AP Calculus BC

7.1 Worksheet

Name: answers/Solutions

Period: ________

Evaluate the definite/indefinite integral.

1. $\int (5x-3)^4 dx$

$\boxed{\frac{1}{25}(5x-3)^5+c}$

2. $\int_0^2 (5x^2+7x-4)\, dx$

$5\int_0^2 x^2 dx + 7\int_0^2 x\, dx - \int_0^2 4 dx$

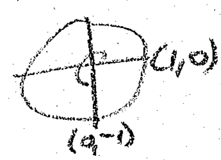
$\left[5\left(\frac{1}{3}\right)x^3 + 7\left(\frac{1}{2}x^2\right) - 4x\right]_0^2$

$\boxed{\left[\frac{5}{3}(2)^3 + \frac{7}{2}(2)^2 - 4(2)\right] - \left[\frac{5}{3}(0)^3 + \frac{7}{2}(0)^2 - 4(0)\right] = \frac{58}{3}}$

3. $\int \frac{2t+1}{t^2+t-4} dx$

$\boxed{\ln|t^2+t-4| + c}$

4. $\int_0^{3\pi/2} \sin(x)\, dx$



$\left[-\cos x\right]_0^{3\pi/2}$

$\left[-\cos\left(\frac{3\pi}{2}\right)\right] - \left[-\cos(0)\right]$

$\left[-(0)\right] - \left[-(1)\right]$

$0+1$

$\boxed{1}$

5. $\int t \cdot sin(t^2)\, dt$

$\boxed{-\frac{1}{2}\cos(t^2)+C}$

6. $\int 2t\sqrt{t^2+7}\, dt$

$u = t^2+7$

$\frac{du}{dt} = 2t$

$du = 2t\,dt$

$\int \sqrt{u}\, du$

$\int u^{1/2} du$

$\frac{2}{3}u^{3/2}+C$

$\boxed{\frac{2}{3}(t^2+7)^{3/2}+C}$

7. $\int \frac{7}{(z-10)^7}\, dz$

$\frac{-7}{6}(z-10)^{-6}+C$

or

$\frac{-7}{6(z-10)^6}+C$

8. $\int [2x + \sin(x)]\, dx$

$2\int x\,dx + \int \sin x\,dx$

$2(\frac{1}{2}x^2)+(-\cos x)+C$

$\boxed{x^2 - \cos x + C}$

9. $\int \left[v + \frac{1}{(3v-1)^3}\right] dv$

$$\boxed{\begin{gathered} \frac{1}{2}v^2 - \frac{1}{6}(3v-1)^{-2} + C \\ \frac{1}{2}v^2 - \frac{1}{6(3v-1)^2} + C \end{gathered}}$$

10. $\int (3e^x + 2x^3)\, dx$

$3\int e^x dx + 2\int x^3 dx$

$3e^x + 2\left(\frac{x^4}{4}\right) + C$

$$\boxed{3e^x + \frac{1}{2}x^4 + C}$$

11. $\int \frac{t^2-3}{-t^3+9t+1}\, dt$

$$\boxed{-\frac{1}{3}\ln\left|-t^3+9t+1\right| + C}$$

12. $\int \frac{\sin x}{\sqrt{\cos x}}\, dx$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x\, dx$

$\sin x\, dx = -du$

$\int \frac{1}{\sqrt{u}}(-du)$

$-\int u^{-1/2} du$

$-\left[\frac{u^{1/2}}{(1/2)}\right] + C$

$$\boxed{\begin{gathered} -2(\cos x)^{1/2} + C \\ -2\sqrt{\cos x} + C \end{gathered}}$$

13. $\int \frac{2}{e^{-x}+1}\,dx$

$\boxed{2x+2\ln|e^{-x}+1|+C}$

14. $\int -\frac{1}{\sqrt{1-(4t+1)^2}}\,dt$

$u=4t+1 \qquad a=1$

$\frac{du}{dt}=4$

$du=4dt$

$dt=\frac{1}{4}du$

$-\int \frac{1}{\sqrt{a^2-u^2}}\left(\frac{1}{4}du\right)$

$-\frac{1}{4}\int \frac{1}{\sqrt{a^2-u^2}}\,du$

$-\frac{1}{4}\arcsin\left(\frac{4t+1}{1}\right)+C$

$\boxed{-\frac{1}{4}\arcsin(4t+1)+C}$

15. $\int x \cdot e^{-x^2}\,dx$

$\boxed{-\frac{1}{2}e^{(-x^2)}+C}$

16. $\int \frac{1}{4+9x^2}\,dx$

$u=3x \qquad a=2$

$\frac{du}{dx}=3$

$du=3dx$

$dx=\frac{1}{3}du$

$\int \frac{1}{a^2+u^2}\left(\frac{1}{3}du\right)$

$\frac{1}{3}\int \frac{1}{a^2+u^2}\,du$

$\frac{1}{3}\left(\frac{1}{2}\arctan\left(\frac{3x}{2}\right)\right)+C$

$\boxed{\frac{1}{6}\arctan\left(\frac{3x}{2}\right)+C}$

7.2 Worksheet Part 1

Identify u and dv for the following integrals. DO NOT EVALUATE.

1. $\int xe^x \, dx$

u = x

dv = $e^x dx$

2. $\int x^2 \sin(x) \, dx$

u = x^2

dv = $\sin x dx$

3. $\int \ln(5x) \, dx$

u = $\ln(5x)$

dv = dx

4. $\int e^x \sin(x) \, dx$

u = $\sin x$

dv = $e^x dx$

Use Integration by Parts to evaluate the integral. The u and dv are given.

5. $\int 2xe^x \, dx$

u = 2x

dv = e^x

$\boxed{2xe^x - 2e^x + C}$

6. $\int x^2 \sin(2x) \, dx$ (twice!)

u = x^2

dv = sin(2x)

$u = x^2$ $dv = \sin(2x)dx$

$\frac{du}{dx} = 2x$ $\int dv = \int \sin(2x)dx$

$du = 2x dx$ $v = -\frac{1}{2}\cos(2x)$

$uv - \int v du$

$(x^2)(-\frac{1}{2}\cos(2x)) - \int(-\frac{1}{2}\cos(2x))2x dx$

$u = x$ $dv = \cos(2x)dx$

$-\frac{1}{2}x^2\cos(2x) + [\int x \cos(2x) dx]$

$\frac{du}{dx} = 1$ $\int dv = \int \cos 2x dx$

$du = dx$ $v = \frac{1}{2}\sin(2x)$

$-\frac{1}{2}x^2\cos(2x) + [uv - \int v du)$

$-\frac{1}{2}x^2\cos(2x) + [(x)(\frac{1}{2}\sin(2x)) - \int \frac{1}{2}\sin(2x)dx]$

$-\frac{1}{2}x^2\cos(2x) + [\frac{1}{2}x\sin(2x) - \frac{1}{2}(-\frac{1}{2}\cos(2x))]$

$\boxed{-\frac{1}{2}x^2\cos(2x) + \frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x) + C}$

7. $\int x^3 Ln(x)\, dx$

u = Ln(x)

dv = x^3

$$\boxed{\frac{1}{4}x^4 \ln x - \frac{1}{16}x^4 + C}$$

8. $\int 3x^2 Ln(x)\, dx$

u = Ln(x)

dv = $3x^2$

$u = \ln x$ $\quad dv = 3x^2 dx$

$\frac{du}{dx} = \frac{1}{x}$ $\quad \int dv = \int 3x^2 dx$

$du = \frac{1}{x}dx$ $\quad v = x^3$

$uv - \int v\,du$

$(\ln x)(x^3) - \int x^3 \frac{1}{x} dx$

$x^3 \ln x - \int x^2 dx$

$$\boxed{x^3 \ln x - \frac{1}{3}x^3 + C}$$

Use the simplest method to evaluate the integral. Some will NOT require Integration by Parts. u-substitution!

9. $\int 2x\, dx$

$$\boxed{x^2 + C}$$

10. $\int x \cdot \sin(x^2)\, dx$

$u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$x\,dx = \frac{1}{2}du$

$\int \sin u \left(\frac{1}{2}du\right)$

$\frac{1}{2}\int \sin u\, du$

$\frac{1}{2}(-\cos u) + C$

$$\boxed{-\frac{1}{2}\cos(x^2) + C}$$

11. $\int x \cdot e^{4x}\, dx$

$$\boxed{\frac{1}{4}xe^{4x} - \frac{1}{16}e^{4x} + C}$$

12. $\int t \cdot Ln(t+1)\, dt$

$u = \ln(t+1)$ $\quad dv = t\,dt$

$\frac{du}{dt} = \frac{1}{t+1}(1)$ $\quad \int dv = \int t\,dt$

$du = \frac{1}{t+1}dt$ $\quad v = \frac{1}{2}t^2$

$uv - \int v\,du$

$(\ln(t+1))(\frac{1}{2}t^2) - \int \frac{1}{2}t^2 \frac{1}{t+1} dt$

continued...

7.2 #12 continued...

$(\ln(t+1))(\frac{1}{2}t^2) - \int \frac{1}{2}t^2 \frac{1}{t+1}\,dt$

$\frac{1}{2}t^2 \ln(t+1) - \frac{1}{2}\int \frac{t^2}{t+1}\,dt$ now, $u = t+1 \rightarrow t = u-1$

$\frac{du}{dt} = 1$

$du = dt$

$-\frac{1}{2}\int t^2 \frac{1}{u}\,du$

$-\frac{1}{2}\int (u-1)^2 \frac{1}{u}\,du$

$-\frac{1}{2}\int (u^2 - 2u + 1)\frac{1}{u}\,du$

$-\frac{1}{2}\int (u - 2 + \frac{1}{u})\,du$

$-\frac{1}{2}\int u\,du + \int 1\,du - \int \frac{1}{u}\,du$

$-\frac{1}{2}(\frac{1}{2}u^2) + u - \ln|u|$

$-\frac{1}{4}(t+1)^2 + t + 1 - \ln|t+1|$

$\boxed{\frac{1}{2}t^2 \ln(t+1) - \frac{1}{4}(t+1)^2 + t + 1 - \ln|t+1| + C}$

13. $\int \frac{[Ln(x)]^2}{x}\, dx$

$\boxed{\frac{1}{3}(\ln x)^3 + c}$

14. $\int \frac{x}{\sqrt{x^2+3}}\, dx$

$u = x^2+3$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$x\,dx = \frac{1}{2}du$

$\int \frac{1}{\sqrt{u}}\left(\frac{1}{2}du\right)$

$\frac{1}{2}\int u^{-1/2}\,du$

$\frac{1}{2}(2u^{1/2}) + C$

$\boxed{\begin{array}{l}(x^2+3)^{1/2} + C \\ \sqrt{x^2+3} + C\end{array}}$

15. $\int x^3 \cdot \sin(x)\, dx$

$\boxed{-x^3\cos x + 3x^2\sin x + 6x\cos x - 6\sin x + C}$

16. $\int 4x^2 \cos(x)\, dx$

$u = 4x^2$ $\quad dv = \cos x\,dx$

$\frac{du}{dx} = 8x$ $\quad \int dv = \int \cos x\,dx$

$du = 8x\,dx$ $\quad v = \sin x$

$uv - \int v\,du$

$(4x^2)(\sin x) - \int \sin x \, 8x\,dx$

$4x^2\sin x - 8\left[\int x\sin x\,dx\right]$

$u = x$ $\quad dv = \sin x\,dx$

$\frac{du}{dx} = 1$ $\quad \int dv = \int \sin x\,dx$

$du = dx$ $\quad v = -\cos x$

$4x^2\sin x - 8\left[uv - \int v\,du\right]$

$4x^2\sin x - 8\left[x(-\cos x) - \int(-\cos x)dx\right]$

$4x^2\sin x + 8x\cos x - 8\int \cos x\,dx$

$\boxed{4x^2\sin x + 8x\cos x - 8\sin x + C}$

17. $\int e^{-3x}\sin(5x)\, dx =$
$\frac{9}{34}\left[-\frac{1}{3}e^{-3x}\sin(5x) - \frac{5}{9}e^{-3x}\cos(5x)\right]$

18. $\int e^{4x}\cos(2x)\, dx$

on next sheet ...

7.2 #18 $\int e^{4x}\cos(2x)\,dx$

$u=\cos(2x)$ $\quad dv=e^{4x}dx$ $\quad$ ← (u-sub w/ $u=4x$)

$\frac{du}{dx}=-2\sin(2x)$ $\quad \int dv=\int e^{4x}dx$

$du=-2\sin(2x)dx$ $\quad v=\frac{1}{4}e^{4x}$

$uv-\int v\,du$

$(\cos(2x))(\frac{1}{4}e^{4x})-\int \frac{1}{4}e^{4x}(-2\sin(2x))dx$

$\frac{1}{4}e^{4x}\cos(2x)+\frac{1}{2}\left[\int e^{4x}\sin(2x)dx\right]$

$u=\sin(2x)$ $\quad dv=e^{4x}dx$

$\frac{du}{dx}=2\cos(2x)$ $\quad \int dv=\int e^{4x}dx$

$du=2\cos(2x)dx$ $\quad v=\frac{1}{4}e^{4x}$

$\frac{1}{4}e^{4x}\cos(2x)+\frac{1}{2}\left[uv-\int v\,du\right]$

$\frac{1}{4}e^{4x}\cos(2x)+\frac{1}{2}\left[(\sin(2x))(\frac{1}{4}e^{4x})-\int \frac{1}{4}e^{4x}2\cos(2x)dx\right]$

$\frac{1}{4}e^{4x}\cos(2x)+\frac{1}{8}e^{4x}\sin(2x)-\frac{1}{4}\int e^{4x}\cos(2x)dx$ this equals the original integral...

$\underline{\int e^{4x}\cos(2x)dx}=\frac{1}{4}e^{4x}\cos(2x)+\frac{1}{8}e^{4x}\sin(2x)\underline{-\frac{1}{4}\int e^{4x}\cos(2x)dx}$ ← combine like terms

$1\int e^{4x}\cos(2x)dx+\frac{1}{4}\int e^{4x}\cos(2x)dx=\frac{1}{4}e^{4x}\cos(2x)+\frac{1}{8}e^{4x}\sin(2x)$

$(1+\frac{1}{4})\int e^{4x}\cos(2x)dx=\frac{1}{4}e^{4x}\cos(2x)+\frac{1}{8}e^{4x}\sin(2x)$

$\frac{5}{4}\int e^{4x}\cos(2x)dx=\frac{1}{4}e^{4x}\cos(2x)+\frac{1}{8}e^{4x}\sin(2x)$

so... $\int e^{4x}\cos(2x)dx=\boxed{\frac{4}{5}\left[\frac{1}{4}e^{4x}\cos(2x)+\frac{1}{8}e^{4x}\sin(2x)\right]}$

Evaluate the definite integral.

19. $\int_0^2 x \cdot e^{\frac{x}{2}}\, dx$

$\boxed{4}$

20. $\int_0^{\pi/4} x \cdot \cos(2x)\, dx$

$u = x \quad dv = \cos(2x)dx$

$\frac{du}{dx} = 1 \quad \int dv = \int \cos(2x)dx$

$du = dx \quad v = \frac{1}{2}\sin(2x)$

$uv - \int v du$

$(x)(\frac{1}{2}\sin(2x)) - \int \frac{1}{2}\sin(2x)dx$

$\frac{1}{2}x\sin(2x) - \frac{1}{2}\int \sin(2x)dx$

$\frac{1}{2}x\sin(2x) - \frac{1}{2}(\frac{1}{2}(-\cos(2x))$

$[\frac{1}{2}x\sin(2x) + \frac{1}{4}\cos(2x)]_0^{\pi/4}$

$[\frac{1}{2}(\frac{\pi}{4})\sin(\frac{\pi}{2}) + \frac{1}{4}\cos(\frac{\pi}{2})] - [\frac{1}{2}(0)\sin(0) + \frac{1}{4}\cos(0)]$

$[\frac{\pi}{8}(1) + \frac{1}{4}(0)] - (0 + \frac{1}{4}(1))$

$\boxed{\frac{\pi}{8} - \frac{1}{4}}$

21. $\int_2^4 x \cdot \text{arcsec}(x)\, dx$

$\boxed{8\text{arcsec}(4) - \frac{\sqrt{15}}{2} - \frac{2\pi}{3} + \frac{\sqrt{3}}{2}}$

(make $u = \text{arcsec}\,x$)

22. $\int_3^4 xLn(x)\, dx$

$u = \ln x \quad dv = x dx$

$\frac{du}{dx} = \frac{1}{x} \quad \int dv = \int x dx$

$uv - \int v du \quad du = \frac{1}{x}dx \quad v = \frac{1}{2}x^2$

$(\ln x)(\frac{1}{2}x^2) - \int \frac{1}{2}x^2 \frac{1}{x}dx$

$\frac{1}{2}x^2\ln x - \frac{1}{2}\int x dx$

$\frac{1}{2}x^2\ln x - \frac{1}{2}(\frac{1}{2}x^2)$

$[\frac{1}{2}x^2\ln x - \frac{1}{4}x^2]_3^4$

$[\frac{1}{2}(4)^2\ln(4) - \frac{1}{4}(4)^2] - [\frac{1}{2}(3)^2\ln(3) - \frac{1}{4}(3)^2]$

$[8\ln(4) - 4] - [\frac{9}{2}\ln(3) - \frac{9}{4}]$

$8\ln(4) - 4 - \frac{9}{2}\ln(3) + \frac{9}{4}$

$\boxed{8\ln(4) - \frac{9}{2}(3) - \frac{7}{4}}$

7.2 Worksheet Part 2

Use the tabular method to evaluate the integral.

1. $\int x^2 e^{2x}\, dx$

$\boxed{\frac{1}{2}e^{2x}x^2 - \frac{1}{2}e^{2x}x + \frac{1}{4}e^{2x} + C}$

2. $\int x^3 e^{-2x}\, dx$

$u = x^3 \quad dv = e^{-2x}dx$

Sign	u (deriv.)	dv (antideriv.)
+	x^3	e^{-2x}
−	$3x^2$	$-\frac{1}{2}e^{-2x}$
+	$6x$	$\frac{1}{4}e^{-2x}$
−	6	$-\frac{1}{8}e^{-2x}$
+	0	$\frac{1}{16}e^{-2x}$

$\boxed{-\frac{1}{2}x^3e^{-2x} - \frac{3}{4}x^2e^{-2x} - \frac{3}{4}xe^{-2x} - \frac{3}{8}e^{-2x} + C}$

3. $\int x^3 \sin(x)\, dx$

$\boxed{-x^3\cos x + 3x^2\sin x + 6x\cos x + 6\sin x + C}$

4. $\int x^3 \cos(x)\, dx$

$u = x^3 \quad dv = \cos x\,dx$

Sign	u (deriv.)	dv (antideriv.)
+	x^3	$\cos x$
−	$3x^2$	$\sin x$
+	$6x$	$-\cos x$
−	6	$-\sin x$
+	0	$\cos x$

$\boxed{x^3\sin x + 3x^2\cos x - 6x\sin x - 6\cos x + C}$

5 – 8 Use Substitution, then Integration by Parts to evaluate the integral.

5. $\int \sin(\sqrt{x})\, dx$

$\boxed{-2\sqrt{x}\cos(\sqrt{x}) + 2\sin(\sqrt{x}) + C}$

6. $\int 2x^3 \cos(x^2)\, dx$

$u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$\int x^2\cos(x^2)(2x\,dx)$

$\int u\cos(u)\,du$ change letter...

$\int w\cos(w)\,dw$ by parts:

$u = w \quad dv = \cos(w)dw$

$\frac{du}{dw} = 1 \quad \int dv = \int \cos(w)dw$

$du = dw \quad v = \sin w$

$uv - \int v\,du$

$(w)(\sin w) - \int \sin w\,dw$

$w\sin(w) - (-\cos(w)) + C$

$w\sin(w) + \cos(w) + C$

$w = \text{orig. } u = x^2$

$\boxed{x^2\sin(x^2) + \cos(x^2) + C}$

7. $\int x^5 e^{x^2}\,dx$

$\boxed{\frac{1}{2}x^4 e^{x^2} - x^2 e^{x^2} + e^{x^2} + c}$

8. $\int e^{\sqrt{x}}\,dx$

$u = \sqrt{x} = x^{1/2}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$

$du = \frac{1}{2\sqrt{x}}dx = \frac{1}{2u}dx, \quad dx = 2u\,du$

$\int e^u (2u\,du)$

$2\int u e^u du = 2\int w e^w dw$

by parts:

$u = w \quad dv = e^w dw$

$\frac{du}{dw} = 1 \quad \int dv = \int e^w dw$

$du = dw \quad v = e^w$

$2[uv - \int v\,du]$

$2(we^w - \int e^w dw)$

$2we^w - 2\int e^w dw$

$2we^w - 2e^w + C$

$w = \text{orig } u = \sqrt{x}$

$\boxed{2\sqrt{x}e^{\sqrt{x}} - 2e^{\sqrt{x}} + C}$

Use the different methods to evaluate the integral. You should get the same answer!

9. $\int \frac{x^3}{\sqrt{4+x^2}}\,dx$

Use Int by Parts with dv = $\frac{x}{\sqrt{4+x^2}}$

$\boxed{x^2\sqrt{4+x^2} - \frac{2}{3}(4+x^2)^{3/2} + C}$

Use Substitution with u = $4 + x^2$

$\boxed{\frac{1}{3}(4+x^2)^{3/2} - 4\sqrt{4+x^2} + C}$

these don't look the same

but graph them in a calculator...

same curve :)

Quiz Review

Evaluate the integral using the simplest method.

1. $\int(3x^4 + 7x^2 + x - 8)dx$

$3\left(\frac{x^5}{5}\right)+7\left(\frac{x^3}{3}\right)+\frac{x^2}{2}-8x+c$

$\boxed{\frac{3}{5}x^5+\frac{7}{3}x^3+\frac{1}{2}x^2-8x+c}$

2. $\int 2\sin(x)\,dx$

$\boxed{-2\cos x+C}$

3. $\int(5x + \cos(x))dx$

$5\left(\frac{x^2}{2}\right)+\sin x+c$

$\boxed{\frac{5}{2}x^2+\sin x+C}$

4. $\int(e^{2x} - 3\sec(x)\tan(x))dx$ $\frac{d}{dx}[\sec x]=\sec x\tan x$

$\int e^{2x}dx-3\int\sec x\tan x\,dx$

$u=2x$

$\frac{du}{dx}=2$

$du=2dx$

$dx=\frac{1}{2}du$

$\frac{1}{2}\int e^u du-3\int\sec x\tan x\,dx$

$\boxed{\frac{1}{2}e^{2x}-3\sec x+C}$

5. $\int\left(\frac{\ln(x)}{x}\right)dx$

u-sub:

$u=\ln x$

$\frac{du}{dx}=\frac{1}{x}$

$du=\frac{1}{x}dx$

$\int \ln x\left(\frac{1}{x}dx\right)$

$\int u\,du$

$\frac{u^2}{2}+c$

$\boxed{\frac{1}{2}(\ln x)^2+c}$

6. $\int\frac{x}{\sqrt{x^2+15}}dx$

u sub:

$u=x^2+15$

$\frac{du}{dx}=2x$

$du=2x\,dx$

$x\,dx=\frac{1}{2}du$

$\int\frac{1}{\sqrt{x^2+15}}(x\,dx)$

$\int\frac{1}{\sqrt{u}}\left(\frac{1}{2}du\right)$

$\frac{1}{2}\int u^{-1/2}du$

$\frac{1}{2}\left(\frac{u^{1/2}}{(1/2)}\right)+c$

$\boxed{(x^2+15)^{1/2}+c}$

$\boxed{\sqrt{x^2+15}+c}$

7. $\int x \sin(x)\, dx$

by parts:

$u = x \qquad dv = \sin x\, dx$

$\frac{du}{dx} = 1 \qquad \int dv = \int \sin x\, dx$

$uv - \int v\, du \qquad du = dx \qquad v = -\cos x$

$(x)(-\cos x) - \int (-\cos x)\, dx$

$-x\cos x + \int \cos x\, dx$

$\boxed{-x\cos x + \sin x + C}$

8. $\int \tan(x)\, dx = \int \frac{\sin x}{\cos x} dx$

u sub:

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$du = -\sin x\, dx$

$\sin x\, dx = -du$

$\int \frac{1}{u}(-du)$

$-\int \frac{1}{u} du$

$-\ln|u| + C$

$\boxed{-\ln|\cos x| + C}$

or

$\ln|(\cos(x))^{-1}| + C$

$\ln|\sec x| + C$

9. $\int x\cos(x^2)\, dx$

u-sub:

$u = x^2$

$\frac{du}{dx} = 2x$

$du = 2x\,dx, \quad x\,dx = \frac{1}{2} du$

$\int \cos u \left(\frac{1}{2} du\right)$

$\frac{1}{2}\int \cos u\, du$

$\frac{1}{2}\sin u + C$

$\boxed{\frac{1}{2}\sin(x^2) + C}$

10. $\int \ln(x)\, dx$

by parts:

$u = \ln x \qquad dv = 1\,dx$

$\frac{du}{dx} = \frac{1}{x} \qquad \int dv = \int 1\,dx$

$du = \frac{1}{x} dx \qquad v = x$

$uv - \int v\, du$

$(\ln x)(x) - \int x \frac{1}{x} dx$

$x\ln x - \int 1\, dx$

$\boxed{x\ln x - x + C}$

11. $\int x^2 e^x \, dx$ by parts or tabular

by parts:

$u = x^2 \quad dv = e^x dx$

$\frac{du}{dx} = 2x \quad \int dv = \int e^x dx$

$du = 2x\,dx \quad v = e^x$

$uv - \int v\,du$

$(x^2)(e^x) - \int e^x 2x\,dx$

$x^2 e^x - 2\int x e^x\,dx$

by parts again!

$u = x \quad dv = e^x dx$

$\frac{du}{dx} = 1 \quad \int dv = \int e^x dx$

$du = dx \quad v = e^x$

$x^2 e^x - 2[uv - \int v\,du]$

$x^2 e^x - 2[x e^x - \int e^x dx]$

$\boxed{x^2 e^x - 2x e^x + 2e^x + c}$

tabular!

Sign	u(deriv)	dv(antideri)
+	x^2	e^x
−	$2x$	e^x
+	2	e^x
−	0	e^x

$\boxed{x^2 e^x - 2x e^x + 2e^x + c}$

12. $\int x^4 \cos(x)\, dx$ by parts 4 times :(or tabular...

Sign	u(deriv)	dv(antideriv)
+	x^4	$\cos x$
−	$4x^3$	$\sin x$
+	$12x^2$	$-\cos x$
−	$24x$	$-\sin x$
+	24	$\cos x$
−	0	$\sin x$

$\boxed{x^4 \sin x + 4x^3 \cos x - 12x^2 \sin x - 24x\cos x + 24\sin x + c}$

13. $\int e^{4x} dx$ u-sub;

$u = 4x$

$\frac{du}{dx} = 4$

$du = 4dx$

$dx = \frac{1}{4}du$

$\int e^u (\frac{1}{4}du)$

$\frac{1}{4}\int e^u du$

$\frac{1}{4}e^u + c$

$\boxed{\frac{1}{4}e^{4x} + c}$

14. $\int \frac{4}{(2x+7)^6} dx$ u-sub:

$u = 2x+7$

$\frac{du}{dx} = 2$

$du = 2dx$

$dx = \frac{1}{2}du$

$4\int \frac{1}{u^6}(\frac{1}{2}du)$

$2\int u^{-6} du$

$2\left(\frac{u^{-5}}{-5}\right) + c$

$\boxed{-\frac{2}{5}(2x+7)^{-5} + c \qquad -\frac{2}{5(2x+7)^5} + c}$

15. $\int \sec^2(3x)\,dx$ $\frac{d}{dx}[\tan x] = \sec^2 x$

usub;
$u = 3x$
$\frac{du}{dx} = 3$
$du = 3dx,\ dx = \frac{1}{3}du$

$\int(\sec^2 u)(\frac{1}{3})du$

$\frac{1}{3}\int \sec^2 u\,du$

$\frac{1}{3}\tan u + c$

$\boxed{\frac{1}{3}\tan(3x) + c}$

16. $\int x^4 \ln(x)\,dx$ by parts:

$u = \ln x$ $dv = x^4 dx$
$\frac{du}{dx} = \frac{1}{x}$ $\int dv = \int x^4 dx$
$du = \frac{1}{x}dx$ $v = \frac{1}{5}x^5$

$uv - \int v\,du$

$(\ln x)(\frac{1}{5}x^5) - \int \frac{1}{5}x^5 \frac{1}{x}dx$

$\frac{1}{5}x^5 \ln x - \frac{1}{5}\int x^4 dx$

$\frac{1}{5}x^5 \ln x - \frac{1}{5}(\frac{1}{5}x^5) + c$

$\boxed{\frac{1}{5}x^5 \ln x - \frac{1}{25}x^5 + c}$

17. $\int e^x \cos(x)\,dx$ cyclic

(doesn't matter which you pick for by parts)... because both cycle

for this one, let's pick:

$u = e^x$ $dv = \cos x\,dx$
$\frac{du}{dx} = e^x$ $\int dv = \int \cos x\,dx$
$du = e^x dx$ $v = \sin x$

$uv - \int v\,du$

$e^x \sin x - \boxed{\int e^x \sin x\,dx}$

again: $u = e^x$ $dv = \sin x\,dx$
$\frac{du}{dx} = e^x$ $\int dv = \int \sin x\,dx$
$du = e^x dx$ $v = -\cos x$

$e^x \sin x - [uv - \int v\,du]$

$e^x \sin x - [-e^x \cos x - \int(-\cos x e^x)dx]$

$e^x \sin x + e^x \cos x - \int e^x \cos x\,dx + C$

continued...

18. $\int e^{2x}\sin(4x)\,dx$

for this one, let's pick:

$u = \sin(4x)$ $dv = e^{2x}dx$
$\frac{du}{dx} = 4\cos(4x)$ $\int dv = \int e^{2x}dx$
$du = 4\cos(4x)dx$ $v = \frac{1}{2}e^{2x}$

$uv - \int v\,du$

$\frac{1}{2}e^{2x}\sin(4x) - \int \frac{1}{2}e^{2x}4\cos(4x)dx$

$\frac{1}{2}e^{2x}\sin(4x) - 2\boxed{\int e^{2x}\cos(4x)dx}$

again: $u = \cos(4x)$ $dv = e^{2x}dx$
$\frac{du}{dx} = -4\sin(4x)$ $\int dv = \int e^{2x}dx$
$du = -4\sin(4x)dx$ $v = \frac{1}{2}e^{2x}$

$\frac{1}{2}e^{2x}\sin(4x) - 2[uv - \int v\,du]$

$\frac{1}{2}e^{2x}\sin(4x) - 2[\frac{1}{2}e^{2x}\cos(4x) - \int \frac{1}{2}e^{2x}(-4\sin(4x))dx]$

$\frac{1}{2}e^{2x}\sin(4x) - e^{2x}\cos(4x) - 4\int e^{2x}\sin(4x)dx$

continued...

#17 continued...

$\int e^x \cos x\,dx = e^x \sin x + e^x \cos x - \int e^x \cos x\,dx$

$1\int e^x \cos x\,dx + 1\int e^x \cos x\,dx = e^x \sin x + e^x \cos x$

$(1+1)$

$2\int e^x \cos x\,dx = e^x \sin x + e^x \cos x$

$\int e^x \cos x\,dx = \boxed{\frac{1}{2}\left[e^x \sin x + e^x \cos x\right] + C}$

#18 continued...

$\int e^{2x} \sin(4x)\,dx = \frac{1}{2}e^{2x}\sin(4x) - e^{2x}\cos(4x) - 4\int e^{2x}\sin(4x)\,dx$

$1\int e^{2x}\sin(4x)\,dx + 4\int e^{2x}\sin(4x)\,dx = \frac{1}{2}e^{2x}\sin(4x) - e^{2x}\cos(4x)$

$(1+4)$

$5\int e^{2x}\sin(4x)\,dx = \frac{1}{2}e^{2x}\sin(4x) - e^{2x}\cos(4x)$

$\int e^{2x}\sin(4x)\,dx = \boxed{\frac{1}{5}\left[\frac{1}{2}e^{2x}\sin(4x) - e^{2x}\cos(4x)\right] + C}$