AP Calc BC Unit 7 AP Packet Solutions

7.1 2012 AB FRQ4 (No Calculator)

(a) $f(x)=\sqrt{25-x^2}=(25-x^2)^{1/2}$

$f'(x)=\frac{1}{2}(25-x^2)^{-1/2}(-2x)=\boxed{\frac{-x}{\sqrt{25-x^2}} \quad (-5\le x\le 5)}$

(b) $f(-3)=\sqrt{25-(-3)^2}=4$, $f'(-3)=\frac{-(-3)}{\sqrt{25-(-3)^2}}=\frac{3}{4}$ $\boxed{(y-4)=\frac{3}{4}(x+3)}$

(c) $g(x)=\begin{cases} f(x) & -5\le x\le -3 \\ x+7 & -3<x\le 5\end{cases}$

1) $g(-3)$ exist? $g(-3)=f(-3)=4$ yes ✓

2) $\lim\limits_{x\to -3} g(x)$ exist?

| LH | | RH |
|---|---|---|
| $\lim\limits_{x\to -3^-} g(x)$ | | $\lim\limits_{x\to -3^+} g(x)$ |
| $\lim\limits_{x\to -3^-}\sqrt{25-x^2}$ | | $\lim\limits_{x\to -3^+}(x+7)$ |
| $\sqrt{25-(-3)^2}$ | | $(-3)+7$ |
| 4 | = | 4 |

yes, $\lim\limits_{x\to -3} g(x)$ exists ✓

3) $g(-3)\stackrel{?}{=}\lim\limits_{x\to -3} g(x)$

$4=4$ yes ✓

$\therefore$ $\boxed{g(x) \text{ is continuous at } x=-3}$

(d) $\int_0^5 x\sqrt{25-x^2}\,dx$ u-sub: $u=25-x^2$

$\frac{du}{dx}=-2x$

$du=-2x\,dx$

$x\,dx=-\frac{1}{2}du$

| $x$ | $\to$ | $u$ |
|---|---|---|
| 0 | $\to$ | $25-0^2=25$ |
| 5 | $\to$ | $25-(5)^2=0$ |

$-\frac{1}{2}\int_{25}^{0}\sqrt{u}\,du$

$\frac{1}{2}\int_0^{25} u^{1/2}\,du$

$\frac{1}{2}\cdot\frac{2}{3}u^{3/2}$

$\left[\frac{1}{3}u^{3/2}\right]_0^{25}$

$\frac{1}{3}(\sqrt{25})^3-\frac{1}{3}(\sqrt{0})^3$

$\boxed{\frac{125}{3}}$

7.1 2011B AB Q4 (No calculator)

(a) $f'(x) = (4-x)x^{-3} \quad (x>0)$

Critical points occur when $f'(x)=0$ or DNE.

$f'(x) = \frac{4-x}{x^3}$ DNE at $x=0$ but $x=0$ is not in the domain $(x>0)$

$f'(x)=0$ when $4-x=0$, at $\boxed{x=4}$

$f'(x)$ sign: 0 (3) 4 (5) $\infty$

$f'(3)$: $\frac{(+)}{(+)}$ $+$ $\qquad f'(5)$: $\frac{(-)}{(+)}$ $-$

f goes from increasing to decreasing at $x=4$, so there is $\boxed{\text{relative maximum at } x=4}$

(b) $f''(x) = (4-x)(-3x^{-4}) + (x^{-3})(-1)$

$f''(x) = \frac{3x-12}{x^4} - \frac{1}{x^3} = \frac{3x-12-x}{x^4} = \frac{2x-12}{x^4} = \frac{2(x-6)}{x^4}$

inflection pts when $f''(x)=0$, at $\underline{x=6}$

0 (5) 6 (7) $\infty$

$f''(5)$: $\frac{(+)(-)}{(+)}$ $-$ $\qquad f''(7)$: $\frac{(+)(+)}{(+)}$ $+$

$\boxed{f \text{ is concave down on } (6,\infty)}$

(c) $f(1)=2$, find $f(x)$

$$\int_1^x f'(t)\,dt = f(x) - f(1)$$

$$\int_1^x (4-t)t^{-3}\,dt = f(x) - 2$$

$$\int_1^x (4t^{-3} - t^{-2})\,dt = f(x) - 2$$

$$\left[\frac{4t^{-2}}{-2} - \frac{t^{-1}}{-1}\right]_1^x = f(x) - 2$$

$$\left[-\frac{2}{t^2} + \frac{1}{t}\right]_1^x = f(x) - 2$$

$$\left[-\frac{2}{x^2} + \frac{1}{x}\right] - \left[\frac{-2}{(1)^2} + \frac{1}{(1)}\right] = f(x) - 2$$

$$-\frac{2}{x^2} + \frac{1}{x} + 2 - 1 = f(x) - 2$$

$$\boxed{f(x) = -\frac{2}{x^2} + \frac{1}{x} + 3}$$

7.1 1999 AB Q1 (calculator allowed)

(a) $v(t) = t\sin(t^2) \quad (t \geq 0)$

$v(1.5) = (1.5)\sin((1.5)^2) = 1.1671 > 0$ (calculator) $\boxed{\text{so the particle is moving in the positive } y \text{ direction, up}}$

(b) $a(t) = v'(t) = t(\cos(t^2)2t) + \sin(t^2)(1)$

$= 2t^2\cos(t^2) + \sin(t^2)$

$a(1.5) = 2(1.5)^2\cos((1.5)^2) + \sin((1.5)^2) = -2.0487$ (calculator)

The velocity of the particle is $\boxed{\text{decreasing}}$ because $a(1.5) < 0$.

(c) $y(0) = 3$ find $y(2)$ $\quad \int_0^2 v(t)\,dt = y(2) - y(0)$

$\int_0^2 t\sin(t^2)\,dt = y(2) - 3$

u sub: $u = t^2$

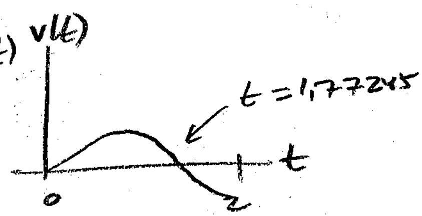
$\frac{du}{dt} = 2t, \quad du = 2t\,dt, \quad t\,dt = \frac{1}{2}du \qquad t = 0 \to u = 0, \quad t = 2 \to u = 4$

$\frac{1}{2}\int_0^4 \sin u\,dt = y(2) - 3$

$\left[-\frac{1}{2}\cos u\right]_0^4 = y(2) - 3$

$\left[-\frac{1}{2}\cos(4)\right] - \left[-\frac{1}{2}\cos(0)\right] = y(2) - 3 \quad \boxed{y(2) = -\frac{1}{2}\cos(4) + \frac{1}{2} + 3 \approx 3.827}$

(d) graph $v(t)$




$t\sin(t^2) = 0$

$\sin(t^2) = 0$

$t^2 = \arcsin(0) = \pi$

$(t = \sqrt{\pi} = 1.77245)$

total distance traveled $= \int_0^2 |v(t)|\,dt$

$= \int_0^{1.77245} t\sin(t^2)\,dt - \int_{1.77245}^{2} t\sin(t^2)\,dt$

using calculator integration function: $= 1 - (-.173178) = \boxed{1.173}$

7.1 MCQ #1 (no calculator)

$$\int_0^K \frac{x}{x^2+4}\,dx = \frac{1}{2}\ln 4$$

u sub $u = x^2+4$ $\quad x=0 \to u=4$

$du = 2x\,dx$ $\quad x=k \to u=k^2+4$

$x\,dx = \frac{1}{2}du$

$$\frac{1}{2}\int_4^{k^2+4} \frac{1}{u}\,du = \frac{1}{2}\ln 4$$

$$\frac{1}{2}\left[\ln|u|\right]_4^{k^2+4} = \frac{1}{2}\ln 4$$

$$\frac{1}{2}\left[\ln|k^2+4| - \ln|4|\right] = \frac{1}{2}\ln 4$$

$$\frac{1}{2}\ln\left|\frac{k^2+4}{4}\right| = \frac{1}{2}\ln 4$$

$$\frac{k^2+4}{4} = 4$$

$$k^2+4 = 16$$

$$k^2 = 12$$

$$\boxed{k = \sqrt{12}}$$

$\boxed{D}$

7.1 MCQ #2 (no calculator)

$$\int_0^{1/2} \frac{\sqrt{x}}{\sqrt{1-x}}\,dx$$

$\sqrt{x} = \sin y$

$x = (\sin y)^2$

$dx = 2(\sin y)\cos y\,dy$

$x=0 \to \sqrt{0} = \sin y$

$\sin y = 0$

$y = 0$

$x = 1/2 \to \sqrt{1/2} = \sin y$

$\sin y = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$y = \pi/4$

$$\int_0^{\pi/4} \frac{\sin y}{\sqrt{1-(\sin y)^2}}\, 2(\sin y)\cos y\,dy$$

$$2\int_0^{\pi/4} \frac{\sin^2 y\cos y}{\sqrt{1-(\sin y)^2}}\,dy$$

$\sin^2 y + \cos^2 y = 1$

$\cos^2 y = 1 - \sin^2 y$

$\cos y = \sqrt{1-\sin^2 y}$

$$2\int_0^{\pi/4} \frac{\sin^2 y\cos y}{\cos y}\,dy$$

$$\boxed{2\int_0^{\pi/4} \sin^2 y\,dy}$$

$\boxed{C}$

7.1 MCQ #3: (no calculator)

$$\int \frac{4}{x^2+4x+8}\,dx$$

complete the square

towards) $\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+C$

$x^2+4x+\underline{4}+8\,\underline{-4}$

$(x+2)^2+4$

$\int \frac{4}{4+(x+2)^2}\,dx \qquad a=2 \quad u=x+2$
$du=dx$

$4\int \frac{1}{a^2+u^2}\,du$

$4\left(\frac{1}{2}\right)\arctan\left(\frac{x+2}{2}\right)+C$

$2\arctan\left(\frac{x+2}{2}\right)+C$

$$\boxed{2\tan^{-1}\left(\frac{x+2}{2}\right)+C}$$

$\boxed{D}$

7.1 MCQ #4: (no calculator)

$$\int \frac{6x^2-4x-25}{x-2}\,dx$$

polynomial (or synthetic) division:

$$\begin{array}{r} 6x+8 \\ x-2\,\overline{)\,6x^2-4x-25} \\ -(6x^2-12x) \\ \hline 8x-25 \\ -(8x-16) \\ \hline -9 \end{array}$$

$$\begin{array}{r|rrr} 2 & 6 & -4 & -25 \\ & & 12 & 16 \\ \hline & 6 & 8 & -9 \end{array}$$

$6x+8 \quad R(-9)$

$\frac{6x^2-4x-25}{x-2} = 6x+8+\frac{-9}{x-2}$

$\int\left(6x+8-\frac{9}{x-2}\right)dx$

$6\int x\,dx+\int 8\,dx-9\int\frac{1}{x-2}\,dx$

usub: $u=x-2$
$du=dx$

$6\int x\,dx+\int 8\,dx-9\int\frac{1}{u}\,du$

$3x^2+8x-9\ln|x-2|+C$

$\boxed{A}$

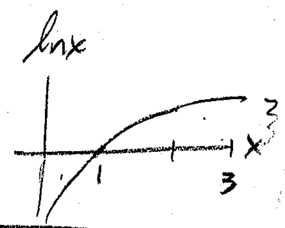
7.2 2007BC FRQ#4 (no calculator)

(a) $f'(x) = x^2 \ln x \,,\quad f(e) = 2$

$f'(e) = e^2 \ln(e) = e^2$

$\boxed{(y-2) = e^2(x-e)}$

(b) $f''(x) = x^2\left(\frac{1}{x}\right) + \ln x(2x) = x + 2x\ln x = \underbrace{x}_{>0}\underbrace{(1+2\ln x)}_{>0 \text{ for } 1<x<3}$




$f''(x) > 0$ over $1<x<3$

so $f$ is concave up over $1<x<3$

(c) $\int_e^x f'(t)\,dt = f(x) - f(e)$

$\int_e^x t^2 \ln t \, dt = f(x) - 2$

by parts: $u = \ln t \quad dv = t^2 dt$

$\frac{du}{dt} = \frac{1}{t} \quad \int dv = \int t^2 dt$

$du = \frac{1}{t} dt \quad v = \frac{1}{3}t^3$

$\left[uv - \int v\,du\right]_e^x = f(x) - 2$

$\left[(\ln t)\left(\frac{1}{3}t^3\right) - \int \frac{1}{3}t^3 \frac{1}{t} dt\right]_e^x = f(x) - 2$

$\left[\frac{1}{3}t^3 \ln t - \frac{1}{3}\int t^2 dt\right]_e^x$

$\left[\frac{1}{3}t^3 \ln t - \frac{1}{9}t^3\right]_e^x = f(x) - 2$

$\left[\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3\right] - \left[\frac{1}{3}e^3 \underbrace{\ln(e)}_{1} - \frac{1}{9}e^3\right] = f(x) - 2$

$\frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 - \frac{2}{9}e^3 = f(x) - 2$

$\boxed{f(x) = \frac{1}{3}x^3 \ln x - \frac{1}{9}x^3 - \frac{2}{9}e^3 + 2}$

7.2 MCQ #1 (no calculator)

$\int_1^e x^4 \ln x\,dx$

by parts $u=\ln x$ $dv=x^4dx$

$\frac{du}{dx}=\frac{1}{x}$ $\int dv=\int x^4dx$

$du=\frac{1}{x}dx$ $v=\frac{1}{5}x^5$

$\left[uv-\int v\,du\right]_1^e$

$\left[\frac{1}{5}x^5\ln x-\int \frac{1}{5}x^5\frac{1}{x}dx\right]_1^e$

$\left[\frac{1}{5}x^5\ln x-\frac{1}{5}\int x^4dx\right]_1^e$

$\left[\frac{1}{5}x^5\ln x-\frac{1}{25}x^5\right]_1^e$

$\left[\frac{1}{5}e^5\underbrace{\ln e}_{1}-\frac{1}{25}e^5\right]-\left[\frac{1}{5}(1)^5\underbrace{\ln 1}_{0}-\frac{1}{25}(1)^5\right]$

$\frac{1}{5}e^5-\frac{1}{25}e^5+\frac{1}{25}$

$\left(\frac{5}{25}-\frac{1}{25}\right)e^5+\frac{1}{25}$

$\boxed{\frac{4e^5+1}{25}}$

$\boxed{B}$

7.2 MCQ #2 (no calculator)

$\int x f(x)\,dx$

by parts $u=f(x)$ $dv=x\,dx$

$\frac{du}{dx}=f'(x)$ $\int dv=\int x\,dx$

$du=f'(x)dx$ $v=\frac{1}{2}x^2$

$uv-\int v\,du$

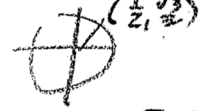
$f(x)\frac{1}{2}x^2-\int \frac{1}{2}x^2f'(x)dx$

$\frac{1}{2}x^2f(x)-\frac{1}{2}\int x^2f'(x)dx$

$\boxed{\frac{x^2f(x)}{2}-\int \frac{x^2}{2}f'(x)dx}$

$\boxed{B}$

## 7.3 MCQ #1: (no calculator)




$$\int_{\frac{\pi}{6}}^{\pi/2} \sin^5(2x)\cos(2x)\,dx \quad \text{using } u=\sin(2x)$$

$x = \pi/6 \to u = \sin(\pi/3) = \sqrt{3}/2$

$x = \pi/2 \to u = \sin(\pi) = 0$

$\frac{du}{dx} = 2\cos(2x)$

$du = 2\cos(2x)\,dx$

$\cos(2x)\,dx = \frac{1}{2}du$

$$\int_{\sqrt{3}/2}^{0} u^5\left(\tfrac{1}{2}du\right)$$

$$\boxed{\frac{1}{2}\int_{\sqrt{3}/2}^{0} u^5\,du}$$

D

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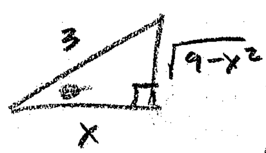
## 7.4 MCQ #1:

$$\int \frac{1}{\sqrt{9-x^2}}\,dx$$




$\cos\theta = \frac{x}{3}$ $\quad \sin\theta = \frac{\sqrt{9-x^2}}{3}$

$x = 3\cos\theta$ $\quad \sqrt{9-x^2} = 3\sin\theta$

$dx = -3\sin\theta\,d\theta$

$$\int \frac{1}{3\sin\theta}(-3)\sin\theta\,d\theta$$

$\cos\theta = \frac{x}{3}$

$\theta = \cos^{-1}\left(\frac{x}{3}\right)$

$$-\int 1\,d\theta$$

$-\theta$

$$\boxed{-\cos^{-1}\left(\frac{x}{3}\right)+c}$$

not a choice...

OKAY, –

attempt #2:

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \arcsin\left(\frac{u}{a}\right)+c$$

$a = 3$ $\quad u = x$

$du = dx$

$$\int \frac{1}{\sqrt{a^2-u^2}}\,du = \arcsin\left(\frac{x}{3}\right)+C$$

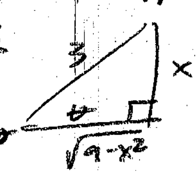
$$\boxed{\sin^{-1}\left(\frac{x}{3}\right)+C}$$

D

← why ~~didn't this work~~?

it would if we just swapped legs.

$\sin\theta = \frac{x}{3}$ $\quad \cos\theta = \frac{\sqrt{9-x^2}}{3}$




$x = 3\sin\theta$ $\quad \sqrt{9-x^2} = 3\cos\theta$

$dx = 3\cos\theta\,d\theta$

$$\int \frac{1}{3\cos\theta}3\cos\theta\,d\theta$$

$$\int 1\,d\theta \qquad \text{now, } \theta = \arcsin\left(\frac{x}{3}\right)$$

$$\boxed{\theta = \sin^{-1}\left(\frac{x}{3}\right)+C}$$

7.5 2015 BC FRQ #5 (no calculator)

(a) $k=3$ $\quad f(x)=\frac{1}{x^2-3x}$ $\quad f'(x)=\frac{3-2x}{(x^2-3x)^2}$ $\quad \boxed{\left(y-\frac{1}{4}\right)=\frac{-5}{16}(x-4)}$

$f(4)=\frac{1}{4^2-3(4)}=\frac{1}{4}$ $\quad f'(4)=\frac{3-2(4)}{(4^2-3(4))^2}=\frac{-5}{16}$

(b) $k=4$ $\quad f'(x)=\frac{4-2x}{(x^2-4x)^2}$, sign of $f'(x)$ $\quad -\infty$ ——— 2 ——— $\infty$

| (1) | 2 | (3) |
|---|---|---|
| $f'(1)$ | $f'(2)$ | $f'(3)$ |
| $\frac{(+)}{(+)}$ | $=0$ | $\frac{(-)}{(+)}$ |
| $+$ | | $-$ |

Because $f'(x)>0$ for $x<2$
& $f'(x)<0$ for $x>2$
$f$ has a relative maximum at $x=2$

(c) $f'(x)=0$ for DNE at critical pt, $f'(x)=\frac{k-2x}{(x^2-kx)^2}$, $f'(-5)=\frac{k-2(-5)}{((-5)^2-k(-5))^2}$

$f'(-5)=\frac{k+10}{(25+5k)^2}$ $\quad$ $f'(-5)=0$ when $k+10=0$ $\quad \boxed{k=-10}$ $\quad$ $f'(-5)$ DNE when $25+5k=0$ $\quad 5k=-25$ $\quad \boxed{k=-5}$

(AP solution only includes this one.)

(d) $k=6$ $\quad f(x)=\frac{1}{x^2-6x}=\frac{1}{x(x-6)}=\frac{A}{x}+\frac{B}{x-6}$

$\frac{1}{x(x-6)}=\frac{A(x-6)}{x(x-6)}+\frac{B(x)}{x(x-6)}$

$Ax-6A+Bx=1$

$(A+B)x+(-6A)=(0)x+(1)$

system: $\begin{cases} A+B=0 \\ -6A=1 \end{cases}$ $\quad A=-\frac{1}{6}$ $\quad B=\frac{1}{6}$

$\boxed{f(x)=\frac{1}{x(x-6)}=\frac{(-1/6)}{x}+\frac{(1/6)}{x-6}}$

$\int f(x)\,dx=-\frac{1}{6}\int\frac{1}{x}dx+\frac{1}{6}\int\frac{1}{x-6}dx$ $\quad$ u sub: $u=x-6$, $du=dx$

$=-\frac{1}{6}\int\frac{1}{x}dx+\frac{1}{6}\int\frac{1}{u}du$

$\boxed{-\frac{1}{6}\ln|x|+\frac{1}{6}\ln|x-6|+C}$

or

$\frac{1}{6}(\ln|x-6|-\ln|x|)+C$

$\boxed{\frac{1}{6}\ln\left|\frac{x-6}{x}\right|+C}$

7.5 MCQ #1

$$\int \frac{4}{x^2-1}\,dx \qquad \frac{4}{x^2-1} = \frac{4}{(x-1)(x+1)} = \frac{A}{x-1} + \frac{B}{x+1}$$

$$\frac{4}{(x-1)(x+1)} = \frac{A(x+1)}{(x-1)(x+1)} + \frac{B(x-1)}{(x-1)(x+1)}$$

$$Ax + A + Bx - B = 4$$

$$(A+B)x + (A-B) = (0)x + (4)$$

<u>System</u>: $\begin{cases} A+B=0 \rightarrow A=-B \\ A-B=4 \end{cases}$

$$(-B)-B=4$$

$$-2B=4$$

$$\underline{B=-2} \qquad \underline{A=2}$$

$$2\int \frac{1}{x-1}\,dx - 2\int \frac{1}{x+1}\,dx$$

$$2\ln|x-1| - 2\ln|x+1| + C$$

$$2(\ln|x-1| - \ln|x+1|) + C$$

$$\boxed{2\ln\left|\frac{x-1}{x+1}\right| + C}$$

$\boxed{B}$

7.8 2001 BC FRQ #5 (No calculator)

$f'(x) = -3xf(x)$, $f(1)=4$, $\lim_{x\to\infty} f(x) = 0$

(a) $\int_1^{\infty} -3xf(x)dx = \lim_{b\to\infty} \int_1^b -3xf(x)dx = \lim_{b\to\infty} \int_1^b f'(x)dx = \lim_{b\to\infty} f(b) - f(1)$

$= 0 - 4 = \boxed{-4}$

(b) Euler's method: $f(x+\Delta x) = f(x) + f'(x)\Delta x = f(x) + (-3)xf(x)\,0.5$

| $(x,y)$ | $f(x+0.5) = f(x) - 3xf(x)(0.5)$ |
|---|---|
| $(1,4)$ | $4 - 3(1)(4)(0.5) = -2$ |
| $(1.5,2)$ | $2 - 3(1.5)(2)(0.5) = -2.5$ |
| $(2,-2.5)$ | |

$\boxed{f(2) \approx -2.5}$

(c) (this is technically unit 8, but we can do it! :) )

$\frac{dy}{dx} = -3xy$ (separate variables to each side)

$\frac{1}{y}dy = -3x\,dx$ (take integral on each side)

$\int \frac{1}{y}dy = \int -3x\,dx$

$\ln y = -\frac{3}{2}x^2 + C_1$

$e^{\ln y} = e^{(-\frac{3}{2}x^2 + C_1)} = e^{-3/2x^2} \underbrace{e^{C_1}}_{\text{new } C} = Ce^{-3/2x^2}$

$y = Ce^{-3/2x^2}$ but $f(1)=4$ so $x=1, y=4$

$4 = Ce^{-3/2(1)^2}$

$\frac{4}{e^{-3/2}} = \frac{Ce^{-3/2}}{e^{-3/2}} \rightarrow C = \frac{4}{e^{-3/2}} = 4e^{3/2}$

and $\boxed{y = (4e^{3/2})e^{-3/2x^2}}$

7.8 2017 BC FRQ #5 (no calculator)

$$f(x)=\frac{3}{2x^2-7x+5}$$

(a) $f'(x)=\frac{(\cancel{2x^2-7x+5})(0)-(3)(4x-7)}{(2x^2-7x+5)^2}$ , $f'(3)=\frac{-3(4(3)-7)}{(2(3)^2-7(3)+5)^2}=\frac{-3(5)}{(18-21+5)^2}=\boxed{\frac{-15}{4}}$

(b) $f'(x)=\frac{-3(4x-7)}{(x-1)(2x-5)}$

$2x^2-7x+5$

$\frac{(2x-2)(2x-5)}{2 \quad\quad 1}$

$(x-1)(2x-5)$

| M | A |
|---|---|
| 10 | -7 |
| (1)(10) | |
| (-1)(-10) | |
| (2)(5) | |
| (-2)(-5) -7 ✓ | |

$\underline{f'(x)=0}$

$-3(4x-7)=0$

$x=\frac{7}{4}$

$\underline{f'(x)\text{ DNE}}$

$(x-1)(2x-5)=0$

$x=1,\ x=\frac{5}{2}$

$\underline{\cancel{2.5}} \quad \underline{2.5}$

on edges of interval

interval $1<x<2.5$

$1 \quad (\tfrac{3}{2}) \quad 7/4 \quad (2) \quad 2.5$

Sign of $f'(x)$: $f'(\tfrac{3}{2})$ $\frac{(-)(-)}{(+)}$ $+$ ; $f'(2)$ $\frac{(-)(+)}{(+)}$ $-$

critical points at: $\boxed{x=7/4}$

there is a relative maximum for $f$ at $x=7/4$ b/c $f'$ changes sign from positive to negative at $x=7/4$

(c) $\frac{3}{2x^2-7x+5}=\frac{2}{2x-5}-\frac{1}{x-1}$

$$\int_5^\infty f(x)\,dx=\lim_{b\to\infty}\int_5^b\left(\frac{2}{2x-5}-\frac{1}{x-1}\right)dx=\lim_{b\to\infty}2\int_5^b\frac{1}{2x-5}dx-\lim_{b\to\infty}\int_5^b\frac{1}{x-1}dx$$

$\underline{\text{u sub}}$: $u_1=2x-5$, $du_1=2dx$, $dx=\frac{1}{2}du_1$

$\underline{\text{u sub}}$: $u_2=x-1$, $du_2=dx$

$$\lim_{b\to\infty}\left[2\left(\tfrac{1}{2}\right)\int_5^b\frac{1}{u_1}du_1-\int_5^b\frac{1}{u_2}du_2\right]$$

$$\lim_{b\to\infty}\left[\ln|2x-5|-\ln|x-1|\right]_5^b$$

$$\lim_{b\to\infty}\left[\ln\left|\frac{2x-5}{x-1}\right|\right]_5^b$$

$$\lim_{b\to\infty}\ln\left|\frac{2b-5}{b-1}\right|-\ln\left|\frac{2(5)-5}{5-1}\right|$$

$\frac{\infty}{\infty}$ (L'Hopital's Rule)

$$\rightarrow \ln\left|\lim_{b\to\infty}\frac{2b-5}{b-1}\right|-\ln\left|\frac{5}{4}\right|$$

$$=\ln\left|\lim_{b\to\infty}\frac{2}{1}\right|-\ln\left|\frac{5}{4}\right|=\ln|2|-\ln\left|\frac{5}{4}\right|=\ln\left|\frac{2}{(5/4)}\right|$$

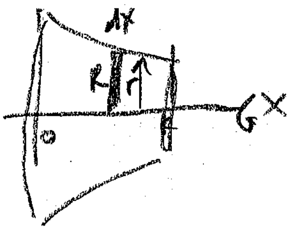
$$\boxed{=\ln\left(\frac{8}{5}\right)}$$

7.8 2005b BC FRQ #6 (no calculator)

(a) $A = \int H\,dw = \int_0^K \frac{1}{x+2}\,dx$ u sub: $u = x+2$, $du = dx$; $x=0 \to u=2$, $x=K \to u=K+2$

$= \int_2^{K+2} \frac{1}{u}\,du = \left[\ln|u|\right]_2^{K+2} = \ln|K+2| - \ln|2| = \boxed{\ln\left|\frac{K+2}{2}\right|}$

(b)




(disc method)

$V = \int_0^K \pi r^2\,dx = \int_0^K \pi\left(\frac{1}{x+2}\right)^2 dx$ u sub: $u = x+2$, $du = dx$; $x=0 \to u=2$, $x=K \to u=K+2$

$\pi \int_2^{K+2} u^{-2}\,du = \left[\pi \frac{u^{-1}}{-1}\right]_2^{K+2} = \left[\frac{-\pi}{u}\right]_2^{K+2}$

$= \frac{-\pi}{K+2} - \left(\frac{-\pi}{2}\right) = \boxed{\frac{-\pi}{K+2} + \frac{\pi}{2}}$

(c) $V_S = \int_K^\infty \pi\left(\frac{1}{x+2}\right)^2 dx = \lim_{b\to\infty} \int_K^b \pi\left(\frac{1}{x+2}\right)^2 dx$ usub: $u = x+2$, $du = dx$; $x=K \to u=K+2$, $x=b \to u=b+2$

$= \lim_{b\to\infty} \pi \int_{K+2}^{b+2} u^{-2}\,du =$

$= \lim_{b\to\infty} \left[\frac{-\pi}{u}\right]_{K+2}^{b+2}$

$= \lim_{b\to\infty} \left[\frac{-\pi}{b+2} - \left(\frac{-\pi}{K+2}\right)\right] = 0 + \frac{\pi}{K+2} = \frac{\pi}{K+2}$

$V_K = V_S$

$\frac{-\pi}{K+2} + \frac{\pi}{2} = \frac{\pi}{K+2}$

$\frac{2\pi}{K+2} = \frac{\pi}{2}$

$\pi(K+2) = 4\pi$

$K+2 = 4$

$\boxed{K=2}$

7.8 MCQ #1 (no calculator)

$g(1) = 0.5, \quad \lim_{x \to \infty} g(x) = 4$

$$\int_1^\infty g'(x)\,dx = \lim_{b \to \infty} \int_1^b g'(x)\,dx = \lim_{b \to \infty} g(b) - g(1)$$

$$= 4 - 0.5$$

$$= \frac{8}{2} - \frac{1}{2}$$

$$= \boxed{\frac{7}{2}} = \boxed{3.5}$$

$\boxed{B}$