1) $\quad \frac{-2}{(0)}\ \frac{(-)}{(+)}$

$= -2-(-\infty)$

$= \infty$ diverges

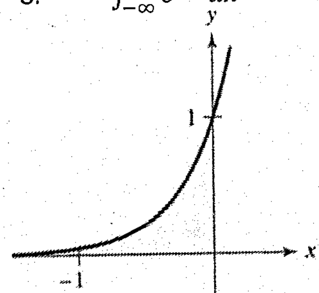
7. $\int_0^2 \frac{1}{(x-1)^2}\,dx$



improper due to vertical asymptote at $x=1$

diverges

8. $\int_{-\infty}^0 e^{3x}\,dx$



improper due to infinite limit of integration

$\int e^{3x}dx = \left[\frac{1}{3}e^{3x}\right]$

$\lim_{b\to-\infty} \int_b^0 e^{3x}dx = \lim_{b\to-\infty} \left[\frac{1}{3}e^{3x}\right]_b^0$

$= \frac{1}{3}e^0 - \lim_{b\to-\infty} \frac{1}{3}e^{3b}$

$= \frac{1}{3} - 0 = \frac{1}{3},$ converges

Determine whether the improper integral converges or diverges. Evaluate it if it converges.

9. $\int_1^\infty \frac{1}{x^3}\,dx$

Converges to $\frac{1}{2}$

10. $\int_1^\infty \frac{6}{x^4}\,dx$

$\lim_{b\to\infty} \int_1^b 6x^{-4}dx$

$\lim_{b\to\infty} \left[-\frac{6}{3}x^{-3}\right]_1^b$

$\lim_{b\to\infty} \left[\frac{-2}{x^3}\right]_1^b$

$\lim_{b\to\infty} \frac{-2}{b^3} - \left(\frac{-2}{(1)^3}\right)$

$0 + 2$

Converges to 2

11. $\int_1^\infty \frac{3}{\sqrt[3]{x}}\,dx$

$\boxed{\text{diverges}}$

12. $\int_0^\infty e^{x/3}\,dx$

$\lim_{b\to\infty} \int_0^b e^{\frac{1}{3}x}\,dx$

$\lim_{b\to\infty} \left[3e^{\frac{1}{3}x}\right]_0^b$

$3\lim_{b\to\infty} e^{\frac{1}{3}b} - 3e^{\frac{1}{3}(0)}$

$\infty - 3$

∞

$\boxed{\text{diverges}}$

Unit 7 Part 2 Review Solutions

(1) $\int e^{6x}\cos(2x)\,dx$ (table #99)

$$\int e^{au}\cos bu\,du = \frac{e^{au}}{a^2+b^2}(a\cos bu + b\sin bu)+C$$

$u=x$ $a=6$
$du=dx$ $b=2$

$$\boxed{\frac{e^{6x}}{36+4}\left(6\cos(2x)+2\sin(2x)\right)+C}$$

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(2) $\int\sqrt{16-9x^2}\,dx$ (closest to table #30)

$$\int\sqrt{a^2-u^2}\,du = \frac{u}{2}\sqrt{a^2-u^2}+\frac{a^2}{2}\sin^{-1}\left(\frac{u}{a}\right)+C$$

$u^2=9x^2$ $a=4$
$u=3x$
$\frac{du}{dx}=3$
$du=3dx$
$\frac{1}{3}du=dx$

$$\int\sqrt{(4)^2-u^2}\,\frac{1}{3}du = \frac{1}{3}\int\sqrt{4^2-u^2}\,du$$

$$\boxed{\frac{1}{3}\left[\frac{3x}{2}\sqrt{16-9x^2}+\frac{16}{2}\sin^{-1}\left(\frac{3x}{4}\right)\right]+C}$$

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(3) $\int xe^{4x}\,dx$ (table #96) $\int ue^{au}\,du = \frac{1}{a^2}(au-1)e^{au}+C$

$u=x$ $a=4$
$du=dx$

$$\boxed{\frac{1}{16}(4x-1)e^{4x}+C}$$

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(4) $\int xe^{3x^2}\cos(3x^2)\,dx$

1st, u-sub: $u=3x^2$
$\frac{du}{dx}=6x$
$du=6x\,dx$
$x\,dx=\frac{1}{6}du$

$$\int e^{u}\cos(u)\,\frac{1}{6}du$$

$$\frac{1}{6}\int e^{u}\cos(u)\,du$$ (table #99)

$$\int e^{au}\cos bu\,du = \frac{e^{au}}{a^2+b^2}(a\cos bu + b\sin bu$$

$u=u$
$a=1$
$b=1$

$$\frac{1}{6}\left[\frac{e^{u}}{1+1}\left(1\cos(u)+(1)\sin(u)\right)\right]+C$$

original $u=3x^2\ldots$

$$\boxed{\frac{1}{6}\left[\frac{e^{3x^2}}{2}\left(\cos(3x^2)+\sin(3x^2)\right)\right]+C}$$

⑤ $\int t^2 \sec^4(t^3)\,dt \rightarrow \int \sec^4(u)\, \underline{t^2 dt}$

1st, u-sub: $u = t^3$

$\frac{du}{dt} = 3t^2$

$du = 3t^2 dt$

$t^2 dt = \frac{1}{3} du$

$\int \sec^4(u)\, \frac{1}{3} du$

$\frac{1}{3}\int \sec^4(u)\,du$ now, (table #77)

$\int \sec^n u\,du = \frac{1}{n-1}\tan u \sec^{n-2} u + \frac{n-2}{n-1}\int \sec^{n-2} u\,du$

$(n=4)$ $u=t^3$

$\frac{1}{3}\left[\frac{1}{3}\tan(t^3)\sec^2(t^3) + \frac{2}{3}\underbrace{\int \sec^2 u\,du}\right]$

this also recuires table #77, w/ $n=2$

$\left[\frac{1}{1}\tan u \cancel{\sec^0(u)}^{1} + \left(\frac{0}{1}\right)^{=0}\cancel{\int \sec^0 (u)\,du}\right]$

$$\boxed{\frac{1}{3}\left[\frac{1}{3}\tan(t^3)\sec^2(t^3) + \frac{2}{3}[\tan(t^3)]\right] + C}$$

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⑥ $\int x\sqrt{9x^4+25}\,dx$ (closest is table #21)

$\int \sqrt{a^2+u^2}\,du = \frac{u}{2}\sqrt{a^2+u^2} + \frac{a^2}{2}\ln(u+\sqrt{a^2+u^2}) + c$

$u^2 = 9x^4$

$u = 3x^2$

$\frac{du}{dx} = 6x$

$du = 6x\,dx$

$x\,dx = \frac{1}{6}du$

$\int \sqrt{u^2+5^2}\,\frac{1}{6}du$

$\frac{1}{6}\int \sqrt{u^2+5^2}\,du$

$u = 3x^2,\ a=5$

$$\boxed{\frac{1}{6}\left[\frac{3x^2}{2}\sqrt{9x^4+25} + \frac{25}{2}\ln\left(3x^2+\sqrt{9x^4+25}\right)\right] + C}$$

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⑦ $\int \sin^4(5t)\,dt$ (table #73)

$\int \sin^n u\,du = -\frac{1}{n}\sin^{n-1} u\cos u + \frac{n-1}{n}\int \sin^{n-2} u\,du$

$n=4,\ u=5t$

$u = 5t$

$\frac{du}{dt} = 5$

$du = 5dt$

$\frac{1}{5}du = dt$

$\frac{1}{5}\left[-\frac{1}{4}\sin^3(5t)\cos(5t) + \frac{3}{4}\int \sin^2 u\,du\right]$

for this, table #63:

$\int \sin^2 u\,du = \frac{1}{2}u - \frac{1}{4}\sin 2u$

$$\boxed{\frac{1}{5}\left[-\frac{1}{4}\sin^3(5t)\cos(5t) + \frac{3}{4}\left(\frac{1}{2}(5t) - \frac{1}{4}\sin(10t)\right)\right] + C}$$

⑧ $\int x^3 \ln x \, dx$ (table #101)

$$\int u^n \ln u \, du = \frac{u^{n+1}}{(n+1)^2}\left[(n+1)\ln u - 1\right] + C$$

$u = x \quad n = 3$

$$\boxed{\frac{x^4}{16}\left[4\ln u - 1\right] + C}$$

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⑨ $\int \sin(3x)\cos(4x)\, dx$ (table #81)

$$\int \sin au \cos bu \, du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$$

$a=3,\ b=4,\ u=x$

$$\boxed{-\frac{\cos(-x)}{2(-1)} - \frac{\cos(7x)}{2(7)} + C}$$

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⑩ $\int x\sqrt{\underbrace{x^4}_{u^2}-4}\, dx \to \int \sqrt{u^2-2^2}\ \frac{1}{2} du$

$u^2 = x^4$

$u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$x\,dx = \frac{1}{2} du$

$$\frac{1}{2}\int \sqrt{u^2-2^2}\, du$$

(table # 39)

$$\int \sqrt{u^2-a^2}\, du = \frac{u}{2}\sqrt{u^2-a^2} - \frac{a^2}{2}\ln\left|u+\sqrt{u^2-a^2}\right| + C$$

$u = x^2,\ a = 2$

$$\boxed{\frac{1}{2}\left[\frac{x^2}{2}\sqrt{x^4-4} - \frac{4}{2}\ln\left|x^2+\sqrt{x^4-4}\right|\right] + C}$$

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⑪ $\int x\tan^3(5x^2)\, dx \to \int \tan^3 u \ \frac{1}{10} du$

$u = 5x^2$

$\frac{du}{dx} = 10x$

$du = 10x\,dx$

$x\,dx = \frac{1}{10} du$

$$\frac{1}{10}\int \tan^3 u \, du$$

(table # 69)

$$\int \tan^3 u\, du = \frac{1}{2}\tan^2 u + \ln|\cos u| + C$$

$$\boxed{\frac{1}{10}\left[\frac{1}{2}\tan^2(5x^2) + \ln\left|\cos(5x^2)\right|\right] + C}$$

(12) $\int \frac{\sqrt{2+9x^2}}{x^2}dx \to \int \frac{\sqrt{(\sqrt{2})^2+u^2}}{(\frac{u}{3})^2}\frac{1}{3}du \to \int \frac{\sqrt{(\sqrt{2})^2+u^2}}{u^2}\left(\frac{\frac{1}{3}}{(1/3)^2}\right)du$

$u=3x \to x=\frac{u}{3}$

$= 3\int \frac{\sqrt{(\sqrt{2})^2+u^2}}{u^2}du$

$\frac{du}{dx}=3$

$du=3dx$

$dx=\frac{1}{3}du$

(table #24) $\int \frac{\sqrt{a^2+u^2}}{u^2}du = \frac{-\sqrt{a^2+u^2}}{u} + \ln\left(u+\sqrt{a^2+u^2}\right)+C$

$a=\sqrt{2}$, $u=3x$

$\boxed{3\left[\frac{-\sqrt{2+9x^2}}{3x}+\ln\left(3x+\sqrt{2+9x^2}\right)\right]+C}$

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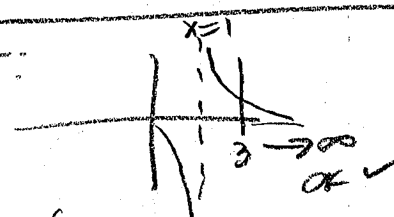
(13) $\int_3^\infty \frac{1}{x(\ln x)^5}dx$ graph 1st:



$u=\ln x$

$\frac{du}{dx}=\frac{1}{x}$

$du=\frac{1}{x}dx$

$\int \frac{1}{x(\ln x)^5}dx = \int u^{-5}du$

$\frac{u^{-4}}{-4} \to \left[\frac{-1}{4(\ln x)^4}\right]_3^\infty$

$= \lim_{b\to\infty}\left[\frac{-1}{4(\ln b)^4}\right]-\left(\frac{-1}{4(\ln 3)^4}\right)$

$-\frac{1}{\infty}+\frac{1}{4(\ln 3)^4}$

$0+\frac{1}{4(\ln 3)^4} = \boxed{\frac{1}{4(\ln 3)^4}}$

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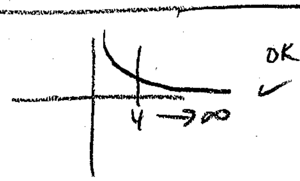
(14) $\int_4^\infty \frac{x}{x^{7/2}}dx = \int_4^\infty x^{-5/2}dx$ graph:



$\frac{x^{-3/2}}{-3/2} = \left[-\frac{2}{3}x^{-3/2}\right]_4^\infty$

$\lim_{b\to\infty}\left[-\frac{2}{3}b^{-3/2}\right]-\left(-\frac{2}{3}(4)^{-3/2}\right)$

$\frac{-2}{3(\infty)^{3/2}}+\frac{2}{3}4^{-3/2}$

$0+\frac{2}{3}(4)^{-3/2} = \boxed{\frac{2}{3}(4)^{-3/2} \approx 0.083333}$

(15) $\int_0^5 \frac{1}{(x-1)^{1/5}}dx$ graph:

$= \int_0^1 \frac{1}{(x-1)^{1/5}}dx + \int_1^5 \frac{1}{(x-1)^{1/5}}dx$

$\int \frac{1}{(x-1)^{1/5}}dx$ $u = x-1$, $\frac{du}{dx} = 1$, $du = dx$

$\int u^{-1/5}du = \frac{5}{4}u^{4/5} = \boxed{\frac{5}{4}(x-1)^{4/5}}$

$\left[\frac{5}{4}(x-1)^{4/5}\right]_0^{1} + \left[\frac{5}{4}(x-1)^{4/5}\right]_{1}^{5}$

$\lim_{b\to 1^-}\left(\frac{5}{4}(b-1)^{4/5}\right) - \frac{5}{4}(0-1)^{4/5} + \frac{5}{4}(5-1)^{4/5} - \lim_{b\to 1^+}\left(\frac{5}{4}(b-1)^{4/5}\right)$

$\sqrt[5]{0^4}$ $0 - \frac{5}{4}(1) + \frac{5}{4}(4)^{4/5} - 0 = \boxed{\frac{5}{4}\left[4^{4/5}-1\right] \approx 2.53929}$

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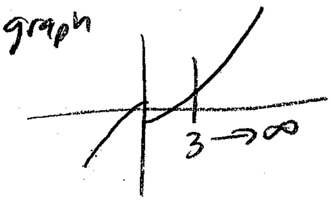
(16) $\int_3^\infty x\ln(x^2)dx$ $u = x^2$, $\frac{du}{dx} = 2x$, $du = 2x\,dx$, $x\,dx = \frac{1}{2}du$

graph



$\frac{1}{2}\int \ln u\, du$

requires by parts:

change letter: $\frac{1}{2}\int \ln y\, dy$

now, by parts: $u = \ln y$ $dv = dy$

$\frac{du}{dy} = \frac{1}{y}$ $\int dv = \int dy$

$du = \frac{1}{y}dy$ $v = y$

$uv - \int v\,du = (\ln y)(y) - \int y\left(\frac{1}{y}dy\right) = y\ln y - \int 1\,dy$

$= \boxed{y\ln y - y}$

back ~~to~~ original letter → $\left[u\ln u - u\right]_3^\infty$

$= \left[x^2\ln(x^2) - x^2\right]_3^\infty$

$\lim_{b\to\infty}\left[b^2\ln(b^2) - b^2\right] - \left[(3)^2\ln(3^2) - (3)^2\right]$

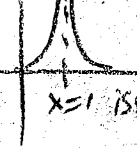
$\infty^2\ln(\infty)^2 - \infty^2$ $- 9\ln 9 - 9$

$\infty^2 \cdot \infty^2 - \infty^2$

$\infty^2 - \infty^2$

$\boxed{\text{diverges}}$

(17) $\int_0^\infty \frac{x^2}{(1-x^3)^2}dx$ graph



x=1 issue, (split here)

$$= \int_0^1 \frac{x^2}{(1-x^3)^2}dx + \int_1^\infty \frac{x^2}{(1-x^3)^2}dx$$

$$\int \frac{x^2}{(1-x^3)^2}dx \qquad u = 1-x^3$$

$$\frac{du}{dx} = -3x^2$$

$$du = -3x^2 dx$$

$$-\frac{1}{3}\int u^{-2}du \qquad x^2 dx = -\frac{1}{3}du$$

$$-\frac{1}{3}\,\frac{u^{-1}}{-1} = \frac{1}{3u} = \left[\frac{1}{3(1-x^3)}\right]$$

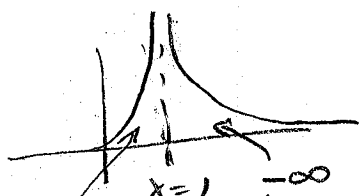
$$\left[\frac{1}{3(1-x^3)}\right]_0^{1} + \left[\frac{1}{3(1-x^3)}\right]_1^{\infty}$$

$$\lim_{b\to 1^-}\left(\frac{1}{3(1-b^3)}\right) - \frac{1}{3(1-0^3)} + \lim_{b\to\infty}\left(\frac{1}{3(1-b^3)}\right) - \lim_{b\to 1^+}\left(\frac{1}{3(1-b^3)}\right)$$

$$\frac{1}{3\,0}\ \begin{matrix}(+)\\(+)\end{matrix} \qquad \frac{1}{-\infty} \qquad \frac{1}{0}\ \begin{matrix}(+)\\(-)\end{matrix}$$

$$+\infty \quad - \quad \frac{1}{3} \quad + \quad 0 \quad - \infty$$

we can just say this $\boxed{\text{diverges}}$



(we can't say $+\infty$ and $-\infty$ cancel)

x=1 $+\infty$ $-\infty$

(likely a larger negative area)