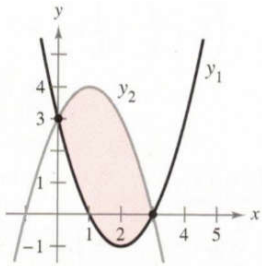


6.1 Worksheet (Odds, 6, 26)

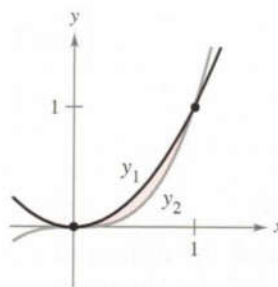
NOTE: For all problems, set up the integral. For selected problems, the instructions say to 'evaluate by hand'. For all other problems, do not evaluate by hand – instead, use MATH 9 to check answers.

Find the definite integral that gives the area of the shaded region.

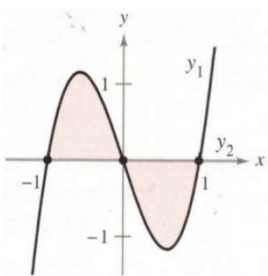
1. $y_1 = x^2 - 4x + 3$
 $y_2 = -x^2 + 2x + 3$



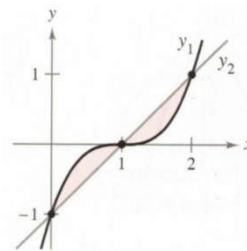
2. $y_1 = x^2$
 $y_2 = x^3$



3. $y_1 = 3(x^3 - x)$
 $y_2 = 0$



4. $y_1 = (x - 1)^3$
 $y_2 = x - 1$

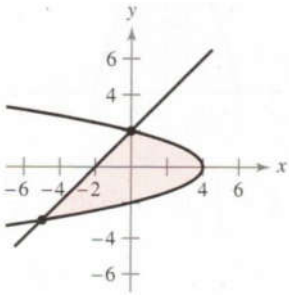


#5 and 6 (evaluate these integrals by hand)

Find the area of the region by integrating (a) with respect to x , (b) with respect to y , and then (c) compare your results. Which method is simpler? In general, will this method always be simpler than the other one? Why or why not?

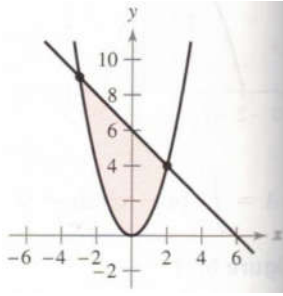
5. $x = 4 - y^2$

$$x = y - 2$$



6. $y = x^2$

$y = 6 - x$



Sketch the region bounded by the graphs of the functions and set up the integral that finds the area of the bounded region. Then use your calculator to find the area of the region.

7. $y = x^2 - 1$, $y = -x + 2$, $x = 0$, $x = 1$

8. $y = -x^3 + 2$, $y = x - 3$, $x = -1$, $x = 1$

9. $y = x$, $y = 2 - x$, $y = 0$

10. $y = \frac{4}{x^3}$, $y = 0$, $x = 1$, $x = 4$

11. (evaluate this one by hand) $f(x) = \sqrt{x} + 3$, $g(x) = \frac{1}{2}x + 3$

12. (evaluate this one by hand) $f(x) = \sqrt[3]{x-1}$, $g(x) = x - 1$

13. $f(y) = y^2, \quad g(y) = y + 2$

14. $f(y) = y(2 - y), \quad g(y) = -y$

15. $f(y) = y^2 + 1, \quad g(y) = 0, \quad y = -1, \quad y = 2$

16. $f(y) = \frac{y}{\sqrt{16-y^2}}, \quad g(y) = 0, \quad y = 3$

17. (evaluate this one by hand) $f(x) = \frac{10}{x}$, $x = 0$, $y = 2$, $y = 10$

18. (evaluate this one by hand) $g(x) = \frac{4}{2-x}$, $y = 4$, $x = 0$

Sketch the region bounded by the graphs of the functions and set up the integral that finds the area of the bounded region. Then use your calculator to find the area of the region.

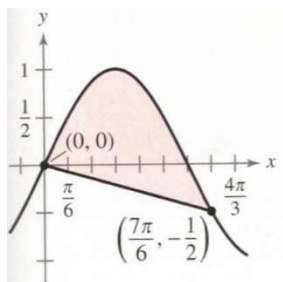
19. $f(x) = \cos(x)$, $g(x) = 2 - \cos(x)$, $0 \leq x \leq 2\pi$

20. $f(x) = \sin(x)$, $g(x) = \cos(2x)$, $-\frac{\pi}{2} \leq x \leq \frac{\pi}{6}$

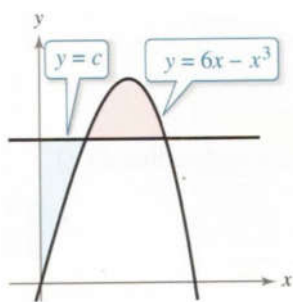
21. $f(x) = 2 \sin(x)$, $g(x) = \tan(x)$, $-\frac{\pi}{3} \leq x \leq \frac{\pi}{3}$

22. $f(x) = \sec\left(\frac{\pi x}{4}\right) \tan\left(\frac{\pi x}{4}\right)$, $g(x) = (\sqrt{2} - 4)x + 4$, $x = 0$

23. Find the area between the graph of $y = \sin(x)$ and the line segment joining the points $(0,0)$ and $(\frac{7\pi}{6}, -\frac{1}{2})$, as shown in the figure. Set up the integral, then use your calculator to find the area.



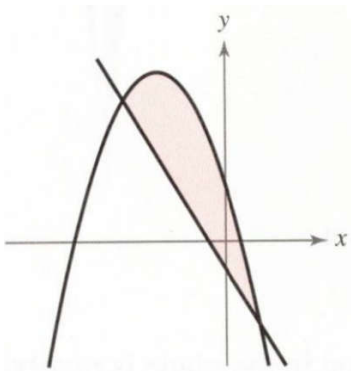
24. (Challenge Problem) The horizontal line $y = c$ intersects the graph of $y = 6x - x^3$ in the first quadrant, as shown in the figure. Find the integer c so that the areas of the two shaded regions are equal.



25. **Multiple Choice:**

The figure below shows the graphs of $y = 2 - 4x - x^2$ and $y = -2x - 1$. What is the area of the shaded region?

- a) $\frac{5}{3}$ b) 10 c) $\frac{32}{3}$ d) $\frac{86}{3}$



26. **Multiple Choice:**

What is the area of the region bounded by the graphs of $x = y^2 - 2y$ and $y = -x + 2$?

- a) $\frac{3}{2}$ b) $\frac{10}{3}$ c) $\frac{9}{2}$ d) $\frac{31}{6}$

27. **Multiple Choice:**

Which integral gives the area A of the region bounded by the graph of $f(x) = x^3 - 2x$ and the tangent line to the graph of f at $(-1, 1)$?

- a) $A = \int_{-1}^2 (x^3 + 3x + 2) dx$
b) $A = \int_{-1}^2 (x^3 - 3x - 2) dx$
c) $A = \int_{-1}^4 (-x^3 + 3x + 2) dx$
d) $A = \int_{-1}^2 (-x^3 + 3x + 2) dx$

6.2 Worksheet (1 – 12, 13 – 27 odds, 28-36)

NOTE: For all problems, set up the integral but do not evaluate by hand – instead, use MATH 9 to check answers.

Sketch the graph of the system. Set up the integral that finds the volume of the figure created by rotating the graph around **the x – axis.** (EASY)

1. $y = 3x + 5, x = 2, x = 7, y = 0$

2. $y = -x^2 + 3x, y = 0$

3. $y = \sin(x), x = \frac{\pi}{6}, x = \frac{5\pi}{6}, y = 0$

Sketch a graph of the system. Set up the integral that finds the volume of the figure created by rotating the graph around **the y – axis.** (EASY)

4. $y = 3x + 5, y = 5, y = 14, x = 0$

5. $y = x^2 + 4, x = 0, y = 8$

Sketch the graph of the system. Set up the integral that finds the volume of the figure created by the graph rotating around the indicated axis. **(MEDIUM/Test Level)**

6. $y = x^2, y = \sqrt{x}$; *AROUND THE X – AXIS.*

7. $y = 2x + 3, x = 0, y = 9$; *AROUND $y = 9$.*

8. $y = x^2, y = \sqrt{x}$; *AROUND $y = 3$.*

9. $y = x^2 - 4x + 9, y = 2x + 1$, *AROUND $x = 1$.*

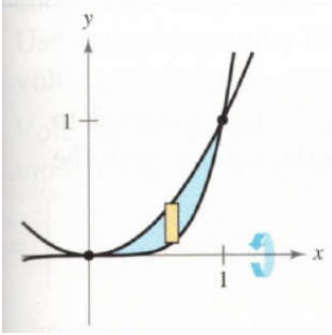
10. $y = -2x + 8, y = 0, x = 0$; *AROUND* $x = 5$.

11. $y = -2x + 8, y = 0, x = 0$; *AROUND* $y = 9$.

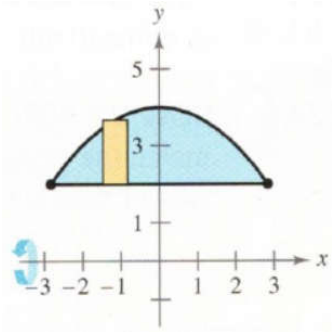
12. $y = 2x^2, y = 8, x = 0$, *AROUND THE Y - AXIS*.

Set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the x-axis.

13. $y = x^2, y = x^5$

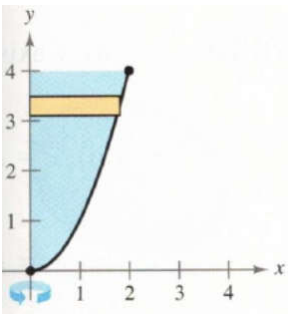


14. $y = 2, y = 4 - \frac{x^2}{4}$

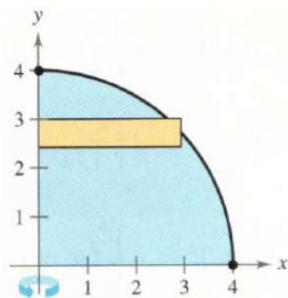


Set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the y-axis.

15. $y = x^2$



16. $y = \sqrt{16 - x^2}$



Find the volumes of the solids generated by revolving the region bounded by the graphs of the equations about the given lines.

17. $y = \sqrt{x}$, $y = 0$, $x = 3$ around... a) x-axis b) y-axis c) $x = 3$ d) $x = 6$

18. $y = 2x^2$, $y = 0$, $x = 2$ around... a) y-axis b) x-axis c) $y = 8$ d) $x = 2$

19. $y = x^2, y = 4x - x^2$ around... a) x-axis b) $y = 6$

20. $y = 4 + 2x - x^2, y = 4 - x$ around... a) x-axis b) $y = 1$

Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis.

21. $y = \frac{1}{x}$, $y = 0$, $x = 1$, $x = 3$

22. $y = \frac{2}{x+1}$, $y = 0$, $x = 0$, $x = 6$

23. $y = e^{-x}$, $y = 0$, $x = 0$, $x = 1$

24. $y = e^{x/4}$, $y = 0$, $x = 0$, $x = 6$

Use your calculator to approximate the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis.

25. $y = e^{-x^2}$, $y = 0$, $x = 0$, $x = 2$

26. $y = \ln(x)$, $y = 0$, $x = 1$, $x = 3$

Find the volume generated by rotating the given region about the specified line.

27. R_1 about $x = 0$

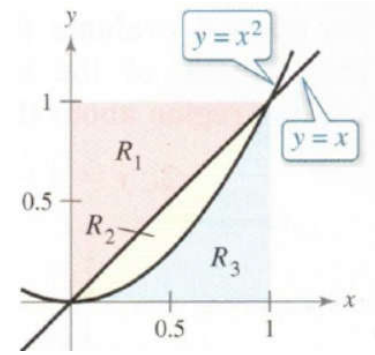
28. R_1 about $x = 1$

29. R_2 about $y = 0$

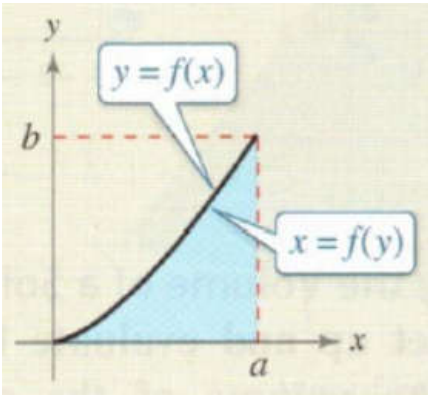
30. R_2 about $y = 1$

31. R_2 about $x = 0$

32. R_2 about $x = 1$



33. Use the graph to match the integral for the volume with the axis of rotation.



- (a) $V = \pi \int_0^b (a^2 - [f(y)]^2) dy$ (i) x-axis
 (b) $V = \pi \int_0^a (b^2 - [b - f(x)]^2) dx$ (ii) y-axis
 (c) $V = \pi \int_0^a [f(x)]^2 dx$ (iii) $x = a$
 (d) $V = \pi \int_0^b [a - f(y)]^2 dy$ (iv) $y = b$

34. Multiple Choice:

Let R be the region bounded by the graphs $y = 3x$ and $y = 4x^2$ for $0 \leq x \leq \frac{3}{4}$. What is the volume of the solid generated when R is revolved about the line $y = 4$?

- a) $\pi \int_0^{3/4} [(4 - 3x)^2 - (4 - 4x^2)^2] dx$ b) $\pi \int_0^{3/4} [(4 - 4x^2)^2 - (4 - 3x)^2] dx$
 c) $\pi \int_0^{3/4} (9x^2 - 16x^4) dx$ d) $\pi \int_0^{9/4} \left[\left(4 - \frac{\sqrt{y}}{2}\right)^2 - \left(4 - \frac{y}{3}\right)^2 \right] dy$

6.3 Worksheet (1 – 10, 11 – 25 odds, 26 – 29)

NOTE: For all problems, set up the integral but do not evaluate by hand – instead, use MATH 9 to check answers.

Sketch the graph of the system. Set up the integral that finds the volume of the figure created by rotating the graph around **the y – axis.** (EASY)

1. $y = 3x + 5, x = 0, x = 7, y = 0$

2. $y = x^2 + 3x, x = 0, x = 3, y = 0$

3. $y = \sin(x), x = 0, x = \frac{5\pi}{6}, y = 0$

Sketch a graph of the system. Set up the integral that finds the volume of the figure created by rotating the graph around **the x – axis.** (EASY)

4. $y = 3x, y = 5, x = 0$

5. $y = x^2 + 4, x = 0, y = 7$

Sketch the graph of the system. Set up the integral that finds the volume of the figure created by the graph rotating around the indicated axis. **(EASY/MEDIUM)**

6. $y = x^2, y = \sqrt{x};$ *AROUND THE Y – AXIS.*

7. $y = 2x + 3, x = 0, y = 9;$ *AROUND $y = 9.$*

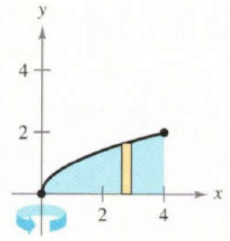
8. $y = x^2, y = \sqrt{x};$ *AROUND $y = 3.$*

9. $y = x^2 - 4x + 9, y = 2x + 1,$ *AROUND $x = 1.$*

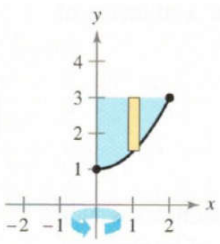
10. $y = -2x + 8, y = 0, x = 0;$ *AROUND $x = 5$*

Use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the y-axis.

11. $y = \sqrt{x}$



12. $y = \frac{1}{2}x^2 + 1$



13. $y = \frac{1}{4}x^2, y = 0, x = 4$

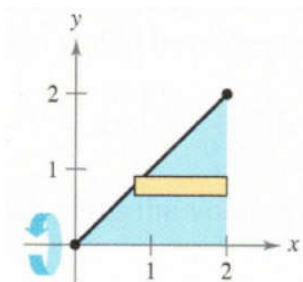
14. $y = \frac{1}{2}x^3, y = 0, x = 3$

15. $y = \sqrt{x-2}$, $y = 0$, $x = 4$

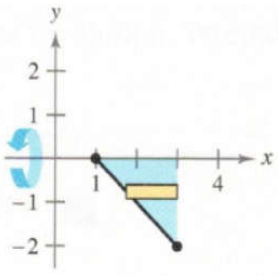
16. $y = -x^2 + 1$, $y = 0$

Use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the x-axis.

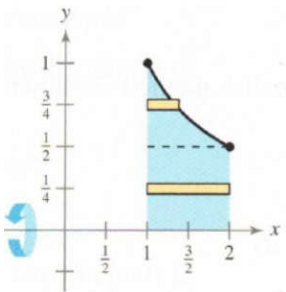
17. $y = x$



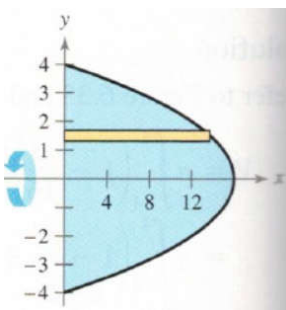
18. $y = 1 - x$



19. $y = \frac{1}{x}$



20. $x + y^2 = 16$



Use the disk method OR the shell method to find the volumes of the solids generated by revolving the region bounded by the graphs of the equations about the given lines.

21. $y = x^3$, $y = 0$, $x = 2$ around... a) x-axis b) y-axis c) $x = 4$

22. $y = \frac{10}{x^2}$, $y = 0$, $x = 1$, $x = 5$ around... a) x-axis b) y-axis c) $y = 10$

23. $x^{1/2} + y^{1/2} = a^{1/2}$, $x = 0$, $y = 0$ around... a) x-axis b) y-axis c) $x = a$

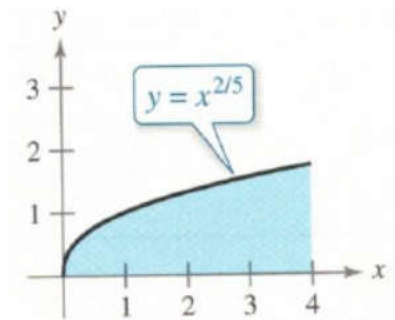
24. $x^{2/3} + y^{2/3} = a^{2/3}$, $a > 0$ (called a hypocycloid) around... a) x-axis b) y-axis

25. Let V_1 and V_2 be the volumes of the solids that result when the plane region bounded by $y = \frac{1}{x}$, $y = 0$, $x = \frac{1}{4}$, and $x = c$ (where $c > \frac{1}{4}$) is revolved about the x-axis and the y-axis, respectively. Find the value of c for which $V_1 = V_2$.

26. **Multiple Choice:**

The region shown in the figure is revolved about the x-axis, the y-axis, and the line $x = 4$. Which of the following orders the volumes of the resulting solids from least to greatest?

- a) x-axis, $x = 4$, y-axis
- b) x-axis, y-axis, $x = 4$
- c) $x = 4$, x-axis, y-axis
- d) $x = 4$, y-axis, x-axis



27. Multiple Choice:

Let A be the region bounded by the graphs of $y = 4\sqrt{x}$, $y = 4$, $x = 0$. What is the volume of the solid formed when R is revolved about the line $x = -\frac{3}{2}$?

- a) $\frac{4\pi}{5}$ b) $\frac{16\pi}{5}$ c) $\frac{24\pi}{5}$ d) $\frac{56\pi}{5}$

28. Multiple Choice:

Let R be the region enclosed by the graphs of $y = \frac{1}{8}(x - 4)^2$ and $y = 2$. What is the volume of the solid formed when R is revolved about the y-axis?

- a) $\frac{13\pi}{3}$ b) $\frac{19\pi}{3}$ c) $\frac{128\pi}{3}$ d) $\frac{256\pi}{3}$

29. Free Response:

Let T be the region bounded by the graphs of $y = \frac{4}{x+2}$ and $y = 2 - \frac{2}{5}x$.

- a. Find the area of the region.
- b. What is the volume of the solid formed when T is revolved about the y-axis?
- c. What is the volume of the solid formed when T is revolved about the x-axis?

6.4 Applications Worksheet

Arc Length Find the arc length of the curve over the given interval. Set up the integral, then use your calculator to find the length, accurate to 4 decimal places. (Easy)

Recall: $L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$ or $L = \int_a^b \sqrt{1 + [f'(y)]^2} dy$

1. $y = 3x^2; -3 \leq x \leq 3$

2. $y = -2x^3 + x - 5; -1 \leq x \leq 5$

3. $y = 4x^3 - 7x^2 + x - 27; -4 \leq x \leq 2$

4. $y = \ln(x) + 8; 3 \leq x \leq 7$

5. $y = \frac{3}{x^2} + x^2; 1 \leq x \leq 4$

6. $x = y^2 + 3y - 4; 0 \leq y \leq 2$

7. $3x + 5 = y^3; 1 \leq x \leq 3$

8. $x^2 - 4x + y^2 + 6y = 62; \text{ for } x \geq 2$

Surface Area Find the surface area of the solid found by rotating the curve over the indicated axis. Set up the integral, then use your calculator to find the area, accurate to 4 decimal places. (Medium)

Recall: $A_{SURFACE} = \int_a^b 2\pi r \sqrt{1 + [f'(x)]^2} dx$ or $A_{SURFACE} = \int_a^b 2\pi r \sqrt{1 + [f'(y)]^2} dy$

9. $y = -3x + 6, x = 0, y = 0$; rotated around x - axis.

10. $y = -3x + 6, x = 0, y = 0$; rotated around y - axis.

11. $y = x^2, x = 0, y = 9$; rotated around y - axis.

12. $y = x^2, x = 4, y = 0$; rotated around x - axis.

13. $2x = y^2 + 6, y = 0, y = 2$; rotated around y - axis.

14. $x^2 + y^2 = 16$, In Quadrant 1; rotated around x - axis.

15. $y = x^2 + 7x + 6, y = 0$; rotated around x - axis.

Volumes With Uniform Cross-sections Find the volume of the shape described in the problem. Set up the integral, and use your calculator to find the volume, accurate to 4 decimal places. (Hard)

Recall: $V = \int_a^b A_{CS} dx$ or $V = \int_a^b A_{CS} dy$

16. *Radius bounded by $y = 2x, y = 0, x = 4$; Cross sections are circles perpendicular to $x - axis$. (Assume that the base in the x - y plane is the diameter of the cross-sectional circle).*

17. *Base bounded by $y = 2x, y = 0, x = 4$; Cross sections are squares perpendicular to $x - axis$.*

18. *Diameters bounded by $y = 2x$, $x = 0$, $y = 4$; Cross sections are semicircles perpendicular to $y - axis$. (Assume base is the diameter of the semicircle).*

19. *Base bounded by $x^2 + y^2 = 4$; Cross sections are squares perpendicular to the $x - axis$.*

20. *Base bounded by $x^2 + y^2 = 4$; Cross sections are squares perpendicular to the $y - axis$.*

21. *Base bounded by $x^2 + y^2 = 4$; Cross sections are rectangles (with heights triple the base) perpendicular to the $x - axis$.*

Average Value Find the average value of the function over the interval. Set up the integral **and evaluate by hand**, leave in simplified form (no calculator). (Easy)

Recall:
$$\text{Ave. Value} = \frac{\int_a^b f(x) dx}{b-a}$$

22. $y = 5x + 3; [1,6]$

23. $y = x^2 + 3x - 2; [-1,5]$

24. $y = \frac{1}{x}; [2,10]$

25. $y = \sin(x) + \cos(x); \left[0, \frac{\pi}{4}\right]$

26. $y = 7x^3 + \sin(x); [0,1]$