

AP Calculus BC – Unit 7 AP Review Packet

7.1: U-substitution and algebra techniques

2012 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (No calculator)

4. The function f is defined by $f(x) = \sqrt{25 - x^2}$ for $-5 \leq x \leq 5$.

(a) Find $f'(x)$.

(b) Write an equation for the line tangent to the graph of f at $x = -3$.

(c) Let g be the function defined by $g(x) = \begin{cases} f(x) & \text{for } -5 \leq x \leq -3 \\ x + 7 & \text{for } -3 < x \leq 5. \end{cases}$

Is g continuous at $x = -3$? Use the definition of continuity to explain your answer.

(d) Find the value of $\int_0^5 x\sqrt{25 - x^2} dx$.

2011 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS (Form B) (No calculator)

4. Consider a differentiable function f having domain all positive real numbers, and for which it is known that $f'(x) = (4 - x)x^{-3}$ for $x > 0$.

(a) Find the x -coordinate of the critical point of f . Determine whether the point is a relative maximum, a relative minimum, or neither for the function f . Justify your answer.

(b) Find all intervals on which the graph of f is concave down. Justify your answer.

(c) Given that $f(1) = 2$, determine the function f .

1999 CALCULUS AB (Calculator allowed)

1. A particle moves along the y -axis with velocity given by $v(t) = t \sin(t^2)$ for $t \geq 0$.

(a) In which direction (up or down) is the particle moving at time $t = 1.5$? Why?

(b) Find the acceleration of the particle at time $t = 1.5$. Is the velocity of the particle increasing at $t = 1.5$? Why or why not?

(c) Given that $y(t)$ is the position of the particle at time t and that $y(0) = 3$, find $y(2)$.

(d) Find the total distance traveled by the particle from $t = 0$ to $t = 2$.

7.1 MCQ #1 (no calculator):

If $\int_0^k \frac{x}{x^2 + 4} dx = \frac{1}{2} \ln 4$, where $k > 0$, then $k =$

- (A) 0
- (B) $\sqrt{2}$
- (C) 2
- (D) $\sqrt{12}$
- (E) $\frac{1}{2} \tan(\ln \sqrt{2})$

7.1 MCQ #2 (no calculator):

If the substitution $\sqrt{x} = \sin y$ is made in the integrand of $\int_0^{1/2} \frac{\sqrt{x}}{\sqrt{1-x}} dx$, the resulting integral is

- (A) $\int_0^{1/2} \sin^2 y dy$
- (B) $2 \int_0^{1/2} \frac{\sin^2 y}{\cos y} dy$
- (C) $2 \int_0^{\pi/4} \sin^2 y dy$
- (D) $\int_0^{\pi/4} \sin^2 y dy$
- (E) $2 \int_0^{\pi/6} \sin^2 y dy$

7.1 MCQ #3 (no calculator):

$$\int \frac{4}{x^2 + 4x + 8} dx =$$

- (A) $4 \ln|x^2 + 4x + 8| + C$
- (B) $\tan^{-1}\left(\frac{x+2}{2}\right) + C$
- (C) $4 \tan^{-1}(x + 2) + C$
- (D) $2 \tan^{-1}\left(\frac{x+2}{2}\right) + C$

7.1 MCQ #4 (no calculator):

$$\int \frac{6x^2 - 4x - 25}{x - 2} dx =$$

- (A) $3x^2 + 8x - 9 \ln|x - 2| + C$
- (B) $3x^2 + 8x + \frac{9}{(x-2)^2} + C$
- (C) $(2x^3 - 2x^2 - 25x) \ln|x - 2| + C$
- (D) $\frac{2x^3 - 2x^2 - 25x}{\frac{x^2}{2} - 2x} + C$

7.2: Integration by Parts

2007 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS (No calculator)

4. Let f be the function defined for $x > 0$, with $f(e) = 2$ and f' , the first derivative of f , given by

$$f'(x) = x^2 \ln x.$$

- Write an equation for the line tangent to the graph of f at the point $(e, 2)$.
- Is the graph of f concave up or concave down on the interval $1 < x < 3$? Give a reason for your answer.
- Use antidifferentiation to find $f(x)$.

7.2 MCQ #1 (no calculator):

$$\int_1^e x^4 \ln x \, dx =$$

(A) $\frac{6e^5 - 1}{25}$

(B) $\frac{4e^5 + 1}{25}$

(C) $\frac{1 - e^3}{3}$

(D) e^4

7.2 MCQ #2 (no calculator):

$$\int x f(x) \, dx =$$

(A) $x f(x) - \int x f'(x) \, dx$

(B) $\frac{x^2}{2} f(x) - \int \frac{x^2}{2} f'(x) \, dx$

(C) $x f(x) - \frac{x^2}{2} f(x) + C$

(D) $x f(x) - \int f'(x) \, dx$

(E) $\frac{x^2}{2} \int f(x) \, dx$

7.3: Trigonometric Integrals

7.3 MCQ #1 (no calculator):

Using the substitution $u = \sin(2x)$, $\int_{\pi/6}^{\pi/2} \sin^5(2x) \cos(2x) dx$ is equivalent to

(A) $-2 \int_{1/2}^1 u^5 du$

(B) $\frac{1}{2} \int_{1/2}^1 u^5 du$

(C) $\frac{1}{2} \int_0^{\sqrt{3}/2} u^5 du$

(D) $\frac{1}{2} \int_{\sqrt{3}/2}^0 u^5 du$

(E) $2 \int_{\sqrt{3}/2}^0 u^5 du$

7.4: Trig Substitution

7.4 MCQ #1 (no calculator):

$$\int \frac{1}{\sqrt{9-x^2}} dx =$$

(A) $\ln(\sqrt{9-x^2}) + C$

(B) $\frac{1}{3}\sin^{-1}\left(\frac{x}{3}\right) + C$

(C) $3\sin^{-1}\left(\frac{x}{3}\right) + C$

(D) $\sin^{-1}\left(\frac{x}{3}\right) + C$

7.5: Partial Fraction Expansion

2015 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS (No calculator)

5. Consider the function $f(x) = \frac{1}{x^2 - kx}$, where k is a nonzero constant. The derivative of f is given by

$$f'(x) = \frac{k - 2x}{(x^2 - kx)^2}.$$

- (a) Let $k = 3$, so that $f(x) = \frac{1}{x^2 - 3x}$. Write an equation for the line tangent to the graph of f at the point whose x -coordinate is 4.
- (b) Let $k = 4$, so that $f(x) = \frac{1}{x^2 - 4x}$. Determine whether f has a relative minimum, a relative maximum, or neither at $x = 2$. Justify your answer.
- (c) Find the value of k for which f has a critical point at $x = -5$.
- (d) Let $k = 6$, so that $f(x) = \frac{1}{x^2 - 6x}$. Find the partial fraction decomposition for the function f .
Find $\int f(x) dx$.

7.5 MCQ #1 (no calculator):

$$\int \frac{4}{x^2 - 1} dx =$$

- (A) $4 \ln|x^2 - 1| + C$
- (B) $2 \ln\left|\frac{x-1}{x+1}\right| + C$
- (C) $4\sin^{-1}x + C$
- (D) $4\tan^{-1}x + C$

7.8: Improper Integrals

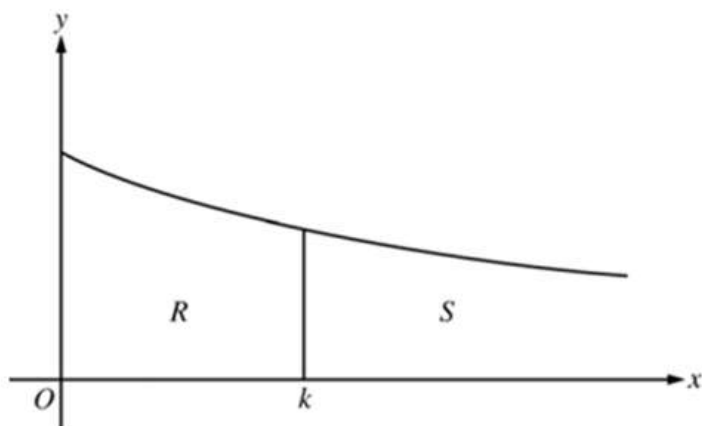
2001 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS (No calculator)

5. Let f be the function satisfying $f'(x) = -3xf(x)$, for all real numbers x , with $f(1) = 4$ and $\lim_{x \rightarrow \infty} f(x) = 0$.
- (a) Evaluate $\int_1^{\infty} -3xf(x)dx$. Show the work that leads to your answer.
- (b) Use Euler's method, starting at $x = 1$ with a step size of 0.5, to approximate $f(2)$.
- (c) Write an expression for $y = f(x)$ by solving the differential equation $\frac{dy}{dx} = -3xy$ with the initial condition $f(1) = 4$.

2017 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS (No calculator)

5. Let f be the function defined by $f(x) = \frac{3}{2x^2 - 7x + 5}$.
- (a) Find the slope of the line tangent to the graph of f at $x = 3$.
- (b) Find the x -coordinate of each critical point of f in the interval $1 < x < 2.5$. Classify each critical point as the location of a relative minimum, a relative maximum, or neither. Justify your answers.
- (c) Using the identity that $\frac{3}{2x^2 - 7x + 5} = \frac{2}{2x - 5} - \frac{1}{x - 1}$, evaluate $\int_5^{\infty} f(x) dx$ or show that the integral diverges.

2005 AP[®] CALCULUS BC FREE-RESPONSE QUESTIONS (Form B) (No calculator)



6. Consider the graph of the function f given by $f(x) = \frac{1}{x+2}$ for $x \geq 0$, as shown in the figure above. Let R be the region bounded by the graph of f , the x - and y -axes, and the vertical line $x = k$, where $k \geq 0$.
- (a) Find the area of R in terms of k .
- (b) Find the volume of the solid generated when R is revolved about the x -axis in terms of k .
- (c) Let S be the unbounded region in the first quadrant to the right of the vertical line $x = k$ and below the graph of f , as shown in the figure above. Find all values of k such that the volume of the solid generated when S is revolved about the x -axis is equal to the volume of the solid found in part (b).

7.8 MCQ #1 (no calculator):

If g is a twice-differentiable function, where $g(1) = 0.5$ and $\lim_{x \rightarrow \infty} g(x) = 4$, then $\int_1^{\infty} g'(x) dx$ is

- (A) -3.5
- (B) 3.5
- (C) 4.5
- (D) nonexistent

ANSWERS:

For FRQ answers, Google 'AP Calculus BC free response' or 'AP Calculus AP free response' and see the published solutions and scoring rubrics on the College Board past exams pages (or see our posted full solutions).

For MCQs...

7.1 MCQ #1: D

7.1 MCQ #2: C

7.1 MCQ #3: D

7.1 MCQ #4: A

7.2 MCQ #1: B

7.2 MCQ #2: B

7.3 MCQ #1: D

7.4 MCQ #1: D

7.5 MCQ #1: B

7.8 MCQ #1: B