AP Calculus BC

7.3 Worksheet

Name: answers/Solutions

Period: ____________

Pythagorean Identities	Other Identities
$\sin^2(x)+\cos^2(x)=1$	$\sin^2(x)=\dfrac{1-\cos(2x)}{2}$
$1+\cot^2(x)=\csc^2(x)$	$\cos^2(x)=\dfrac{1+\cos(2x)}{2}$
$\tan^2(x)+1=\sec^2(x)$	

Evaluate the indefinite integral.

1. $\int \sin^3(x)\cos^4(x)\,dx$

$$\boxed{-\frac{1}{5}\cos^5 x+\frac{1}{7}\cos^7 x+C}$$

2. $\int \sin^5(x)\cos^2(x)\,dx$

$$\int \sin^4 x\cos^2 x\sin x\,dx$$

$$\int (\sin^2 x)^2\cos^2 x\sin x\,dx$$

$$\int (1-\cos^2 x)^2\cos^2 x\sin x\,dx$$

$$\int (1-2\cos^2 x+\cos^4 x)\cos^2 x\sin x\,dx$$

$$\int \cos^2 x\sin x\,dx-2\int \cos^4 x\sin x\,dx+\int \cos^6 x\sin x\,dx$$

$u=\cos x$, $\frac{du}{dx}=-\sin x$, $du=-\sin x\,dx$, $\sin x\,dx=-du$

$$-\int u^2\,du-2\int u^4\,du+\int u^6\,du$$

$$-\frac{1}{3}u^3-\frac{2}{5}u^5+\frac{1}{7}u^7+C$$

$$\boxed{-\frac{1}{3}\cos^3 x-\frac{2}{5}\cos^5 x+\frac{1}{7}\cos^7 x+C}$$

3. $\int \cos^5(x)\sin(x)\,dx$

$\boxed{\frac{1}{2}\sin^2 x - \frac{1}{2}\sin^4 x + \frac{1}{6}\sin^6 x + C}$

4. $\int \cos^3(x)\sin^5(x)\,dx$

$\int \cos^2 x \sin^5 x \cos x\,dx$

$\int (1-\sin^2 x)\sin^5 x \cos x\,dx$

$\int \sin^5 x\cos x\,dx - \int \sin^7 x \cos x\,dx$

$u = \sin x,\ \frac{du}{dx} = \cos x,\ du = \cos x\,dx$

$\int u^5\,du - \int u^7\,du$

$\frac{1}{6}u^6 - \frac{1}{8}u^8 + C$

$\boxed{\frac{1}{6}\sin^6 x - \frac{1}{8}\sin^8 x + C}$

5. $\int \sin^7(2x)\cos(2x)\,dx$

$\boxed{-\frac{1}{4}\cos^2(2x) + \frac{3}{8}\cos^4(2x) - \frac{1}{4}\cos^6(2x) + \frac{1}{16}\cos^8(2x) + C}$

(hint: use the Binomial Theorem to expand $(1-\cos^2(2x))^3$)

6. $\int \sin^5(3x)\cos(3x)\,dx$

$\int \sin^4(3x)\cos(3x)\sin(3x)\,dx$

$\int (\sin^2(3x))^2\cos(3x)\sin(3x)\,dx$

$\int (1-\cos^2(3x))^2\cos(3x)\sin(3x)\,dx$

$\int (1-2\cos^2(3x)+\cos^4(3x))\cos(3x)\sin(3x)\,dx$

$\int \cos(3x)\sin(3x)\,dx - 2\int \cos^3(3x)\sin(3x)\,dx + \int \cos^5(3x)\sin(3x)\,dx$

$u = \cos(3x),\ \frac{du}{dx} = -3\sin(3x),\ du = -3\sin(3x)\,dx$

$\sin(3x)\,dx = -\frac{1}{3}du$

$-\frac{1}{3}\int u\,du + \frac{2}{3}\int u^3\,du - \frac{1}{3}\int u^5\,du$

$-\frac{1}{3}\frac{1}{2}u^2 + \frac{2}{3}\frac{1}{4}u^4 - \frac{1}{3}\frac{1}{6}u^6 + C$

$\boxed{-\frac{1}{6}\cos^2(3x) + \frac{1}{6}\cos^4(3x) - \frac{1}{18}\cos^6(3x) + C}$

7. $\int \sin^3(2x)\sqrt{\cos(2x)}dx$

$$\boxed{-\frac{1}{3}(\cos(2x))^{3/2}+\frac{1}{5}(\cos(2x))^{5/2}+C}$$

8. $\int \sin^5(3x)\sqrt{\cos(3x)}dx$

$$\int \sin^4(3x)\sqrt{\cos(3x)}\,\sin(3x)dx$$

$$\int (\sin^2(3x))^2\cos^{1/2}(3x)\sin(3x)dx$$

$$\int (1-\cos^2(3x))^2\cos^{1/2}(3x)\sin(3x)dx$$

$$\int (1-2\cos^2(3x)+\cos^4(3x))\cos^{1/2}(3x)\sin(3x)dx$$

$$\int \cos^{1/2}(3x)\sin(3x)dx-2\int\cos^{5/2}(3x)\sin(3x)dx+\int\cos^{9/2}(3x)\sin(3x)dx$$

$u=\cos(3x)$, $du=-3\sin(3x)dx$, $\sin(3x)dx=-\frac{1}{3}du$

$$-\frac{1}{3}\int u^{1/2}du+\frac{2}{3}\int u^{5/2}du-\frac{1}{3}\int u^{9/2}du$$

$$-\frac{1}{3}\cdot\frac{2}{3}u^{3/2}+\frac{2}{3}\cdot\frac{2}{7}u^{7/2}-\frac{1}{3}\cdot\frac{2}{11}u^{11/2}+C$$

$$\boxed{-\frac{2}{9}(\cos(3x))^{3/2}+\frac{4}{21}(\cos(3x))^{7/2}-\frac{2}{33}(\cos(3x))^{11/2}+C}$$

9. $\int \cos^2(3x)\,dx$

$$\boxed{\frac{1}{2}x+\frac{1}{12}\sin(6x)+C}$$

10. $\int 4\cos^2(2x)\,dx$

$$4\int\left(\frac{1+\cos(4x)}{2}\right)dx$$

$u=4x$

$\frac{du}{dx}=4$

$du=4dx$

$dx=\frac{1}{4}du$

$$\int 2dx+2\int\cos(4x)dx$$

$$\int 2dx+2\int\cos u\left(\frac{1}{4}du\right)$$

$$2x+\frac{1}{2}\sin u+C$$

$$\boxed{2x+\frac{1}{2}\sin(4x)+C}$$

11. $\int x \sin^2(x)\,dx$

$$\boxed{\frac{1}{4}x^2-\frac{1}{4}x\sin(2x)-\frac{1}{8}\cos(2x)+C}$$

12. $\int x \sin^2(2x)\,dx$

$$\int x\left(\frac{1-\cos(4x)}{2}\right)dx$$

u-sub: $u=4x$, $du=4dx$, $x=\frac{1}{4}u$, $dx=\frac{1}{4}du$

$$\frac{1}{2}\int x\,dx-\frac{1}{2}\int x\cos(4x)dx$$

$$-\frac{1}{2}\int\left(\frac{1}{4}u\right)\cos u\left(\frac{1}{4}du\right)$$

$$-\frac{1}{32}\int u\cos u\,du \text{ (change letter)..}$$

$$-\frac{1}{32}\int w\cos w\,dw$$

by parts: $u=w$, $dv=\cos w\,dw$, $\frac{du}{dw}=1$, $\int dv=\int\cos w\,dw$, $du=dw$, $v=\sin w$

$$\frac{1}{2}\int x\,dx-\frac{1}{32}\left[uv-\int v\,du\right]$$

$$-\frac{1}{32}\left[w\sin w-\int\sin w\,dw\right]$$

$$\frac{1}{2}\frac{1}{2}x^2-\frac{1}{32}w\sin w+\frac{1}{32}\left[-\cos w\right]+C \quad \left(\begin{matrix}w=\text{orig } u\\ w=4x\end{matrix}\right)$$

$$\frac{1}{4}x^2-\frac{1}{32}(4x)\sin(4x)-\frac{1}{32}\cos(4x)+C$$

$$\boxed{\frac{1}{4}x^2-\frac{1}{8}x\sin(2x)-\frac{1}{32}\cos(4x)+C}$$

13. $\int \sec(4x)\,dx$

$$\boxed{\frac{1}{4}\ln\left|\sec(4x)+\tan(4x)\right|+C}$$

14. $\int 6\sec(x)\,dx$

$$=6\int\sec(x)dx$$

$$=\boxed{6\ln\left|\sec x+\tan x\right|+C} \text{ (by shortcut)}$$

to derive shortcut:

$$\int\sec x\,dx$$

$$\int\sec x\left(\frac{\sec x+\tan x}{\sec x+\tan x}\right)dx$$

$$\int\frac{\sec^2x+\sec x\tan x}{\sec x+\tan x}dx$$

u-sub: $u=\sec x+\tan x$

$\frac{du}{dx}=\sec x\tan x+\sec^2x$

$du=(\sec^2x+\sec x\tan x)dx$

$$\int\frac{1}{u}du$$

$$\ln|u|+C$$

$$\left(\int\sec x\,dx=\ln\left|\sec x+\tan x\right|+C\right)$$

15. $\int \sec^3(\pi x)\,dx$

$$\boxed{\frac{1}{2\pi}\sec(\pi x)\tan(\pi x)+\frac{1}{2\pi}\ln|\sec(\pi x)+\tan(\pi x)|+C}$$

16. $\int \sec^5(2x)\,dx$

(on separate page)...

17. $\int \tan^5\left(\frac{x}{2}\right)dx$

$$\boxed{\frac{1}{8}\tan(2x)\sec^3(2x)+\frac{3}{16}\sec(2x)\tan(2x)+\frac{3}{8}\ln|\sec(2x)+\tan(2x)|+C}$$

18. $\int \tan^3(2x)\,dx$

u-sub: $u=2x$

$du=2dx,\ dx=\frac{1}{2}du$

$\frac{1}{2}\int \tan^3 u\,du$ (letter change)

$\frac{1}{2}\int \tan^3 w\,dw$

$\frac{1}{2}\int \tan^2 w\tan w\,dw$ $(\tan^2 w=\sec^2 w-1)$

$\frac{1}{2}\int(\sec^2 w-1)\tan w\,dw$

$\frac{1}{2}\int \sec^2 w\tan w\,dw-\frac{1}{2}\int \tan w\,dw$

$\frac{1}{2}\int \sec w\,\underline{\sec w\tan w\,dw}-\frac{1}{2}\int\frac{\sin x}{\cos x}dw$

2 different usubs:

$u=\sec w$

$du=\sec w\tan w\,dw$

$u=\cos x$

$du=-\sin x\,dw$

$\frac{1}{2}\int u\,du \quad +\frac{1}{2}\int\frac{1}{u}du$

$\quad +\frac{1}{2}\ln|u|+C$

$\frac{1}{2}\frac{1}{2}u^2$

$\frac{1}{4}\sec^2 w \quad +\frac{1}{2}\ln|\cos w|+C$

$w=$ orig $u=2x$ ...

$$\boxed{\frac{1}{4}\sec^2(2x)+\frac{1}{2}\ln|\cos(2x)|+C}$$

7.3 #16

$\int \sec^5(2x)\,dx$ u-sub: $u=2x$

$du=2dx$, $dx=\frac{1}{2}du$

$=\frac{1}{2}\int \sec^5 u\,du$ (letter change)

$=\frac{1}{2}\int \sec^5 w\,dw$ by parts

$u=\sec^3 w$ $dv=\sec^2 w\,dw$

$\frac{du}{dw}=3\sec^2 w\sec w\tan w$ $\int dv=\int \sec^2 w\,dw$

$du=3\sec^3 w\tan w\,dw$ $v=\tan w$

$=\frac{1}{2}\left[uv-\int v\,du\right]$

$=\frac{1}{2}\left[\tan w\sec^3 w-\int \tan w\,3\sec^3 w\tan w\,dw\right]$

$=\frac{1}{2}\tan w\sec^3 w-\frac{3}{2}\int \tan^2 w\sec^3 w\,dw$ $\tan^2 w=\sec^2 w-1$

$=\frac{1}{2}\tan w\sec^3 w-\frac{3}{2}\int(\sec^2 w-1)\sec^3 w\,dw$

$\frac{1}{2}\int\sec^5 w\,dw=\frac{1}{2}\tan w\sec^3 w-\frac{3}{2}\int\sec^5 w\,dw+\frac{3}{2}\int\sec^3 w\,dw$

$\left(\frac{1}{2}+\frac{3}{2}\right)\int\sec^5 w\,dw=\frac{1}{2}\tan w\sec^3 w+\frac{3}{2}\int\sec^3 w\,dw$

$2\int\sec^5 w\,dw=\frac{1}{2}\tan w\sec^3 w+\frac{3}{2}\cdot\frac{1}{2}\left[\sec w\tan w+\ln|\sec w+\tan w|\right]$

$\int\sec^5 w\,dw=\frac{1}{4}\tan w\sec^3 w+\frac{3}{8}\sec w\tan w+\frac{3}{8}\ln|\sec w+\tan w|$

side problem: $\int\sec^3 w\,dw$

by parts:

$u=\sec w$ $dv=\sec^2 w\,dw$

$\frac{du}{dw}=\sec w\tan w$ $\int dv=\int\sec^2 w\,dw$

$du=\sec w\tan w\,dw$ $v=\tan x$

$\int\sec^3 w\,dw=uv-\int v\,du$

$\int\sec^3 w\,dw=\sec w\tan w-\int\tan^2 w\sec w\,dw$

$\int\sec^3 w\,dw=\sec w\tan w-\int(\sec^2 w-1)\sec w\,dx$

$\int\sec^3 w\,dw=\sec w\tan w-\int\sec^3 w\,dw+\int\sec w\,dw$

$2\int\sec^3 w\,dw=\sec w\tan w+\ln|\sec w+\tan w|$

$\int\sec^3 w\,dw=\frac{1}{2}\left[\sec w\tan w+\ln|\sec w+\tan w|\right]$

and

$\int\sec^5(2x)\,dx=\frac{1}{2}\int\sec^5 w\,dw$

with $w=$ original $u=2x$

so

$$\boxed{\int\sec^5(2x)\,dx=\frac{1}{8}\tan(2x)\sec^3(2x)+\frac{3}{16}\sec(2x)\tan(2x)+\frac{3}{8}\ln|\sec(2x)+\tan(2x)|+C}$$

19. $\int \tan^3(2x)\sec^3(2x)\,dx$

$$\boxed{\frac{1}{10}\sec^5(2x) - \frac{1}{6}\sec^3(2x) + c}$$

20. $\int \tan^3(3x)\sec^3(3x)\,dx$

u-sub: $u = 3x$, $du = 3dx$, $dx = \frac{1}{3}du$

$\frac{1}{3}\int \tan^3 w \sec^3 w\, dw$

$\frac{1}{3}\int \tan^2 w \sec^2 w \sec w \tan w\, dw$ $(\tan^2 w = \sec^2 w - 1)$

$\frac{1}{3}\int (\sec^2 w - 1)\sec^2 w \sec w \tan w\, dw$

$\frac{1}{3}\int \sec^4 w \sec w \tan w\, dw - \frac{1}{3}\int \sec^2 w \sec w \tan w\, dw$

usub: $u = \sec w$, $du = \sec w \tan w\, dw$

$\frac{1}{3}\int u^4\, du - \frac{1}{3}\int u^2\, du$

$\frac{1}{3}\frac{1}{5}u^5 - \frac{1}{3}\frac{1}{3}u^3 + c$

$\frac{1}{15}\sec^5 w - \frac{1}{9}\sec^3 w + c$

$w =$ orig. $u = 3x$

$$\boxed{\frac{1}{15}\sec^5(3x) - \frac{1}{9}\sec^3(3x) + c}$$

21. $\int \sec^6(4x)\tan(4x)\,dx$

$$\boxed{\frac{1}{24}\tan^6(4x) + \frac{1}{8}\tan^4(4x) + \frac{1}{8}\tan^2(4x) + c}$$

22. $\int \sec^6(2x)\tan(2x)\,dx$

usub: $u = 2x$, $du = 2dx$, $dx = \frac{1}{2}du$

$\frac{1}{2}\int \sec^6 w \tan w\, dw$

$\frac{1}{2}\int \sec^4 w \tan w \sec^2 w\, dw$ $(\sec^2 w = \tan^2 w + 1)$

$\frac{1}{2}\int (\sec^2 w)^2 \tan w \sec^2 w\, dw$

$\frac{1}{2}\int (\tan^2 w + 1)^2 \tan w \sec^2 w\, dw$

$\frac{1}{2}\int (\tan^4 w + 2\tan^2 w + 1)\tan w \sec^2 w\, dw$

$\frac{1}{2}\int \tan^5 w \sec^2 w\, dw + \int \tan^3 w \sec^2 w\, dw + \frac{1}{2}\int \tan w \sec^2 w\, dw$

u-sub: $u = \tan w$, $du = \sec^2 w\, dw$

$\frac{1}{2}\int u^5\, du + \int u^3\, du + \frac{1}{2}\int u\, du$

$\frac{1}{2}\frac{1}{6}u^6 + \frac{1}{4}u^4 + \frac{1}{2}\frac{1}{2}u^2 + c$

$\frac{1}{12}\tan^6 w + \frac{1}{4}\tan^4 w + \frac{1}{4}\tan^2 w + c$

$w =$ orig. $u = 2x$

$$\boxed{\frac{1}{12}\tan^6(2x) + \frac{1}{4}\tan^4(2x) + \frac{1}{4}\tan^2(2x) + c}$$

23. $\int \sec^5(x)\tan^3(x)\,dx$

$$\boxed{\frac{1}{7}\sec^7 x - \frac{1}{5}\sec^5 x + C}$$

24. $\int 4\sec^5(2x)\tan^3(2x)\,dx$

$\underline{u\text{sub}}$
$u = 2x$
$du = 2dx$
$dx = \frac{1}{2}du$

$4\left(\frac{1}{2}\right)\int \sec^5 u \tan^3 u \, du$

$2\int \sec^5 w \tan^3 w \, dw$

$2\int \sec^4 w \tan^2 w \sec w \tan w \, dw$ $(\tan^2 w = \sec^2 w - 1)$

$2\int \sec^4 w(\sec^2 w - 1)\underline{\sec w \tan w \, dw}$

$2\int \sec^6 w \sec w \tan w \, dw - 2\int \sec^4 w \sec w \tan w \, dw$

$\underline{u\text{-sub}}$: $u = \sec w$
$du = \sec w \tan w \, dw$

$2\int u^6 du - 2\int u^4 du$

$2\left(\frac{1}{7}\right)u^7 - 2\left(\frac{1}{5}\right)u^5 + C$

$\frac{2}{7}\sec^7 w - \frac{2}{5}\sec^5 w + C$

$w = \text{orig } u = 2x$

$$\boxed{\frac{2}{7}\sec^7(2x) - \frac{2}{5}\sec^5(2x) + C}$$

25. $\int \frac{\tan^2(x)}{\sec(x)}\,dx$

$$\boxed{\ln|\sec x + \tan x| - \sin x + C}$$

26. $\int 2\frac{\tan^2(2x)}{\sec(2x)}\,dx$

$\underline{u\text{sub}}$
$u = 2x$
$du = 2dx$
$dx = \frac{1}{2}du$

$2\left(\frac{1}{2}\right)\int \frac{\tan^2 u}{\sec u}\,du$

$\int \frac{\tan^2 w}{\sec w}\,dw$ $(\tan^2 w = \sec^2 w - 1)$

$\int \frac{\sec^2 w - 1}{\sec w}\,dw$

$\int \frac{\sec^2 w}{\sec w}\,dw - \int \frac{1}{\sec w}\,dw$

$\int \sec w \, dw - \int \cos w \, dw$

$\ln|\sec w + \tan w| - \sin w + C$

$w = \text{orig } u = 2x$

$$\boxed{\ln|\sec(2x) + \tan(2x)| - \sin(2x) + C}$$

7.4 Worksheet

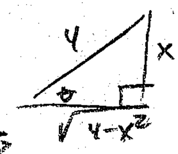
State the substitution you would use to find the indefinite integral. DO NOT INTEGRATE.

1. $\int \frac{1}{9+x^2}\,dx$

$a=3 \quad u=x$
$du=dx$

$\rightarrow \int \frac{1}{a^2+u^2}\,du$

2. $\int \sqrt{4-x^2}\,dx$



$\sin\theta = \frac{x}{4} \quad \cos\theta = \frac{\sqrt{4-x^2}}{4}$

$x = 4\sin\theta \quad \sqrt{4-x^2} = 4\cos\theta$
$dx = 4\cos\theta\, d\theta$

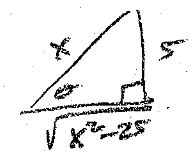
$\rightarrow \int 4\cos\theta\; 4\cos\theta\, d\theta$

3. $\int \frac{x^2}{\sqrt{25-x^2}}\,dx$

$x = 25\sin\theta \quad \sqrt{25-x^2} = 25\cos\theta$
$dx = 25\cos\theta\, d\theta$

$\rightarrow \int \frac{1}{25\cos\theta}(25\sin\theta)^2\, 25\cos\theta\, d\theta$

4. $\int x^2(x^2-25)^{3/2}\,dx$



$\sin\theta = \frac{5}{x} \quad \tan\theta = \frac{5}{\sqrt{x^2-25}}$

$x = \frac{5}{\sin\theta} = 5\csc\theta \quad \sqrt{x^2-25} = \frac{5}{\tan\theta} = 5\cot\theta$
$dx = -5\csc\theta\cot\theta\, d\theta$

$\rightarrow \int (5\csc\theta)^2(5\cot\theta)^3(-5)\csc\theta\cot\theta\, d\theta$

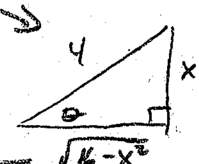
Find the indefinite integral using the given substitution.
Use $x = 4\sin(\theta)$

5. $\int \frac{\sqrt{16-x^2}}{x}\,dx$

$$4\ln\left|\frac{4-\sqrt{16-x^2}}{x}\right| + \sqrt{16-x^2} + C$$

$\sin\theta = \frac{x}{4} = \frac{opp}{hyp}$

6. $\int \frac{4}{x^2\sqrt{16-x^2}}\,dx$



$x = 4\sin\theta \quad \cos\theta = \frac{\sqrt{16-x^2}}{4}$ ← form another trig ratio w/ constant & variable sides

$\frac{dx}{d\theta} = 4\cos\theta$

$dx = 4\cos\theta\, d\theta \quad \sqrt{16-x^2} = 4\cos\theta$

$\int \frac{4}{(4\sin\theta)^2 4\cos\theta} 4\cos\theta\, d\theta$

$\frac{1}{4}\int \frac{1}{\sin^2\theta}\,d\theta = \frac{1}{4}\int \csc^2\theta\, d\theta$ from triangle:

$= -\frac{1}{4}\cot\theta + C \quad \cot\theta = \frac{adj}{opp} = \frac{\sqrt{16-x^2}}{x}$

$$\boxed{-\frac{1}{4}\frac{\sqrt{16-x^2}}{x} + C}$$

Use $x = 5\sec(\theta)$

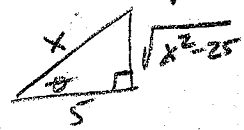
7. $\int \frac{1}{\sqrt{x^2-25}}\,dx$

$$\boxed{\ln\left|\frac{x}{5}+\frac{\sqrt{x^2-25}}{5}\right|+C}$$

8. $\int \frac{\sqrt{x^2-25}}{x}\,dx$

$\sec\theta = \frac{1}{\cos\theta} = \frac{x}{5}$

$\cos\theta = \frac{5}{x} = \frac{adj}{hyp}$



$x = 5\sec\theta$ $\quad \tan\theta = \frac{\sqrt{x^2-25}}{5}$

$dx = 5\sec\theta\tan\theta\,d\theta$ $\quad \sqrt{x^2-25} = 5\tan\theta$

$\int \frac{5\tan\theta}{5\sec\theta} 5\sec\theta\tan\theta\,d\theta$

$5\int \tan^2\theta\,d\theta$ $\quad (\tan^2\theta = \sec^2\theta - 1)$

$5\int (\sec^2\theta - 1)\,d\theta$

$5\int \sec^2\theta\,d\theta - \int 5\,d\theta$ $\quad x = 5\sec\theta = \frac{5}{\cos\theta}$

$5\tan\theta - 5\theta + C$ $\quad \cos\theta = \frac{5}{x}$

$\theta = \arccos\left(\frac{5}{x}\right)$

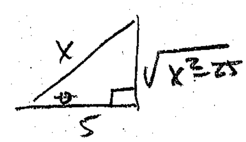
$5\frac{\sqrt{x^2-25}}{5} - 5\arccos\left(\frac{5}{x}\right) + C$

$$\boxed{\sqrt{x^2-25} - 5\arccos\left(\frac{5}{x}\right) + C}$$

9. $\int x^3\sqrt{x^2-25}\,dx$

$$\boxed{625\left(\frac{\sqrt{x^2-25}}{5}\right)^5 + \frac{3125}{3}\left(\frac{\sqrt{x^2-25}}{5}\right)^3 + C}$$

10. $\int \frac{x^3}{\sqrt{x^2-25}}\,dx$



$\cos\theta = \frac{5}{x}$ $\quad \tan\theta = \frac{\sqrt{x^2-25}}{5}$

$x = \frac{5}{\cos\theta} = 5\sec\theta$ $\quad \sqrt{x^2-25} = 5\tan\theta$

$dx = 5\sec\theta\tan\theta\,d\theta$

$\int \frac{(5\sec\theta)^3}{5\tan\theta} 5\sec\theta\tan\theta\,d\theta = 125\int \sec^2\theta\sec^2\theta\,d\theta$

$(\sec^2\theta = \tan^2\theta + 1)$

$125\int (\tan^2\theta + 1)\sec^2\theta\,d\theta$

$125\int \tan^2\theta\sec^2\theta\,d\theta + 125\int \sec^2\theta\,d\theta$

u sub $\quad u = \tan\theta$

$du = \sec^2\theta\,d\theta$

$125\int u^2\,du \quad + 125\tan\theta + C$

$\frac{125}{3}\tan^3\theta + 125\tan\theta + C$

$$\boxed{\frac{125}{3}\left(\frac{\sqrt{x^2-25}}{5}\right)^3 + 125\left(\frac{\sqrt{x^2-25}}{5}\right) + C}$$

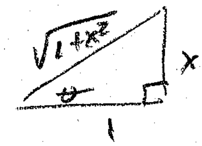
$\tan\theta = \frac{x}{1} = \frac{opp}{adj}$

Use $x = \tan(\theta)$

11. $\int x\sqrt{1+x^2}\,dx$

$$\boxed{\frac{1}{3}\left(\sqrt{1+x^2}\right)^3 + C}$$

12. $\int \frac{1}{(1+x^2)^2}\,dx$



$\tan\theta = \frac{x}{1}$ $\quad \cos\theta = \frac{1}{\sqrt{1+x^2}}$

$x = \tan\theta$ $\quad \sqrt{1+x^2} = \frac{1}{\cos\theta} = \sec\theta$ $\quad (1+x^2 = \sec^2\theta)$

$dx = \sec^2\theta\,d\theta$

$\int \frac{1}{(\sec^2\theta)^2}\sec^2\theta\,d\theta = \int \frac{1}{\sec^2\theta}\,d\theta = \int \cos^2\theta\,d\theta$

$\int \left(\frac{1+\cos(2\theta)}{2}\right)d\theta$ $\quad$ u sub $\quad u = 2\theta$, $du = 2\,d\theta$, $d\theta = \frac{1}{2}du$

$\int \frac{1}{2}\,d\theta + \frac{1}{2}\int \cos(2\theta)\,d\theta$

$\frac{1}{2}\theta + \frac{1}{2}\frac{1}{2}\int \cos u\,du$ $\quad (\sin(2\theta) = 2\sin\theta\cos\theta)$

$\frac{1}{2}\theta + \frac{1}{4}\sin(2\theta) + C$ $\quad$ from triangle:

$\frac{1}{2}\theta + \frac{1}{4}(2\sin\theta\cos\theta) + C$ $\quad \sin\theta = \frac{x}{\sqrt{1+x^2}}$

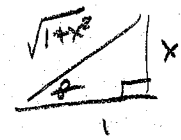
$\frac{1}{2}\arctan x + \frac{1}{2}\frac{x}{\sqrt{1+x^2}}\frac{1}{\sqrt{1+x^2}} + C$ $\quad \theta = \arctan x$

$$\boxed{\frac{1}{2}\arctan x + \frac{1}{2}\frac{x}{1+x^2} + C}$$

13. $\int \frac{9x^3}{\sqrt{1+x^2}}\,dx$

$$\boxed{3\left(\sqrt{1+x^2}\right)^3 - 9\sqrt{1+x^2} + C}$$

14. $\int \frac{x^2}{(1+x^2)^2}\,dx$



$\tan\theta = \frac{x}{1}$ $\quad \cos\theta = \frac{1}{\sqrt{1+x^2}}$

$x = \tan\theta$ $\quad \sqrt{1+x^2} = \frac{1}{\cos\theta} = \sec\theta$, $1+x^2 = \sec^2\theta$

$dx = \sec^2\theta\,d\theta$

$\int \frac{(\tan\theta)^2}{(\sec^2\theta)^2}\sec^2\theta\,d\theta$

$\int \frac{\tan^2\theta}{1}\frac{1}{\sec^2\theta}\,d\theta = \int \frac{\sin^2\theta}{\cos^2\theta}\frac{\cos^2\theta}{1}\,d\theta = \int \sin^2\theta\,d\theta$

$\int \left(\frac{1-\cos(2\theta)}{2}\right)d\theta = \frac{1}{2}\int d\theta - \frac{1}{2}\int \cos(2\theta)\,d\theta$ $\quad$ u sub: $u = 2\theta$, $du = 2\,d\theta$, $d\theta = \frac{1}{2}du$

$\frac{1}{2}\int d\theta - \frac{1}{2}\frac{1}{2}\int \cos u\,du$

$\frac{1}{2}\theta - \frac{1}{4}\sin(2\theta) + C$ $\quad (\sin(2\theta) = 2\sin\theta\cos\theta)$

$\cos\theta = \frac{1}{\sqrt{1+x^2}}$

$\theta = \arctan x$

$\frac{1}{2}\arctan x - \frac{1}{2}\frac{x}{\sqrt{1+x^2}}\frac{1}{\sqrt{1+x^2}} + C$ $\quad$ from Δ: $\sin\theta = \frac{x}{\sqrt{1+x^2}}$

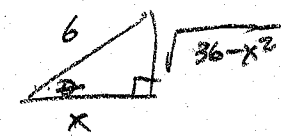
$$\boxed{\frac{1}{2}\arctan x - \frac{1}{2}\frac{x}{1+x^2} + C}$$

Figure out the trigonometric substitution and evaluate the indefinite integral.

15. $\int \frac{1}{\sqrt{16-x^2}}\,dx$

$$\boxed{\arccos\left(\frac{x}{4}\right)+C}$$

16. $\int \frac{x^2}{\sqrt{36-x^2}}\,dx$



pair constant side w/ each of the other sides!

$\cos\theta = \frac{x}{6}$ $\quad\quad$ $\sin\theta = \frac{\sqrt{36-x^2}}{6}$

$x = 6\cos\theta$ $\quad\quad$ $\sqrt{36-x^2} = 6\sin\theta$

$dx = -6\sin\theta\,d\theta$

$\int \frac{(6\cos\theta)^2}{6\sin\theta}(-6)\sin\theta\,d\theta$

$-36\int \cos^2\theta\,d\theta$ $\quad\quad$ $\left[\cos^2\theta = \left(\frac{1+\cos(2\theta)}{2}\right)\right]$

$-36\int\left(\frac{1+\cos(2\theta)}{2}\right)d\theta$ $\quad\quad$ uSub: $u = 2\theta$, $du = 2d\theta$, $d\theta = \frac{1}{2}du$

$-18\int 1\,d\theta - 18\int \cos(2\theta)\,d\theta$

$-18\int 1\,d\theta - 9\int \cos u\,du$ $\quad\quad$ $[\sin(2\theta) = 2\sin\theta\cos\theta]$

$-18\theta - 9\sin(2\theta) + C$ $\quad\quad$ $\theta = \arccos\left(\frac{x}{6}\right)$

$-18\arccos\left(\frac{x}{6}\right) - 9(2\sin\theta\cos\theta) + C$

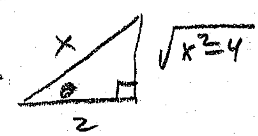
$-18\arccos\left(\frac{x}{6}\right) - 18\frac{\sqrt{36-x^2}}{6}\frac{x}{6} + C$

$$\boxed{-18\arccos\left(\frac{x}{6}\right) - \frac{1}{2}x\sqrt{36-x^2} + C}$$

17. $\int \sqrt{16-4x^2}\,dx$

$$\boxed{-4\arccos\left(\frac{x}{2}\right) + \frac{1}{2}x\sqrt{16-4x^2} + C}$$

18. $\int \frac{1}{\sqrt{x^2-4}}\,dx$



$\cos\theta = \frac{2}{x}$ $\quad\quad$ $\tan\theta = \frac{\sqrt{x^2-4}}{2}$

$x = \frac{2}{\cos\theta} = 2\sec\theta$ $\quad\quad$ $\sqrt{x^2-4} = 2\tan\theta$

$dx = 2\sec\theta\tan\theta\,d\theta$

$\int \frac{1}{2\tan\theta} 2\sec\theta\tan\theta\,d\theta$ $\quad\quad$ $\sec\theta = \frac{1}{\cos\theta} = \frac{x}{2}$

$\int \sec\theta\,d\theta$ $\quad\quad$ $\tan\theta = \frac{\sqrt{x^2-4}}{2}$

$\ln|\sec\theta + \tan\theta| + C$

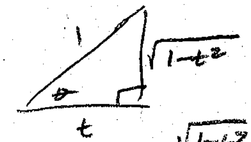
$$\boxed{\ln\left|\frac{x}{2} - \frac{\sqrt{x^2-4}}{2}\right| + C}$$

Figure out the trigonometric substitution and evaluate the definite integral.

19. $\int_0^{\sqrt{3}/2} \frac{t^2}{(1-t^2)^{3/2}}\, dt$

$$\boxed{-\frac{\pi}{2}+\sqrt{3}+\frac{\pi}{6}}$$

20. $\int_0^{\sqrt{3}/2} \frac{1}{(1-t^2)^{5/2}}\, dt$



$\cos\theta = \frac{t}{1}$ $\sin\theta = \frac{\sqrt{1-t^2}}{1}$

$t = \cos\theta$ $\sqrt{1-t^2} = \sin\theta$

$dt = -\sin\theta\, d\theta$

$\theta = \arccos t$

$t = \sqrt{3}/2 \to \theta = \pi/6$

$t = 0 \to \theta = \pi/2$

reverse limits

$$\int_{\pi/2}^{\pi/6} \frac{1}{\sin^5\theta}(-1)\sin\theta\, d\theta$$

$$\int_{\pi/6}^{\pi/2} \csc^4\theta\, d\theta = \int_{\pi/6}^{\pi/2} \csc^2\theta \csc^2\theta\, d\theta$$

$$(\csc^2\theta = 1+\cot^2\theta)$$

$$\int_{\pi/6}^{\pi/2} (1+\cot^2\theta)\csc^2\theta\, d\theta$$

$$\int_{\pi/6}^{\pi/2} \csc^2\theta\, d\theta + \int_{\pi/6}^{\pi/2} \cot^2\theta \csc^2\theta\, d\theta$$

u sub: $u = \cot\theta$
$du = -\csc^2\theta\, d\theta$

$$\left[-\cot\theta\right]_{\pi/6}^{\pi/2} - \int_{\pi/6}^{\pi/2} u^2\, du = \left[-\cot\theta - \frac{1}{3}\cot^3\theta\right]_{\pi/6}^{\pi/2}$$

$$\left[-\frac{\cos\pi/2}{\sin\pi/2} - \frac{1}{3}\left(\frac{\cos\pi/2}{\sin\pi/2}\right)^3\right] - \left[-\frac{\cos\pi/6}{\sin\pi/6} - \frac{1}{3}\left(\frac{\cos\pi/6}{\sin\pi/6}\right)^3\right]$$

$$\left[-\frac{0}{1} - \frac{1}{3}\left(\frac{0}{1}\right)^3\right] - \left[-\frac{\sqrt{3}/2}{1/2} - \frac{1}{3}\left(\frac{\sqrt{3}/2}{1/2}\right)^3\right]$$

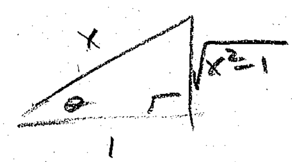
$$[0-0) - [-\sqrt{3} - \frac{1}{3}(3\sqrt{3})] = \boxed{2\sqrt{3}}$$

State the method of integration that you would use to perform each integration. Explain why you chose it. DO Not INTEGRATE.

21. $\int x\sqrt{x^2+1}\, dx$

u-Sub, because the x needed for the du appears in the integrand.

22. $\int x^2\sqrt{x^2-1}\, dx$



trig substitution

$\cos\theta = \frac{1}{x}$ $\tan\theta = \frac{\sqrt{x^2-1}}{1}$

$x = \frac{1}{\cos\theta} = \sec\theta$ $\sqrt{x^2-1} = \tan\theta$

$dx = \sec\theta\tan\theta\, d\theta$

$\int \sec^2\theta \tan\theta \sec\theta\tan\theta\, d\theta$

... because x^2 (not x) is in the integrand, so direct u-sub leaves an extra x, and there is a radical which could be the leg of a triangle.

7.5 Worksheet

Use Partial Fractions to evaluate the indefinite integral.

1. $\int \frac{1}{x^2-9}\,dx$

$$\boxed{\frac{1}{6}\ln|x-3| - \frac{1}{6}\ln|x+3| + C}$$

2. $\int \frac{2}{9x^2-1}\,dx = \int \frac{2}{(3x-1)(3x+1)}\,dx$

$9x^2-1 = (3x)^2 - (1)^2 \quad (a^2-b^2=(a-b)(a+b))$

$= (3x-1)(3x+1)$

$\frac{2}{(3x-1)(3x+1)} = \frac{A}{3x-1} + \frac{B}{3x+1}$

$\frac{2}{(3x-1)(3x+1)} = \frac{A(3x+1)}{(3x-1)(3x+1)} + \frac{B(3x-1)}{(3x-1)(3x+1)}$

$A(3x+1) + B(3x-1) = 2$

$3Ax + A + 3Bx - B = 2$

$(3A+3B)x + (A-B) = (0)x + (2)$

system: $\begin{cases} 3A+3B=0 \\ A-B=2 \end{cases} \rightarrow A = 2+B$

$3(2+B)+3B=0$

$6+6B=0, \quad \underline{B=-1}$

$\underline{A=2+(-1)=1}$

(or can use RREF)
here's how you show work:

$\left[\begin{array}{cc|c} 3 & 3 & 0 \\ 1 & -1 & 2 \end{array}\right) \text{rref} \left[\begin{array}{cc|c} 1 & 0 & 1 \\ 0 & 1 & -1 \end{array}\right] \begin{matrix} =A \\ =B \end{matrix}$

$1\int \frac{1}{3x-1}\,dx - 1\int \frac{1}{3x+1}\,dx$

usub $u=3x-1$
$du=3dx$
$dx=\frac{1}{3}du$

usub $u=3x+1$
$du=3dx$
$dx=\frac{1}{3}du$

(differential)

$\frac{1}{3}\int \frac{1}{u}\,du - \frac{1}{3}\int \frac{1}{u}\,du$

$$\boxed{\frac{1}{3}\ln|3x-1| - \frac{1}{3}\ln|3x+1| + C}$$

3. $\int \frac{5}{x^2+3x-4}\,dx$

$$\boxed{\begin{gathered} \ln|x-1| - \ln|x+4| + C \\ \text{or} \\ \ln\left|\frac{x-1}{x+4}\right| + C \end{gathered}}$$

4. $\int \frac{3-x}{3x^2-2x-1}\,dx$

$$\int \frac{3-x}{(3x+1)(x-1)}\,dx$$

$$-\frac{5}{2}\int \frac{1}{3x+1}\,dx + \frac{1}{2}\int \frac{1}{x-1}\,dx$$

u sub
$u = 3x+1$
$du = 3dx$
$dx = \frac{1}{3}du$

u sub
$u = x-1$
$du = dx$

$$-\frac{5}{2}\left(\frac{1}{3}\right)\int \frac{1}{u}\,du + \frac{1}{2}\int \frac{1}{u}\,du$$

$$\boxed{-\frac{5}{6}\ln|3x+1| + \frac{1}{2}\ln|x-1| + C}$$

$3x^2 - 2x - 1$

$$\frac{(3x+1)}{1}\frac{(3x-3)}{3}$$

m	a
-3	-2
(1)(-3)	-2 ✓
(-1)(3)	2

$(3x+1)(x-1)$

$$\frac{3-x}{(3x+1)(x-1)} = \frac{A}{3x+1} + \frac{B}{x-1}$$

$$\frac{3-x}{(3x+1)(x-1)} = \frac{A(x-1)}{(3x+1)(x-1)} + \frac{B(3x+1)}{(3x+1)(x-1)}$$

$$Ax - A + 3Bx + B = 3 - x$$
$$(A+3B)x + (-A+B) = (-1)x + (3)$$

System: $\begin{cases} A + 3B = -1 \\ -A + B = 3 \end{cases}$

$$\left[\begin{array}{cc|c} 1 & 3 & -1 \\ -1 & 1 & 3 \end{array}\right] \text{rref} \left[\begin{array}{cc|c} 1 & 0 & -5/2 \\ 0 & 1 & 1/2 \end{array}\right] \begin{matrix} =A \\ =B \end{matrix}$$

5. $\int \frac{x^2+12x+12}{x^3-4x}\,dx$

$$\boxed{-\ln|x| + 6\ln|x-2| - 4\ln|x+2| + c \quad \text{or} \quad \ln\left|\frac{(x-2)^6}{x(x+2)^4}\right| + c}$$

6. $\int \frac{x^3+3x^2-x-3}{x^2+x-1}\,dx$

degree larger in numerator, polynomial division first...

$$\begin{array}{r|l} & x+2 \\ x^2+x-1 & x^3+3x^2-x-3 \\ & -(x^3+x^2-x) \\ & 2x^2 \quad -3 \\ & -(2x^2+2x-2) \\ & -2x-1 \end{array}$$

$$\frac{x^3+3x^2-x-3}{x^2+x-1} = x+2 + \frac{-2x-1}{x^2+x-1}$$

$$= x+2 - \frac{2x+1}{x^2+x-1}$$

$\int \left(x+2 - \frac{2x+1}{x^2+x-1}\right)dx$

$\int x\,dx + \int 2\,dx - \int \frac{2x+1}{x^2+x-1}\,dx$

Just usub:

$u = x^2+x-1$

$du = (2x+1)\,dx$

$\int x\,dx + \int 2\,dx - \int \frac{1}{u}\,du$

$$\boxed{\tfrac{1}{2}x^2 + 2x - \ln|x^2+x-1| + c}$$

Unit 7 Practice Test

Period: ____________

Evaluate each indefinite integral.

1. $\int \cos(2x+5)\,dx$

u-sub

$u = 2x+5$

$du = 2dx$

$dx = \frac{1}{2}du$

$\frac{1}{2}\int \cos u\,du$

$\frac{1}{2}\sin u + C$

$\boxed{\frac{1}{2}\sin(2x+5)dx}$

2. $\int \ln x\,dx$

by parts

$u = \ln x \qquad dv = dx$

$\frac{du}{dx} = \frac{1}{x} \qquad \int dv = \int dx$

$du = \frac{1}{x}dx \qquad v = x$

$uv - \int v\,du$

$(\ln x)(x) - \int x \frac{1}{x}dx$

$x\ln x - \int 1\,dx$

$\boxed{x\ln x - x + C}$

3. $\int x e^x\,dx$

by parts

(select u to get 'easier' if possible)

$u = x \qquad dv = e^x$

$\frac{du}{dx} = 1 \qquad \int dv = \int e^x$

$du = dx \qquad v = e^x$

$uv - \int v\,du$

$xe^x - \int e^x dx$

$\boxed{xe^x - e^x + C}$

4. $\int \frac{1}{(x+3)(x-2)}\,dx$

partial fraction expansion

$$\frac{1}{(x+3)(x-2)} = \frac{A}{x+3} + \frac{B}{x-2}$$

$$\frac{1}{(x+3)(x-2)} = \frac{A(x-2)}{(x+3)(x-2)} + \frac{B(x+3)}{(x+3)(x-2)}$$ ← (you don't need to show this step on tests)

$A(x-2) + B(x+3) = 1$

$Ax - 2A + Bx + 3B = 1$

$(A+B)x + (-2A+3B) = (0)x + (1)$

system $\begin{cases} A+B=0 \\ -2A+3B=1 \end{cases}$ ← (practice evaluating systems by hand)

$A = -B$

$-2(-B) + 3B = 1$

$5B = 1$, $B = \frac{1}{5}$ $A = -\frac{1}{5}$

$$-\frac{1}{5}\int \frac{1}{x+3}\,dx + \frac{1}{5}\int \frac{1}{x-2}\,dx$$

usub
$u = x+3$
$du = dx$

usub
$u = x-2$
$du = dx$

$$-\frac{1}{5}\int \frac{1}{u}\,du + \frac{1}{5}\int \frac{1}{u}\,du$$

$$\boxed{-\frac{1}{5}\ln|x+3| + \frac{1}{5}\ln|x-2| + C}$$

5. $\int x\cos^2 x\;dx$

by parts

$u = x$
$\frac{du}{dx} = 1$
$du = dx$

$dv = \cos^2 x\,dx$

$\int dv = \int \cos^2 x\,dx$ $\left(\cos^2 x = \frac{1+\cos(2x)}{2}\right)$

$v = \int \left(\frac{1+\cos(2x)}{2}\right)dx$

$v = \int \frac{1}{2}dx + \frac{1}{2}\int \cos(2x)\,dx$

(u sub)
$u = 2x$
$du = 2dx$
$dx = \frac{1}{2}du$

$v = \frac{1}{2}x + \frac{1}{2}\frac{1}{2}\int \cos u\,du$

$v = \frac{1}{2}x + \frac{1}{4}\sin u$

$v = \frac{1}{2}x + \frac{1}{4}\sin(2x)$

$uv - \int v\,du$

$(x)\left(\frac{1}{2}x + \frac{1}{4}\sin(2x)\right) - \int\left(\frac{1}{2}x + \frac{1}{4}\sin(2x)\right)dx$

$\frac{1}{2}x^2 + \frac{1}{4}x\sin(2x) - \frac{1}{2}\int x\,dx - \frac{1}{4}\int \sin(2x)\,dx$

$\frac{1}{2}\frac{1}{2}x^2$

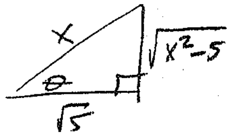
(u sub)
$u = 2x$
$du = 2dx$
$dx = \frac{1}{2}du$

$\frac{1}{2}x^2 + \frac{1}{4}x\sin(2x) - \frac{1}{4}x^2 - \frac{1}{4}\frac{1}{2}\int \sin u\,du$

$\frac{1}{4}x^2 + \frac{1}{4}x\sin(2x) - \frac{1}{8}(-\cos u) + C$

$$\boxed{\frac{1}{4}x^2 + \frac{1}{4}x\sin(2x) + \frac{1}{8}\cos(2x) + C}$$

6. $\int \frac{2}{\sqrt{x^2-5}}\,dx$ $\boxed{\text{trig substitution}}$



check
$a^2+b^2=c^2$
$(\sqrt{5})^2+(\sqrt{x^2-5})^2=(x)^2$
$5+x^2-5=x^2$ ✓

pair constant with each of other sides:

$\cos\theta=\frac{\sqrt{5}}{x}$ $\tan\theta=\frac{\sqrt{x^2-5}}{\sqrt{5}}$

$x=\frac{\sqrt{5}}{\cos\theta}=\sqrt{5}\sec\theta$ $\sqrt{x^2-5}=\sqrt{5}\tan\theta$

$dx=\sqrt{5}\sec\theta\tan\theta\,d\theta$

$\int \frac{2}{\sqrt{5}\tan\theta}\sqrt{5}\sec\theta\tan\theta\,d\theta$

$2\int \sec\theta\,d\theta$ ← memorize this as a shortcut

$2\ln|\sec\theta+\tan\theta|+C$ ← use triangle to replace:

$\sec\theta=\frac{1}{\cos\theta}=\frac{hyp}{adj}=\frac{x}{\sqrt{5}}$

$\tan\theta=\frac{opp}{adj}=\frac{\sqrt{x^2-5}}{\sqrt{5}}$

$$\boxed{2\ln\left|\frac{x}{\sqrt{5}}+\frac{\sqrt{x^2-5}}{\sqrt{5}}\right|+C}$$

(answer must use original variable, x)

7. $\int (x+5)(x^3+5x-7)\,dx$ (distribute) $(x+5)(x^3+5x-7)$

$x(x^3+5x-7)+5(x^3+5x-7)$

$x^4+5x^2-7x+5x^3+25x-35$

$x^4+5x^3+5x^2+18x-35$

$\int (x^4+5x^3+5x^2+18x-35)\,dx$

$$\boxed{\frac{1}{5}x^5+\frac{5}{4}x^4+\frac{5}{3}x^3+9x^2-35x+C}$$

8. $\int \frac{\sin^2 x}{\cos x}\, dx$ (trig identity) $\sin^2 x = 1-\cos^2 x$

$\int \frac{1-\cos^2 x}{\cos x} dx$

$\int \frac{1}{\cos x} dx - \int \frac{\cos^2 x}{\cos x} dx$

$\int \sec x\, dx - \int \cos x\, dx$

$\boxed{\ln|\sec x+\tan x| - \sin x + C}$

9. $\int \csc^4 x \cot x\, dx$ $\boxed{\text{trig integrals}}$ $\underbrace{\frac{d}{dx}[\cot x] = -\csc^2 x}_{\text{we this one}}$ or $\frac{d}{dx}[\csc x] = -\csc x \cot x$

$\left(\frac{\sin^2 x}{\sin^2 x} + \frac{\cos^2 x}{\sin^2 x} = \frac{1}{\sin^2 x} \right.$

$\left. 1 + \cot^2 x = \csc^2 x \right)$

$\int \csc^2 x \cot x \underline{\csc^2 x dx}$ ← for du

$\int (1+\cot^2 x)\cot x \csc^2 x dx$

$\int \cot x \csc^2 x dx + \int \cot^3 x \csc^2 x dx$

u sub: $u = \cot x$

$du = -\csc^2 x dx$

$\csc^2 x dx = -du$

$-\int u\, du - \int u^3 du$

$-\frac{1}{2}u^2 - \frac{1}{4}u^4 + C$

$\boxed{-\frac{1}{2}\cot^2 x - \frac{1}{4}\cot^4 x + C}$

10. $\int \cot x\, dx = \int \frac{\cos x}{\sin x} dx$ u sub $u = \sin x$

$du = \cos x\, du$

$= \int \frac{1}{u} du$

$= \ln|u| + C$

$= \boxed{\ln|\sin x| + C}$