AP Calculus BC

Name: answers/solutions

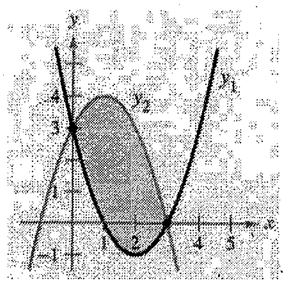
6.1 Worksheet (Odds, 6, 26)

NOTE: For all problems, set up the integral. For selected problems, the instructions say to 'evaluate by hand'. For all other problems, <u>do not evaluate by hand</u> – instead, use MATH 9 to check answers.

Find the definite integral that gives the area of the shaded region.

1. $y_1 = x^2 - 4x + 3$

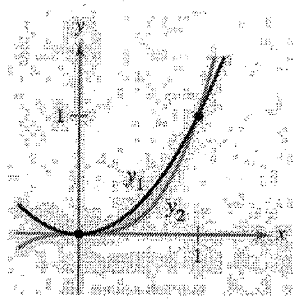
   $y_2 = -x^2 + 2x + 3$



$$\boxed{A = \int_0^3 [(-x^2+2x+3) - (x^2-4x+3)]\,dx}$$

2. $y_1 = x^2$

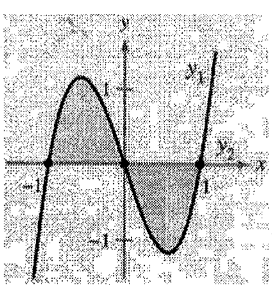
   $y_2 = x^3$



$$\boxed{A = \int_0^1 [x^2 - x^3]\,dx}$$

3. $y_1 = 3(x^3 - x)$

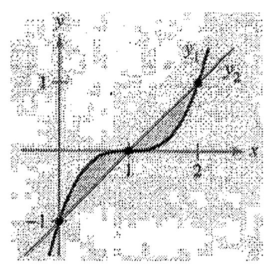
   $y_2 = 0$



$$\boxed{A = \int_{-1}^0 3(x^3-x)\,dx - \int_0^1 3(x^3-x)\,dx}$$

4. $y_1 = (x-1)^3$

   $y_2 = x - 1$



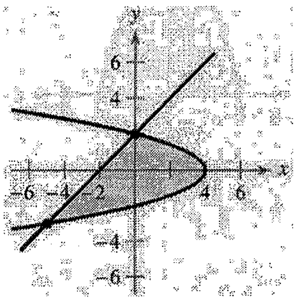
$$\boxed{A = \int_0^1 [(x-1)^3 - (x-1)]\,dx + \int_1^2 [(x-1) - (x-1)^3]\,dx}$$

#5 and 6 (evaluate these integrals by hand)

Find the area of the region by integrating (a) with respect to x, (b) with respect to y, and then (c) compare your results. Which method is simpler? In general, will this method always be simpler than the other one? Why or why not?

5. $x = 4 - y^2$

$x = y - 2$

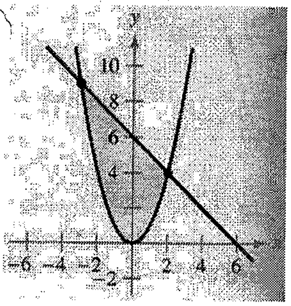


both (a) & (b) evaluate to $\frac{125}{6} u^2$

integrating with respect to y is better because there is a single 'top' and 'bottom' curve for the entire region, so only one integral is required.

6. $y = x^2$

$y = 6 - x$



intersections:

$$\begin{cases} y = x^2 \\ y = 6 - x \end{cases}$$

$x^2 = 6 - x$

$x^2 + x - 6 = 0$

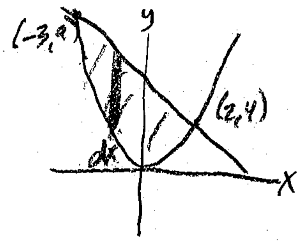
$(x+3)(x-2) = 0$

$x = -3 \quad x = 2$

$y = 9 \quad y = 4$

$(-3, 9) \quad (2, 4)$

(a) with respect to x:



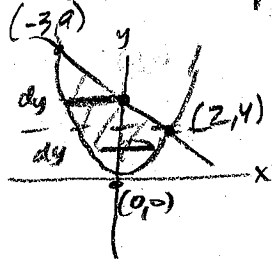
$$A = \int_{-3}^{2} [(6-x)-(x^2)]\,dx$$

$$= \left[6x - \tfrac{1}{2}x^2 - \tfrac{1}{3}x^3\right]_{-3}^{2}$$

$$= \left\{6(2) - \tfrac{1}{2}(2)^2 - \tfrac{1}{3}(2)^3\right\} - \left[6(-3) - \tfrac{1}{2}(-3)^2 - \tfrac{1}{3}(-3)^3\right]$$

$$= \boxed{\frac{125}{6}\ u^2}$$

(b) with respect to y:



$$A = \int_0^4 [\sqrt{y} - (-\sqrt{y})]\,dy + \int_4^9 [(6-y) - (-\sqrt{y})]\,dy$$

$$= 2\int_0^4 y^{1/2}\,dy + \int_4^9 (6 - y + y^{1/2})\,dy$$

$$= 2\left\{\tfrac{2}{3} y^{3/2}\right\}_0^4 + \left[6y - \tfrac{1}{2}y^2 + \tfrac{2}{3}y^{3/2}\right]_4^9$$

$$= \tfrac{4}{3}\left[(\sqrt{4})^3 - (\sqrt{0})^3\right] + \left[(6(9) - \tfrac{1}{2}(9)^2 + \tfrac{2}{3}(\sqrt{9})^3 - (6(4) - \tfrac{1}{2}(4)^2 + \tfrac{2}{3}(\sqrt{4})^3)\right]$$

$$= \boxed{\frac{125}{6}\ u^2}$$

(c) both are equal (same area), but integrating with respect to x is easier because there is a single 'top' and 'bottom' curve for the entire region, so only one integral is required.

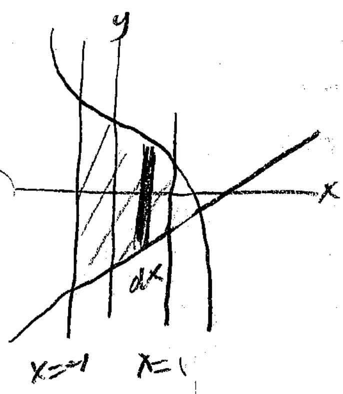
Sketch the region bounded by the graphs of the functions and set up the integral that finds the area of the bounded region. Then use your calculator to find the area of the region.

7. $y = x^2 - 1, \quad y = -x + 2, \quad x = 0, \quad x = 1$

$\boxed{\frac{13}{6} \approx 2.1667\ u^2}$

(plus sketch)

8. $y = -x^3 + 2, \quad y = x - 3, \quad x = -1, \quad x = 1$



$A = \int_{-1}^{1} [(-x^3+2) - (x-3)]dx$

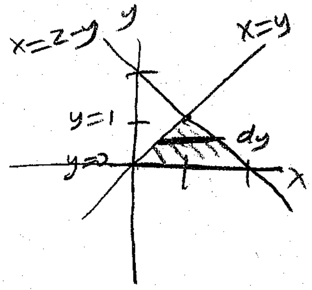
$= \int_{-1}^{1} (-x^3 - x + 5)dx = \boxed{10u^2}$ (math 9)

9. $y = x, \quad y = 2 - x, \quad y = 0$

$\boxed{1u^2}$

(plus sketch)

9. $y = x, \quad y = 2 - x, \quad y = 0$

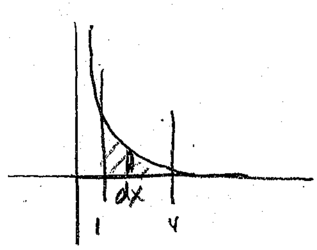


$$A = \int_0^1 [(2-y)-(y)]\,dy = \int_0^1 (-2y+2)\,dy$$

$$= \left[-y^2+2y\right]_0^1$$

$$= (-(1)^2+2(1)) - 0 = \boxed{1\,u^2}$$

10. $y = \frac{4}{x^3}, \quad y = 0, \quad x = 1, \quad x = 4$



$$A = \int_1^4 \left[\left(\frac{4}{x^3}\right) - 0\right] dx = 4\int_1^4 x^{-3}\,dx$$

$$= 4\left[\frac{x^{-2}}{-2}\right]_1^4$$

$$= -2\left[\frac{1}{(4)^2} - \frac{1}{(1)^2}\right] = \boxed{\frac{15}{8}\,u^2}$$

11. $f(x) = \sqrt{x} + 3, \quad g(x) = \frac{1}{2}x + 3$

Intersections:

$$\begin{cases} y = \sqrt{x}+3 \\ y = \frac{1}{2}x+3 \end{cases}$$

$\sqrt{x}+3 = \frac{1}{2}x+3$

$\sqrt{x} = \frac{1}{2}x$

$\sqrt{x} - \frac{1}{2}x = 0$

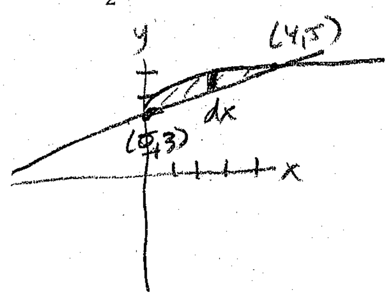
$\sqrt{x} - \frac{1}{2}\sqrt{x}\sqrt{x} = 0$

$\sqrt{x}\left(1 - \frac{1}{2}\sqrt{x}\right) = 0$

$x = 0 \qquad \frac{1}{2}\sqrt{x} = 1, \ \sqrt{x} = 2, \ x = 4$

$(0,3)$

$(4,5)$



$$A = \int_0^4 \left[(\sqrt{x}+3) - \left(\tfrac{1}{2}x+3\right)\right] dx$$

$$= \int_0^4 \left(x^{1/2} - \tfrac{1}{2}x\right) dx = \left[\tfrac{2}{3}x^{3/2} - \tfrac{1}{4}x^2\right]_0^4$$

$$= \left(\tfrac{2}{3}(\sqrt{4})^3 - \tfrac{1}{4}(4)^2\right) - 0$$

$$= \boxed{\frac{4}{3}\,u^2}$$

12. $f(x) = \sqrt[3]{x - 1}, \quad g(x) = x - 1$

Intersections:

$$\begin{cases} y = \sqrt[3]{x-1} \\ y = x-1 \end{cases}$$

$\sqrt[3]{x-1} = x-1$

$x-1 = (x-1)^3$

$(x-1) - (x-1)^3 = 0$

$(x-1)\left[1-(x-1)^2\right] = 0$

$x = 1 \qquad 1-(x-1)^2 = 0$

$(x-1)^2 = 1$

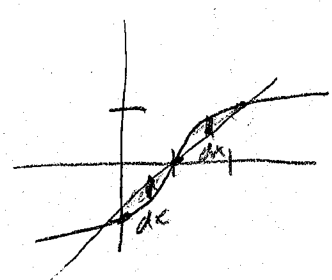
$x-1 = \pm 1$

$x = 1 \pm 1, \ x = 0, \ x = 2$

$(1,0)$

$(0,-1)$

$(2,1)$



$$A = \int_0^1 \left[(x-1) - (x-1)^{1/3}\right] dx + \int_1^2 \left[(x-1)^{1/3} - (x-1)\right] dx$$

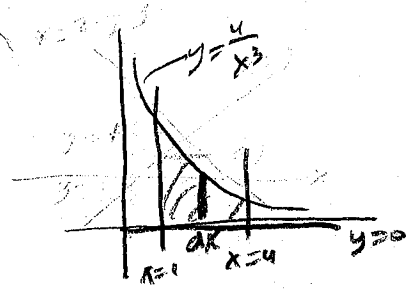
$$= \int_0^1 (x-1)\,dx - \int_0^1 (x-1)^{1/3}\,dx + \int_1^2 (x-1)^{1/3}\,dx - \int_1^2 (x-1)\,dx$$

$u = x-1, \ du = dx$

$$= \int_0^1 (x-1)\,dx - \int_{-1}^0 u^{1/3}\,du + \int_0^1 u^{1/3}\,du - \int_1^2 (x-1)\,dx$$

continued...

10. $y = \frac{4}{x^3}$, $y = 0$, $x = 1$, $x = 4$

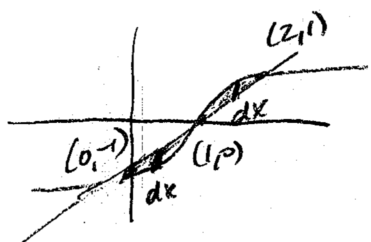


$$A = \int_1^4 \left[\left(\frac{4}{x^3}\right) - 0\right] dx = 4\int_1^4 x^{-3}\,dx = \boxed{\frac{15}{8} \approx 1.875\ u^2}$$

11. (evaluate this one by hand) $f(x) = \sqrt{x} + 3$, $g(x) = \frac{1}{2}x + 3$

$$\boxed{\frac{4}{3} \approx 1.333\ u^2}$$

12. (evaluate this one by hand) $f(x) = \sqrt[3]{x-1}$, $g(x) = x - 1$



Intersections: $\begin{cases} y = \sqrt[3]{x-1} \\ y = x-1 \end{cases}$

$\sqrt[3]{x-1} = x-1$

$x-1 = (x-1)^3$

$(x-1)^3 - (x-1) = 0$

$(x-1)((x-1)^2 - 1) = 0$

$(x-1)^2 - 1 = 0$

$(x-1)^2 = 1$

$x-1 = \pm\sqrt{1} = \pm 1$

$x = 1 \pm 1$ $x = 0$, $x = 2$

$(1,0)$ $(0,-1)$ $(2,1)$

$$A = \int_0^1 [(x-1) - (x-1)^{1/3}]\,dx + \int_1^2 [(x-1)^{1/3} - (x-1)]\,dx$$

$$= \int_0^1 (x-1)\,dx - \underbrace{\int_0^1 (x-1)^{1/3}\,dx + \int_1^2 (x-1)^{1/3}\,dx}_{u = x-1,\ du = dx} - \int_1^2 (x-1)\,dx$$

$$= \int_0^1 (x-1)\,dx - \int_{-1}^0 u^{1/3}\,du + \int_0^1 u^{1/3}\,du - \int_1^2 (x-1)\,dx$$

$$= \left[\tfrac{1}{2}x^2 - x\right]_0^1 - \left[\tfrac{3}{4}u^{4/3}\right]_{-1}^0 + \left[\tfrac{3}{4}u^{4/3}\right]_0^1 - \left[\tfrac{1}{2}x^2 - x\right]_1^2$$

$$= \left(\tfrac{1}{2} - 1 - 0\right) - \left(0 - \tfrac{3}{4}\right) + \left(\tfrac{3}{4} - 0\right) - \left(\left(\left(\tfrac{1}{2}\right)(2)^2 - 2\right) - \left(\tfrac{1}{2} - 1\right)\right)$$

$$\left(-\tfrac{1}{2}\right) + \tfrac{3}{4} + \tfrac{3}{4} - \tfrac{1}{2}$$

$$= \boxed{\tfrac{1}{2}u^2}$$

#12 continued...

$$= \int_0^1 (x-1)\,dx - \int_{-1}^0 u^{1/3}\,du + \int_0^1 u^{1/3}\,du - \int_1^2 (x-1)\,dx$$

$$= \left[\tfrac{1}{2}x^2 - x\right]_0^1 - \left[\tfrac{3}{4}u^{4/3}\right]_{-1}^0 + \left[\tfrac{3}{4}u^{4/3}\right]_0^1 - \left[\tfrac{1}{2}x^2 - x\right]_1^2$$

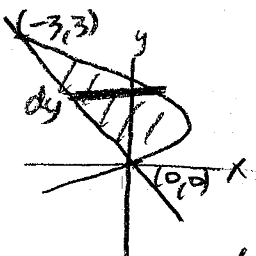
$$= \left(\tfrac{1}{2} - 1 - (0)\right) - \left(0 - \tfrac{3}{4}\right) + \left(\tfrac{3}{4} - 0\right) - \left(\left(\tfrac{1}{2}(2)^2 - 2\right) - \left(\tfrac{1}{2} - 1\right)\right)$$

$$\left(-\tfrac{1}{2}\right) + \tfrac{3}{4} + \tfrac{3}{4} - \tfrac{1}{2} = \boxed{\tfrac{1}{2}\ u^2}$$

13. $f(y) = y^2, \quad g(y) = y + 2$

$\boxed{\frac{9}{2} = 4{,}5u^2}$

14. $f(y) = y(2 - y), \quad g(y) = -y$



intersections:

$\begin{cases} x = y(2-y) \\ x = -y \end{cases}$

$-y = y(2-y)$

$y(2-y) + y = 0$

$y(2-y+1) = 0$

$y = 0 \quad y = 3$

$x = 0 \quad x = -3$

$(0,0) \quad (-3,3)$

$A = \int_0^3 [y(2-y) - (-y)]\,dy$

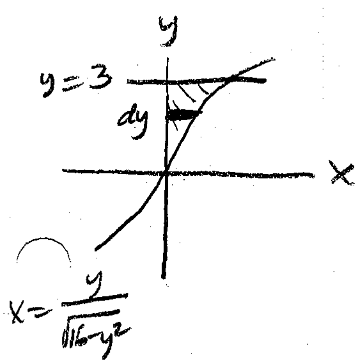
$= \int_0^3 [3y - y^2]\,dy$

$= \boxed{4{,}5u^2}$ (math9)

15. $f(y) = y^2 + 1, \quad g(y) = 0, \quad y = -1, \quad y = 2$

$\boxed{6u^2}$

16. $f(y) = \frac{y}{\sqrt{16-y^2}}, \quad g(y) = 0, \quad y = 3$



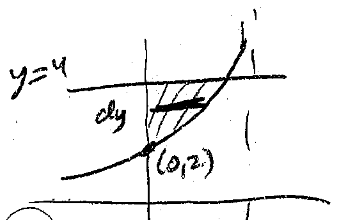
$A = \int_0^3 \left[\frac{y}{\sqrt{16-y^2}} - 0\right] dy$

$= \int_0^3 \left[\frac{y}{\sqrt{16-y^2}}\right] dy = \boxed{1{,}3542\ u^2}$ (math9)

17. (evaluate this one by hand) $f(x) = \frac{10}{x}$, $x = 0$, $y = 2$, $y = 10$

$$\boxed{10\ln(5) \approx 16.094\ u^2}$$

18. (evaluate this one by hand) $g(x) = \frac{4}{2-x}$, $y = 4$, $x = 0$



$$A = \int_2^4 \left(2 - \frac{4}{y}\right) dy = \int_2^4 2\,dy - 4\int_2^4 \frac{1}{y}\,dy$$

$$= \left[2y - 4\ln|y|\right]_2^4$$

$$\boxed{[2(4) - 4\ln(4)] - [2(2) - 4\ln|2|]\ u^2}$$

$$= 4 - 4\ln 4 - 4\ln 2$$

$$= 4 - 4\ln\left(\frac{4}{2}\right)$$

$$= 4 - 4\ln(2)$$

$$= \boxed{4(1 - \ln 2) \approx 1.2274\ u^2}$$

$g(0) = \frac{4}{2-0} = 2$

$(0,2)$

$y = \frac{4}{2-x}$

solve for x...

$y(2-x) = 4$

$2 - x = \frac{4}{y}$

$x = 2 - \frac{4}{y}$

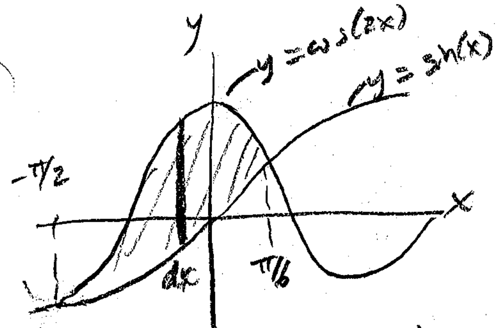
Sketch the region bounded by the graphs of the functions and set up the integral that finds the area of the bounded region. Then use your calculator to find the area of the region.

19. $f(x) = \cos(x)$, $g(x) = 2 - \cos(x)$, $0 \le x \le 2\pi$

$$\boxed{4\pi \approx 12.5664\ u^2}$$

(plus sketch)

20. $f(x) = \sin(x)$, $g(x) = \cos(2x)$, $-\frac{\pi}{2} \le x \le \frac{\pi}{6}$



(by calculator graph)

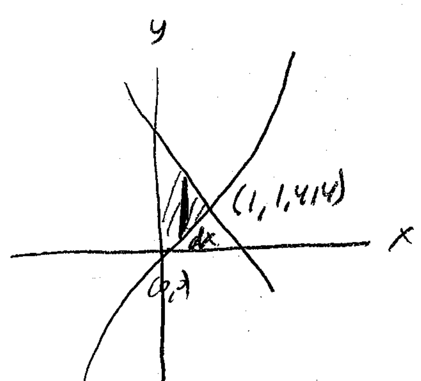
$$A = \int_{-\pi/2}^{\pi/6} [\cos(2x) - \sin(x)]\,dx$$

$$= \boxed{1.299\ u^2} \text{ (math 9)}$$

21. $f(x) = 2\sin(x)$, $g(x) = \tan(x)$, $-\frac{\pi}{3} \le x \le \frac{\pi}{3}$

$$2\ln(\tfrac{1}{2}) + 2 \approx \boxed{0.6137\ u^2}$$

22. $f(x) = \sec\left(\frac{\pi x}{4}\right)\tan\left(\frac{\pi x}{4}\right)$, $g(x) = (\sqrt{2} - 4)x + 4$, $x = 0$



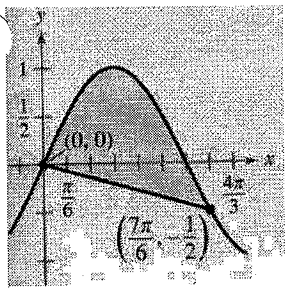
by calculator graph
for $\sec(\frac{\pi x}{4})$, enter $\frac{1}{\cos(\frac{\pi x}{4})}$

$$A = \int_0^1 \left[((\sqrt{2}-4)x+4) - \sec\left(\tfrac{\pi x}{4}\right)\tan\left(\tfrac{\pi x}{4}\right)\right]dx$$

$$= \int_0^1 \left[(\sqrt{2}-4)x+4 - \frac{1}{\cos(\frac{\pi x}{4})}\tan\left(\tfrac{\pi x}{4}\right)\right]dx$$

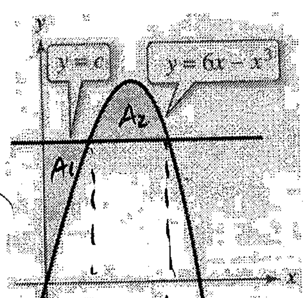
$$= \boxed{2.1797\ u^2} \text{ math 9}$$

23. Find the area between the graph of $y = \sin(x)$ and the line segment joining the points $(0,0)$ and $\left(\frac{7\pi}{6}, -\frac{1}{2}\right)$, as shown in the figure. Set up the integral, then use your calculator to find the area.



$\boxed{2.7823u^2}$

24. (Challenge Problem) The horizontal line $y = c$ intersects the graph of $y = 6x - x^3$ in the first quadrant, as shown in the figure. Find the integer c so that the areas of the two shaded regions are equal.



a b

can't easily find a, b

$A_1 = \int_0^a [c - (6x - x^3)]\,dx$ $\quad A_2 = \int_a^b [(6x - x^3) - c]\,dx$

$A_1 = [cx - 3x^2 + \frac{1}{4}x^4]_0^a$ $\quad A_2 = [3x^2 - \frac{1}{4}x^4 - cx]_a^b$

$A_1 = [ca - 3a^2 + \frac{1}{4}a^4] - [0]$ $\quad A_2 = [3b^2 - \frac{1}{4}b^4 - cb] - [3a^2 - \frac{1}{4}a^2 - ca]$

$A_1 = A_2$

$\cancel{ca} - \cancel{3a^2} + \cancel{\frac{1}{4}a^4} = 3b^2 - \frac{1}{4}b^4 - cb - \cancel{3a^2} + \cancel{\frac{1}{4}a^4} + \cancel{ca}$

$-4\left(-\frac{1}{4}b^4 + 3b^2 - cb\right) = (0)\cdot 4$

$b^4 - 12b^2 + 4cb = 0$

$b(\underbrace{b^3 - 12b + 4c}) = 0$

find a, b w/ calculator graph: then mult a A_1 & A_2

$b^3 - 12b + 4c = 0$ guess/~~synthetic~~ synthetic division

guess 3 | 1 0 −12 4c
 3 9 −9
 1 3 −3 | 4c − 9

remainder = 0
4c − 9 = 0
$c = \frac{9}{4}$

try it: $c = \frac{9}{4} \Rightarrow$ a = .38447, b = 2.2345 $\quad A_1 = .427$, $A_2 = 4.0856$ no

guess 2 | 1 0 −12 4c
 2 4 −16
 1 2 −8 | 4c − 16

remainder = 0
4c − 16 = 0
c = 4

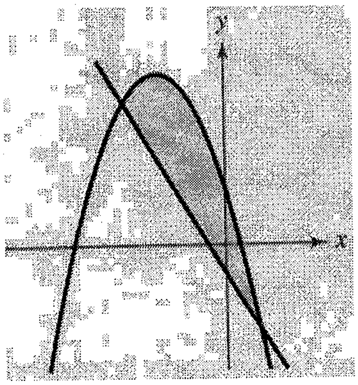
try it: c = 4 a = .732, b = 4 $\quad A_1 = 1.392$, $A_2 = 1.392$ yes!

$\boxed{c = 4}$

25. **Multiple Choice:**

The figure below shows the graphs of $y = 2 - 4x - x^2$ and $y = -2x - 1$. What is the area of the shaded region?

a) $\frac{5}{3}$ b) 10 $\boxed{\text{c) } \frac{32}{3}}$ d) $\frac{86}{3}$



26. **Multiple Choice:**

What is the area of the region bounded by the graphs of $x = y^2 - 2y$ and $y = -x + 2$?

a) $\frac{3}{2}$ b) $\frac{10}{3}$ $\boxed{\text{c) } \frac{9}{2}}$ d) $\frac{31}{6}$

complete square to graph:

$y^2 - 2y \overset{+1}{=} x + 1 + 1$

$(y-1)^2 = x + 2$

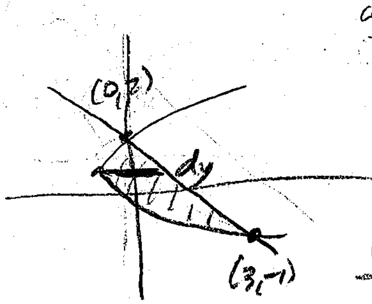
vertex at $(-2,1)$

intersections $\begin{cases} x = y^2 - 2y \\ y = -x + 2 \end{cases}$ $x = 2 - y$

$2 - y = y^2 - 2y$ $y = -1$ $y = 2$

$y^2 - y - 2 = 0$ $x = 3$ $x = 0$

$(y+1)(y-2) = 0$ $(3,-1)$ $(0,2)$



$\int_{-1}^{2} [(2-y) - (y^2 - 2y)]\, dy$

math 9 $= 9/2$

27. **Multiple Choice:**

Which integral gives the area A of the region bounded by the graph of $f(x) = x^3 - 2x$ and the tangent line to the graph of f at $(-1,1)$?

- take $f'(x)$ derivative $= m$, use to build tangent line equation
- sketch along with $f(x)$ to see region

a) $A = \int_{-1}^{2} (x^3 + 3x + 2)dx$

b) $A = \int_{-1}^{2} (x^3 - 3x - 2)dx$

c) $A = \int_{-1}^{4} (-x^3 + 3x + 2)dx$

$\boxed{\text{d)}}$ $A = \int_{-1}^{2} (-x^3 + 3x + 2)dx$

6.2 Worksheet (1 – 12, 13 – 27 odds; 28-36)

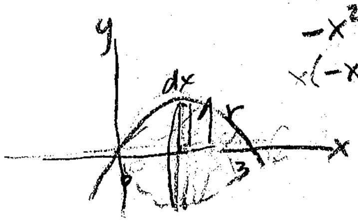
NOTE: For all problems, set up the integral but <u>do not evaluate by hand</u> – instead, use MATH 9 to check answers.

Sketch the graph of the system. Set up the integral that finds the volume of the figure created by rotating the graph around **<u>the x – axis. (EASY)</u>**

1. $y = 3x + 5, x = 2, x = 7, y = 0$

(sketch) $\boxed{V = \pi \int_2^7 [(3x+5)^2 - (0)^2]\,dx} = (1805\pi \approx 5670.575)$

2. $y = -x^2 + 3x,\ y = 0$



$-x^2+3x=0$

$x(-x+3)=0$

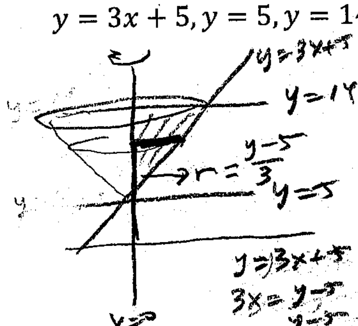
$V = \int_{x_1}^{x_2} \pi r^2 dx = \boxed{\int_0^3 \pi(-x^2+3x)^2 dx} = \left(\frac{81\pi}{10} \approx 25.447\right)$

3. $y = \sin(x), x = \frac{\pi}{6}, x = \frac{5\pi}{6}, y = 0$

(sketch) $\boxed{V = \pi \int_{\pi/6}^{5\pi/6} [(\sin x)]^2 dx} \approx (1.48021\pi \approx 4.650)$

Sketch a graph of the system. Set up the integral that finds the volume of the figure created by rotating the graph around **<u>the y – axis. (EASY)</u>**

4. $y = 3x + 5, y = 5, y = 14, x = 0$



$V = \int_{y_1}^{y_2} \pi r^2 dy = \boxed{\int_5^{14} \pi\left(\frac{y-5}{3}\right)^2 dy} \approx 84.823\ u^3$

$y=3x+5$

$3x = y-5$

$x = \frac{y-5}{3}$

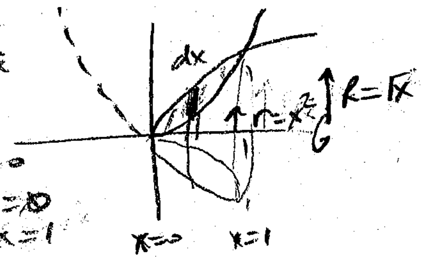
5. $y = x^2 + 4, x = 0, y = 8$

(sketch) $V = \boxed{\int_4^8 \pi(\sqrt{y-4})^2 dy} = (8\pi \approx 25.133)$

Sketch the graph of the system. Set up the integral that finds the volume of the figure created by the graph rotating around the indicated axis. **(MEDIUM/Test Level)**

6. $y = x^2, y = \sqrt{x}\,; AROUND\ THE\ X - AXIS.$

$x^2 = \sqrt{x}$
$x^4 = x$
$x^4 - x = 0$
$x(x^3 - 1) = 0$
$x = 0 \quad x = 1$



$$V = \int_{x_1}^{x_2} \pi R^2 dx - \int_{x_1}^{x_2} \pi r^2 dx$$

$$= \int_0^1 \pi (\sqrt{x})^2 dx - \int_0^1 \pi (x^2)^2 dx$$

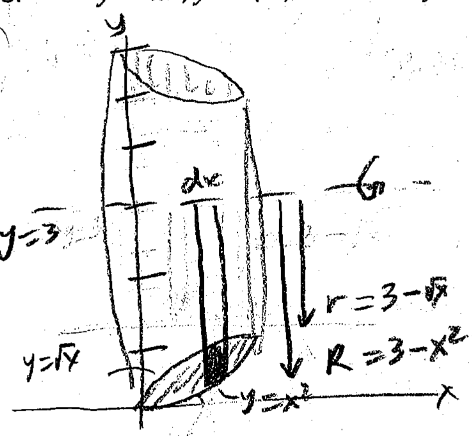
$$= \boxed{\pi \int_0^1 (x - x^4) dx} \approx 0.942$$

7. $y = 2x + 3, x = 0, y = 9\,; AROUND\ y = 9.$

(sketch)

$$V = \boxed{\int_0^3 \pi \left(9 - (2x+3)\right)^2 dx} = 36\pi \approx 113.097$$

8. $y = x^2, y = \sqrt{x}; AROUND\ y = 3.$



$$V = \int \pi R^2 dx - \int \pi r^2 dx$$

$$= \int_0^1 \pi (3 - x^2)^2 dx - \int_0^1 (3 - \sqrt{x})^2 dx$$

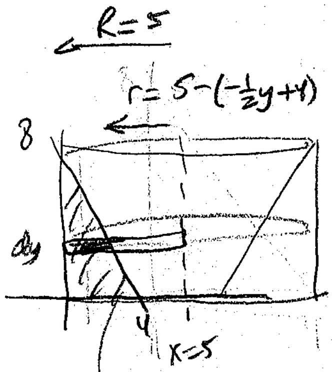
$$= \boxed{\pi \int_0^1 \left[(3 - x^2)^2 - (3 - \sqrt{x})^2\right] dx} = 5.3407$$

9. $y = x^2 - 4x + 9, y = 2x + 1, AROUND\ x = 1.$

(sketch)

$$V = \boxed{\int_5^9 \pi\left[\left(2 + \sqrt{y-5}\right) - 1\right]^2 dy - \int_5^9 \pi\left[\left(\frac{y-1}{2}\right) - 1\right]^2 dy} \approx 16.755$$

10. $y = -2x + 8, y = 0, x = 0;$ *AROUND* $x = 5$.



$y=-2x+8$

$-2x=y-8$

$x=\frac{y-8}{-2}=-\frac{1}{2}y+4$

$V=\int \pi R^2 dh - \int \pi r^2 dh$

$= \boxed{\int_0^8 \pi[5]^2 dy - \int_0^8 \pi\left[5-\left(\frac{8-y}{2}\right)\right]^2 dy} \approx 368.614$

11. $y = -2x + 8, y = 0, x = 0;$ *AROUND* $y = 9$.

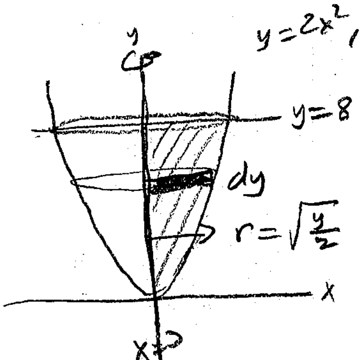
(sketch)

$V = \boxed{\int_0^4 \pi[9]^2 dx - \int_0^4 \pi[9-(-2x+8)]^2 dx} \approx 636.696\ u^3$

12. $y = 2x^2, y = 8, x = 0,$ *AROUND THE Y − AXIS.*

$y=2x^2, \quad x^2=\frac{y}{2}$

$x=\pm\sqrt{\frac{y}{2}}$

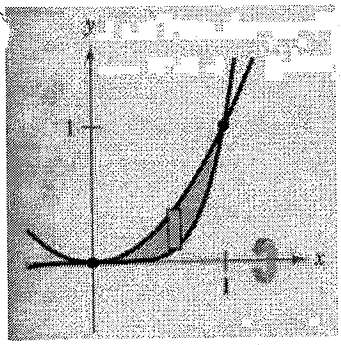


$V=\int \pi r^2 dy$

$= \boxed{\int_0^8 \pi\left(\sqrt{\frac{y}{2}}\right)^2 dy} \approx 50.265\ u^3$

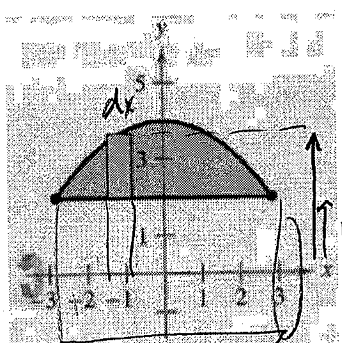
Set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the x-axis.

13. $y = x^2,\ y = x^5$



$$\boxed{V = \pi \int_0^1 [(x^2)^2 - (x^5)^2]\,dx} \approx 0.3427\,u^3$$

14. $y = 2,\ y = 4 - \frac{x^2}{4}$



$R = 4 - \frac{x^2}{4}$

$r = 2$

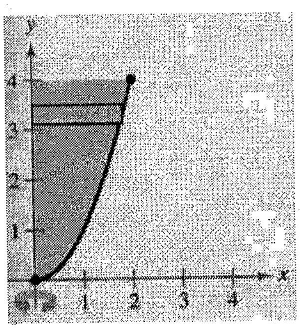
$$V = \int \pi R^2\,dx - \int \pi r^2\,dx$$

$$= \int_{-3}^{3} \pi \left(4 - \frac{x^2}{4}\right)^2 dx - \int_{-3}^{3} \pi (2)^2\,dx$$

$$= \boxed{\pi \int_{-3}^{3} \left[\left(4 - \frac{x^2}{4}\right)^2 - (2)^2\right] dx} = 132.1825\,u^3$$

Set up and evaluate the integral that gives the volume of the solid formed by revolving the region about the y-axis.

15. $y = x^2$



$$\boxed{V = \pi \int_0^4 [(\sqrt{y})^2]\,dy} = 8\pi \approx 25.133$$

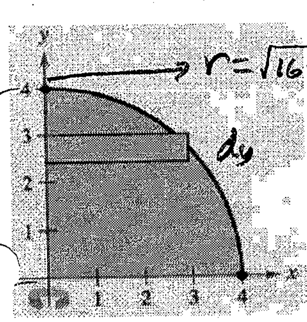
16. $y = \sqrt{16 - x^2}$

$y^2 = 16 - x^2$

$x^2 = 16 - y^2$

$x = \pm\sqrt{16 - y^2}$

$r = \sqrt{16 - y^2}$



$$V = \int \pi r^2\,dy$$

$$= \int_0^4 \pi \left(\sqrt{16 - y^2}\right)^2 dy$$

$$= \boxed{\pi \int_0^4 (16 - y^2)\,dy} = 134.0413\,u^3$$

Find the volumes of the solids generated by revolving the region bounded by the graphs of the equations about the given lines.

17. $y = \sqrt{x},\ y = 0,\ x = 3$ around... a) x-axis b) y-axis c) x = 3 d) x = 6

a) x-axis: $\boxed{V = \pi \int_0^3 [(\sqrt{x})^2 dx} = \frac{9\pi}{2} (14.1372)$ (a sketch)

b) y-axis: $\boxed{V = \int_0^{\sqrt{3}} \pi [3]^2 dy - \int_0^{\sqrt{3}} \pi [3 - \sqrt{y}]^2 dy}$ $\approx (23.933)$

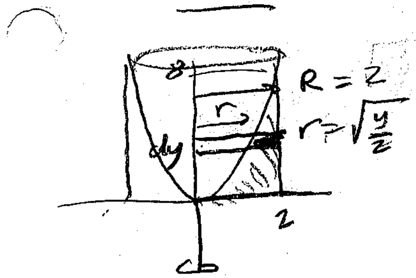
c) x = 3: $\boxed{V = \int_0^{\sqrt{3}} \pi [3 - \sqrt{y}]^2 dy}$ $\approx (25.0398)$

d) x = 6: $\boxed{V = \int_0^{\sqrt{3}} \pi [6 - \sqrt{y}]^2 dy - (\int_0^{\sqrt{3}} \pi [3]^2 dy}$ $\approx (94.3399)$

$x^2=\frac{1}{2}y$, $x=\pm\sqrt{\frac{1}{2}y}=\pm\sqrt{\frac{y}{2}}$

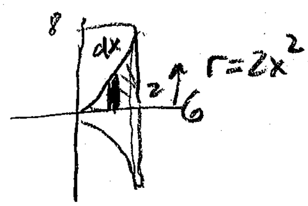
18. $y = 2x^2$, $y = 0$, $x = 2$ around... a) y-axis b) x-axis c) y = 8 d) x = 2

a) y-axis



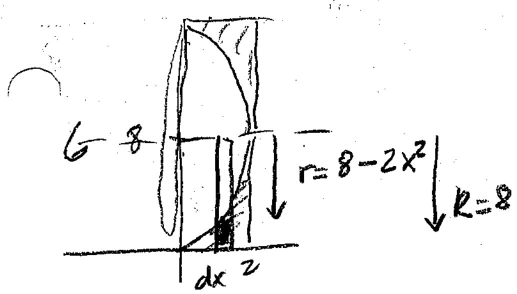
$$V=\int \pi R^2\,dy-\int \pi r^2\,dy$$
$$=\int_0^8 \pi(2)^2\,dy-\int \pi\left(\sqrt{\tfrac{y}{2}}\right)^2 dy$$
$$=\boxed{\pi\int_0^8\left[(2)^2-\left(\sqrt{\tfrac{y}{2}}\right)^2\right]dy}=16\pi\approx 50.2655\ u^3$$

b) x-axis



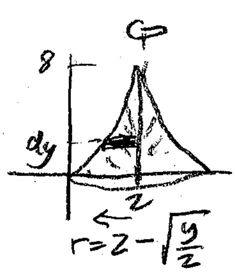
$$V=\int \pi r^2\,dx$$
$$=\boxed{\int_0^2 \pi(2x^2)^2\,dx}=\frac{128}{5}\pi\approx 80.4248\ u^3$$

c) y=8



$$V=\int \pi R^2\,dx-\int \pi r^2\,dx$$
$$=\int_0^2 \pi(8)^2\,dx-\int_0^2 \pi(8-2x^2)^2\,dx$$
$$=\boxed{\pi\int_0^2\left[(8)^2-(8-2x^2)^2\right]dx}=\frac{896\pi}{15}\approx 187.658\ u^3$$

d) x=2



$$V=\int \pi r^2\,dy$$
$$=\boxed{\int_0^8 \pi\left(2-\sqrt{\tfrac{y}{2}}\right)^2 dy}\approx 16.755\ u^3$$

19. $y = x^2,\ y = 4x - x^2$ around... a) x-axis b) y = 6

a) x-axis ; $V = \boxed{\int_0^2 \pi[4x-x^2]^2dx - \int_0^2 \pi (x^2)^2 dx} \approx (33.510)$ (& sketch)

b) y=6 ; $V = \boxed{\int_0^2 \pi[6-x^2]^2dx - \int_0^2 \pi[6-(4x-x^2)]^2dx} \approx 67.021$

20. $y = 4 + 2x - x^2,\ y = 4 - x$ around... a) x-axis b) y = 1

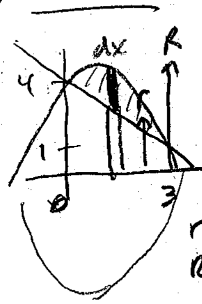
a) x-axis

$\begin{cases} y = 4+2x-x^2 \\ y = 4-x \end{cases}$

$4-x = 4+2x-x^2$

$x^2-3x = 0$

$x(x-3) = 0$



$r = 4-x$

$R = 4+2x-x^2$

$V = \int \pi R^2 dx - \int \pi r^2 dx$

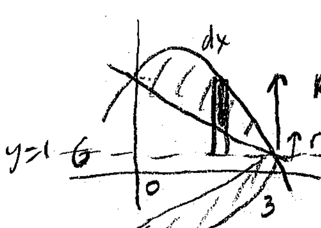
$= \int_0^3 \pi(4+2x-x^2)^2 dx - \int_0^3 \pi(4-x)^2 dx$

$= \boxed{\pi \int_0^3 [(4+2x-x^2)^2-(4-x)^2]\,dx} = \frac{153\pi}{5} \approx 96.132\,u^3$

b) y=1

$V = \int \pi R^2 dx - \int \pi r^2 dx$

$= \int_0^3 \pi((4+2x-x^2)-1)^2 dx - \int_0^3 \pi((4-x)-1)^2 dx$



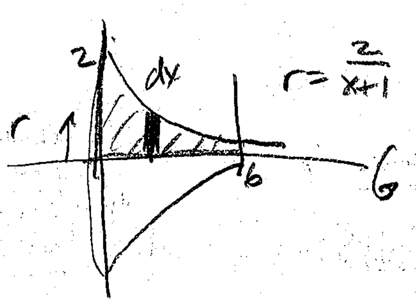
$= \boxed{\pi \int_0^3 [((4+2x-x^2)-1)^2-((4-x)-1)^2]\,dx} = \frac{108\pi}{5} \approx 67.858\,u^3$

Find the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis.

21. $y = \frac{1}{x}$, $y = 0$, $x = 1$, $x = 3$

(sketch) $\boxed{V = \pi \int_1^3 \left[\left(\frac{1}{x}\right)^2 dx\right]} = \frac{2\pi}{3} \approx 2.0944 u^3$

22. $y = \frac{2}{x+1}$, $y = 0$, $x = 0$, $x = 6$



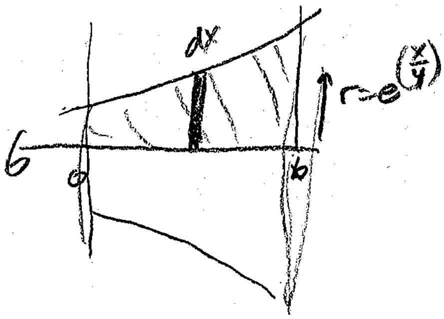
$V = \int \pi r^2 dx$

$= \boxed{\int_0^6 \pi \left(\frac{2}{x+1}\right)^2 dx} = \frac{24\pi}{7} = 10.771 u^3$

23. $y = e^{-x}$, $y = 0$, $x = 0$, $x = 1$

(sketch) $\boxed{V = \pi \int_0^1 (e^{-x})^2 dx} \approx 1.3582 u^2$

24. $y = e^{x/4}, \ y = 0, \ x = 0, \ x = 6$



$V = \int \pi r^2 dx$

$= \boxed{\int_0^6 \pi (e^{(\frac{x}{4})})^2 dx}$ $\approx 119.9796 u^3$

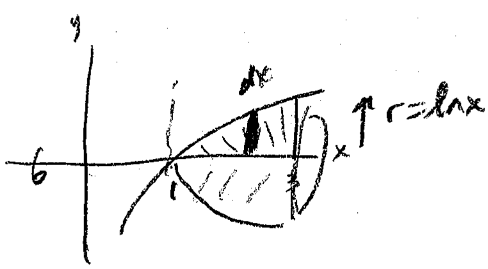
$(= 2\pi(e^3 - 1))$

Use your calculator to approximate the volume of the solid generated by revolving the region bounded by the graphs of the equations about the x-axis.

25. $y = e^{-x^2}, \ y = 0, \ x = 0, \ x = 2$

(sketch) $\boxed{V = \pi \int_0^2 [(e^{-x^2})^2 dx}$ $\approx$ $1.9686 u^3$

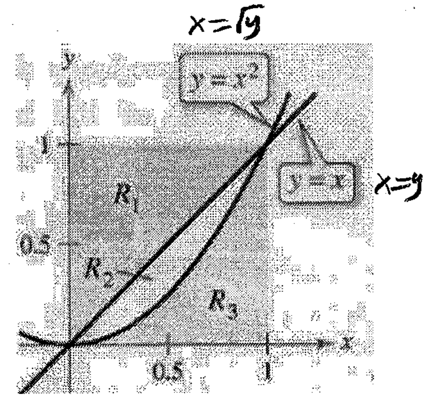
26. $y = \ln(x), \ y = 0, \ x = 1, \ x = 3$



$V = \int \pi r^2 dx$

$= \boxed{\int_1^3 \pi (\ln x)^2 dx}$ $\approx 3.2332 u^3$

Find the volume generated by rotating the given region about the specified line.



27. R_1 about $x = 0$ 28. R_1 about $x = 1$

29. R_2 about $y = 0$ 30. R_2 about $y = 1$

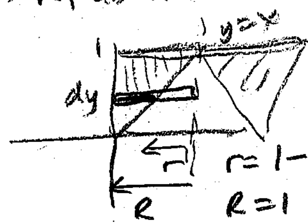
31. R_2 about $x = 0$ 32. R_2 about $x = 1$

(27) R_1 about $x=0$
(sketch)

$$\boxed{V=\int_0^1 \pi (y)^2\,dy}$$

$=\pi/3 = 1.047\,u^3$

(28) R_1 about $x=1$



$r=1-y$
$R=1$

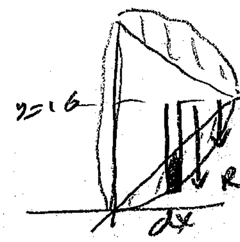
$$V=\int \pi R^2 dy - \int \pi r^2 dy$$
$$=\int_0^1 \pi(1)^2 dy - \int_0^1 \pi(1-y)^2 dy$$
$$=\boxed{\pi\int_0^1 [(1)^2-(1-y)^2]\,dy}$$
$$=\frac{2\pi}{3} \approx 2.0944\,u^3$$

(29) R_2 about $y=0$
(sketch)

$$\boxed{V=\pi\int_0^1 [(x)^2-(x^2)^2]\,dx}$$

$=\frac{2\pi}{15} \simeq 0.4189\,u^3$

(30) R_2 about $y=1$



$r=1-x$
$R=1-x^2$

$$V=\int \pi R^2 dx - \int \pi r^2 dx$$
$$=\int_0^1 \pi(1-x^2)^2 dx - \int_0^1 \pi(1-x)^2 dx$$
$$=\boxed{\pi\int_0^1 [(1-x^2)^2-(1-x)^2]\,dx}$$
$$=\frac{\pi}{5} = 0.6283\,u^3$$

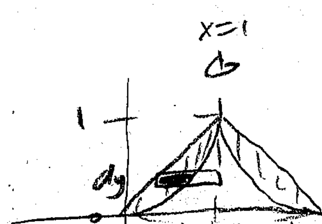
(31) R_2 about $x=0$

(sketch)

$$\boxed{V=\pi\int_0^1 [(\sqrt{y})^2-(y)^2]\,dy}$$

$=\frac{\pi}{6} = 0.5236\,u^3$

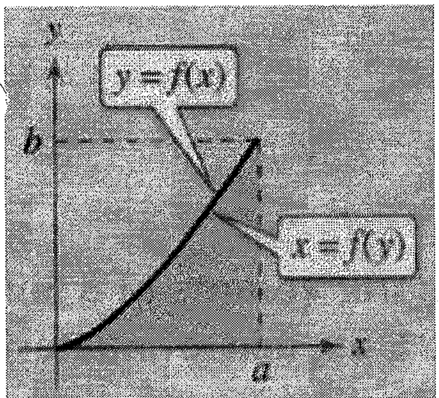
(32) R_2 about $x=1$



$r=1-\sqrt{y}$
$R=1-y$

$$V=\int \pi R^2 dy - \int \pi r^2 dy$$
$$=\int_0^1 \pi(1-y)^2 dy - \int_0^1 \pi(1-\sqrt{y})^2 dy$$
$$=\boxed{\pi\int_0^1 [(1-y)^2-(1-\sqrt{y})^2]\,dy}$$
$$=\frac{\pi}{6} \simeq 0.524\,u^3$$

33. Use the graph to match the integral for the volume with the axis of rotation.



(a) $V = \pi \int_0^b (a^2 - [f(y)]^2)\, dy$ (i) x-axis

(b) $V = \pi \int_0^a (b^2 - [b - f(x)]^2)\, dx$ (ii) y-axis

(c) $V = \pi \int_0^a [f(x)]^2\, dx$ (iii) $x = a$

(d) $V = \pi \int_0^b [a - f(y)]^2\, dy$ (iv) $y = b$

a) ii
b) iv
c) i
d) iii

34. **Multiple Choice:**

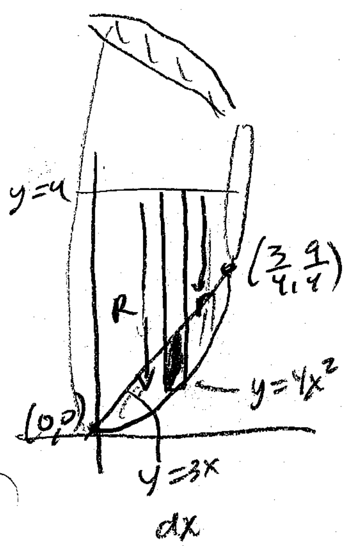
Let R be the region bounded by the graphs $y = 3x$ and $y = 4x^2$ for $0 \le x \le \frac{3}{4}$. What is the volume of the solid generated when R is revolved about the line $y = 4$?

a) $\pi \int_0^{3/4} [(4-3x)^2 - (4-4x^2)^2]dx$

b) $\boxed{\pi \int_0^{3/4} [(4-4x^2)^2 - (4-3x)^2]dx}$

c) $\pi \int_0^{3/4} (9x^2 - 16x^4)dx$

d) $\pi \int_0^{9/4} \left|\left(4-\frac{\sqrt{y}}{2}\right)^2 - \left(4-\frac{y}{3}\right)^2\right| dy$



$\begin{cases} y=3x \\ y=4x^2 \end{cases}$

$4x^2=3x$

$4x^2-3x=0$

$x(4x-3)=0$

$x=0 \quad x=\frac{3}{4}$

$y=\frac{9}{4}$

$r=4-3x$

$R=4-4x^2$

$V=\int \pi R^2 dx - \int \pi r^2 dx$

$= \int_0^{3/4} \pi(4-4x^2)^2 dx - \int_0^{3/4} \pi(4-3x)^2 dx$

$= \pi \int_0^{3/4} [(4-4x^2)^2 - (4-3x)^2] dx$

6.3 Worksheet (1 – 10, 11 – 25 odds, 26 – 29)

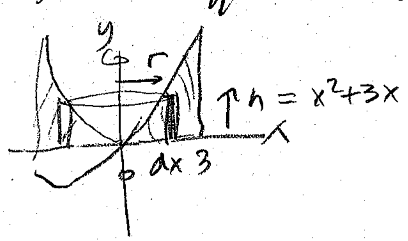
NOTE: For all problems, set up the integral but do not evaluate by hand – instead, use MATH 9 to check answers.

Sketch the graph of the system. Set up the integral that finds the volume of the figure created by rotating the graph around **the y – axis. (EASY)**

1. $y = 3x + 5, x = 0, x = 7, y = 0$

(sketch) $\boxed{\int_0^7 2\pi(x)(3x+5)\,dx} \approx 2924.823\ u^3$

2. $y = x^2 + 3x$ ~~4/4~~$, x = 0, x = 3, y = 0$



$V = \int 2\pi r h\,dr$

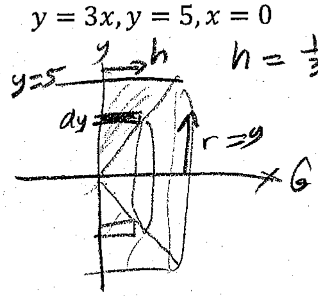
$= \boxed{\int_0^3 2\pi(x)(x^2+3x)\,dx} \approx 296.881\ u^3$

3. $y = \sin(x), x = 0, x = \frac{5\pi}{6}, y = 0$

(sketch) $V = \boxed{\int_0^{5\pi/6} 2\pi(x)(\sin x)\,dx} \approx 17.387\ u^3$

Sketch a graph of the system. Set up the integral that finds the volume of the figure created by rotating the graph around **the x – axis. (EASY)**

4. $y = 3x, y = 5, x = 0$



$h = \frac{1}{3}y$

$V = \int 2\pi r h\,dr$

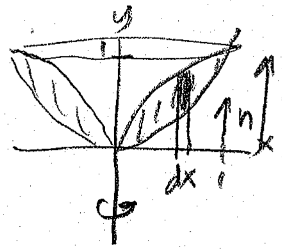
$= \boxed{\int_0^5 2\pi(y)\left(\tfrac{1}{3}y\right)dy} \approx 87.266\ u^3$

5. $y = x^2 + 4, x = 0, y = 7$

(sketch) $\boxed{V = \int_4^7 2\pi(y)(\sqrt{y-4})\,dy} \approx 126.240\ u^3$

Sketch the graph of the system. Set up the integral that finds the volume of the figure created by the graph rotating around the indicated axis. **(EASY/MEDIUM)**

6. $y = x^2, y = \sqrt{x}$; *AROUND THE Y – AXIS.*



$h = x^2$

$H = \sqrt{x}$

$V = \int 2\pi r H dr - \int 2\pi r h dr$

$= \int_0^1 2\pi(x)(\sqrt{x})dx - \int_0^1 2\pi(x)(x^2)dx$

$= \boxed{2\pi \int_0^1 x(\sqrt{x} - x^2)\,dx} \approx 0.942u^3$

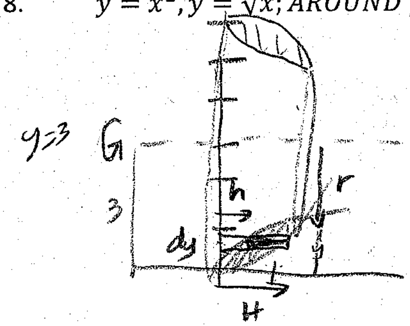
7. $y = 2x + 3, x = 0, y = 9$; *AROUND* $y = 9$.

(sketch)

$\boxed{V = \int_3^9 2\pi(9-y)\left(\frac{y-3}{2}\right)dy} \approx 113.097u^3$

$x = \pm\sqrt{y}, x = y^2$

8. $y = x^2, y = \sqrt{x}$; *AROUND* $y = 3$.



$r = 3 - y$

$h = y^2$

$H = \sqrt{y}$

$V = \int 2\pi r H dr - \int 2\pi r h dr$

$= \int_0^1 2\pi(3-y)(\sqrt{y})dy - \int_0^1 2\pi(3-y)(y^2)dy$

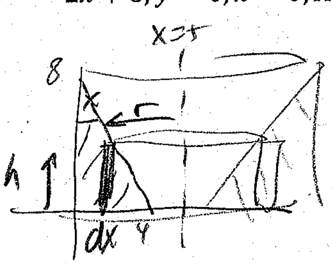
$= \boxed{2\pi \int_0^1 (3-y)(\sqrt{y} - y^2)dy} \approx 5.341u^3$

9. $y = x^2 - 4x + 9, y = 2x + 1$, *AROUND* $x = 1$.

(sketch)

$V = \boxed{\int_2^4 2\pi x(2x+1)dx - \int_2^4 2\pi x(x^2-4x+9)dx} \approx (25.133)$

10. $y = -2x + 8, y = 0, x = 0$; *AROUND* $x = \cancel{6}\,5$



$r = 5 - x$

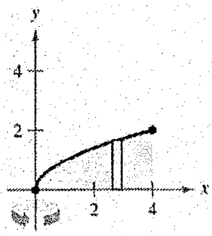
$h = -2x + 8$

$V = \int 2\pi r h dr$

$= \boxed{\int_0^4 2\pi(5-x)(-2x+8)\,dx} \approx 368.614u^3$

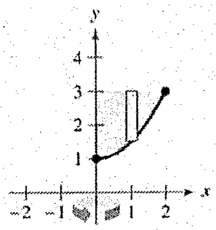
Use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the y-axis.

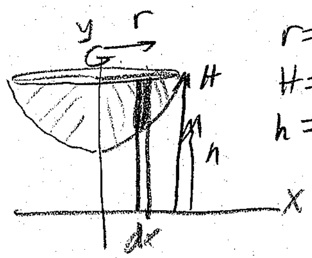
11. $y = \sqrt{x}$



(sketch) $\boxed{V = \int_0^4 2\pi(x)(\sqrt{x})\,dx} \approx 80.425u^3$

12. $y = \frac{1}{2}x^2 + 1$





$r = x$
$H = 3$
$h = \frac{1}{2}x^2 + 1$

$V = \int 2\pi r H\,dr - \int 2\pi r h\,dr$

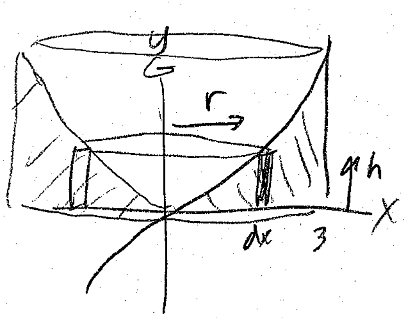
$= \boxed{\int_0^2 2\pi(x)(3)\,dx - \int_0^2 2\pi(x)\left(\frac{1}{2}x^2+1\right)dx}$

$\approx (12.566)$

13. $y = \frac{1}{4}x^2, \ y = 0, \ x = 4$

(sketch) $\boxed{V = \int_0^4 2\pi(x)\left(\frac{1}{4}x^2\right)dx} \approx 100.531u^3$

14. $y = \frac{1}{2}x^3, \ y = 0, \ x = 3$



$r = x$
$h = \frac{1}{2}x^3$

$V = \int 2\pi r h\,dr$

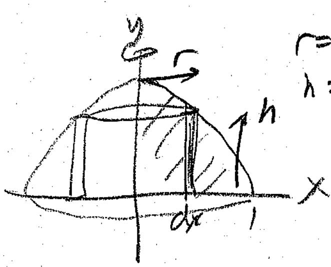
$= \boxed{\int_0^3 2\pi(x)\left(\frac{1}{2}x^3\right)dx} \approx 152.681u^3$

15. $y = \sqrt{x-2},\ y = 0,\ x = 4$

(sketch)

$\boxed{V = \int_2^4 2\pi(x)(\sqrt{x-2})\,dx}$ $\approx 37.913\ u^3$

16. $y = -x^2 + 1,\ y = 0$ (in quadrant I)



$r = x$

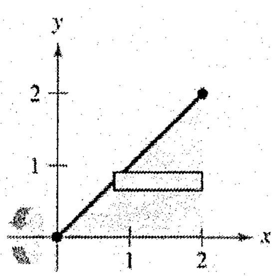
$h = -x^2+1$

$V = \int 2\pi r h\, dr$

$= \boxed{\int_0^1 2\pi(x)(-x^2+1)\,dx}$ $\approx 1.571\ u^3$

Use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the x-axis.

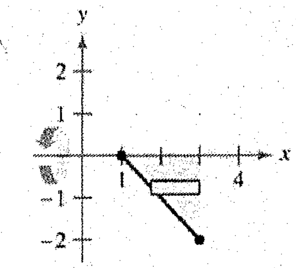
17. $y = x$

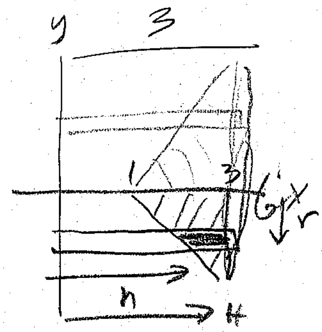


(sketch)

$\boxed{V = 2\pi \int_0^2 y(2-y)\,dy}$ $\approx 8.378\ u^3$

18. $y = 1 - x$





$r = |y| = -y$

$h = 3 - (1-y)$

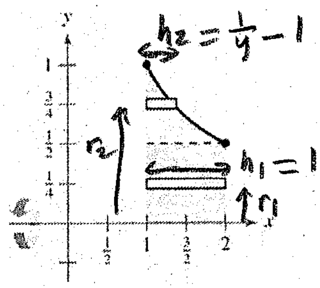
$H = 3$

$V = \int 2\pi r H\,dr - \int 2\pi r h\,dr$

$= \boxed{\int_{-2}^{0} 2\pi(-y)(3)\,dy - \int_{-2}^{0} 2\pi(-y)(3-(1-y))\,dy}$

$\approx (29.3215)$

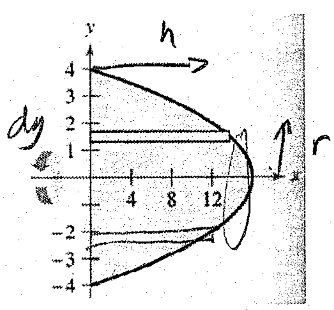
19. $y = \frac{1}{x}$, $x = \frac{1}{y}$



$V = \boxed{\int_{0}^{1/2} 2\pi(y)(1)\,dy + \int_{1/2}^{1} 2\pi(y)\left(\frac{1}{y} - 1\right)dy}$

$\frac{\pi}{4} + \frac{\pi}{4} \approx (1.5708)$

20. $x + y^2 = 16$



$V = \int 2\pi r h\,dr$

$= \boxed{\int_{0}^{4} 2\pi(y)(16 - y^2)\,dy} \approx 402.124\ u^3$

Use the disk method OR the shell method to find the volumes of the solids generated by revolving the region bounded by the graphs of the equations about the given lines.

21. $y = x^3,\ y = 0,\ x = 2$ around... a) x-axis b) y-axis c) x = 4

(sketches)

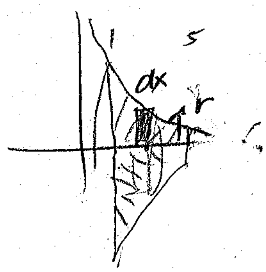
a) $\boxed{V = \int_0^2 \pi (x^3)^2 dx} \approx 57{,}446 u^3$

b) $\boxed{V = \int_0^2 2\pi (x)(x^3) dx} \approx 40{,}212 u^3$

c) $\boxed{V = \int_0^2 2\pi (4-x)(x^3) dx} \approx 60{,}319 u^3$

22. $y = \frac{10}{x^2},\ y = 0,\ x = 1,\ x = 5$ around... a) x-axis b) y-axis c) y = 10

a) x-axis

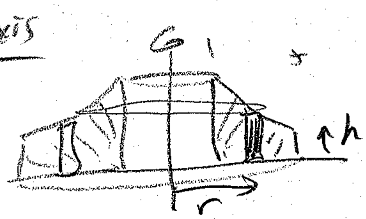


disc

$V = \int \pi r^2 dh$

$= \boxed{\int_1^5 \pi \left(\frac{10}{x^2}\right)^2 dx} \approx 103.882$

b) y-axis



shell

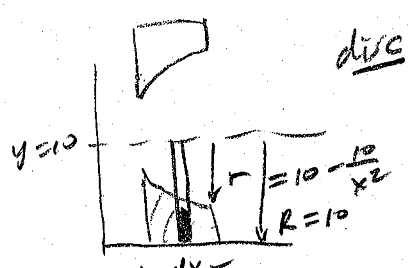
$V = \int 2\pi r h\, dr$

$= \boxed{\int_1^5 2\pi (x)\left(\frac{10}{x^2}\right) dx} \approx 101{,}124$

$r = x$

$h = \frac{10}{x^2}$

c)



disc

$V = \int \pi R^2 dh - \int \pi r^2 dh$

$= \boxed{\int_1^5 \pi (10)^2 dx - \int_1^5 \pi \left(10 - \frac{10}{x^2}\right)^2 dx} \approx 398{,}773$

23. $x^{1/2} + y^{1/2} = a^{1/2},\ x = 0,\ y = 0$ around... a) x-axis b) y-axis c) x = a

(sketches) a) $\boxed{V = \int_0^a \pi\left(\left(\sqrt{a} - \sqrt{x}\right)^2\right)^2 dx}$

b) $\boxed{V = \int_0^a \pi\left(\left(\sqrt{a} - \sqrt{y}\right)^2\right)^2 dy}$

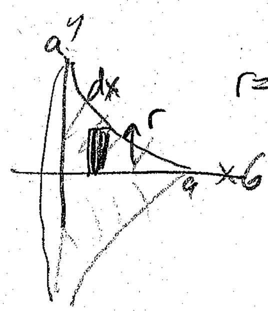
c) $\boxed{V = \int_0^a 2\pi(a-x)\left(\sqrt{a} - \sqrt{x}\right)^2 dx}$

24. $x^{2/3} + y^{2/3} = a^{2/3},\ a > 0$ (in quadrant I) (called a hypocycloid) around... a) x-axis b) y-axis

$y^{2/3} = a^{2/3} - x^{2/3}$

$y = \left(a^{2/3} - x^{2/3}\right)^{3/2}$

a) x-axis



$r = \left(a^{2/3} - x^{2/3}\right)^{3/2}$

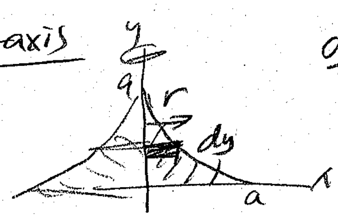
disc

$V = \int \pi r^2 db$

$= \int_0^a \pi\left(\left(a^{2/3} - x^{2/3}\right)^{3/2}\right)^2 dx$

$= \boxed{\int_0^a \pi\left(a^{2/3} - x^{2/3}\right)^3 dx}$

b) y-axis



disc

$r = \left(a^{2/3} - y^{2/3}\right)^{3/2}$

$V = \int \pi r^2 dh$

$= \int_0^a \pi\left(\left(a^{2/3} - y^{2/3}\right)^{3/2}\right)^2 dy$

$= \boxed{\int_0^a \pi\left(a^{2/3} - y^{2/3}\right)^3 dy}$

25. Let V_1 and V_2 be the volumes of the solids that result when the plane region bounded by $y = \frac{1}{x}$, $y = 0$, $x = \frac{1}{4}$, and $x = c$ (where $c > \frac{1}{4}$) is revolved about the x-axis and the y-axis, respectively. Find the value of c for which $V_1 = V_2$.

$\boxed{c=2}$

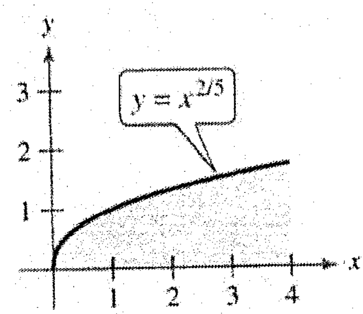
(use disc for V_1, shell for V_2)

check each of the two c solutions

(only one is non-trivial))

26. **Multiple Choice:**

The region shown in the figure is revolved about the x-axis, the y-axis, and the line x = 4. Which of the following orders the volumes of the resulting solids from least to greatest?



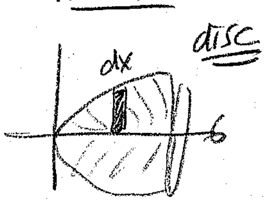
$\boxed{\text{a)}}$ x-axis, x = 4, y-axis

b) x-axis, y-axis, x = 4

c) x = 4, x-axis, y-axis

d) x = 4, y-axis, x-axis

X-axis

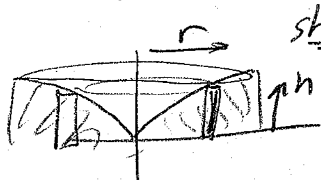


$V = \int \pi r^2 \, dh$

$= \int_0^4 \pi (x^{2/5})^2 \, dx$

$= 21.163396$

y-axis



$V = \int 2\pi r h \, dr$

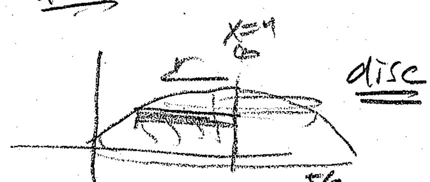
$= \int_0^4 2\pi (x)(x^{2/5}) \, dx$

$= 72.93107$

least → greatest

x-axis, x=4, y-axis

x=4



$r = 4 - y^{5/2}$

$V = \int \pi r^2 \, dh$

$= \int_0^2 \pi (4 - y^{5/2})^2 \, dy$

$= 52.7999$

27. **Multiple Choice:**

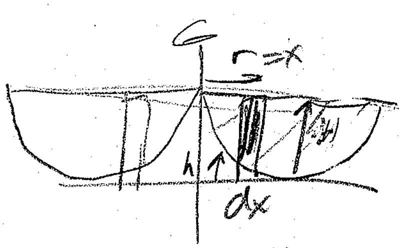
Let A be the region bounded by the graphs of $y = 4\sqrt{x}$, $y = 4$, $x = 0$. What is the volume of the solid formed when R is revolved about the line $x = -\frac{3}{2}$?

a) $\frac{4\pi}{5}$ b) $\frac{16\pi}{5}$ $\boxed{\text{c)}}$ $\frac{24\pi}{5}$ d) $\frac{56\pi}{5}$

28. **Multiple Choice:**

Let R be the region enclosed by the graphs of $y = \frac{1}{8}(x-4)^2$ and $y = 2$. What is the volume of the solid formed when R is revolved about the y-axis?

a) $\frac{13\pi}{3}$ b) $\frac{19\pi}{3}$ c) $\frac{128\pi}{3}$ $\boxed{\text{d)}}$ $\frac{256\pi}{3}$



$H = 2$

$h = \frac{1}{8}(x-4)^2$

$V = \int 2\pi r H\,dr - \int 2\pi r h\,dr$

$= \int_0^8 2\pi(x)(2)\,dx - \int_0^8 2\pi(x)\left(\frac{1}{8}(x-4)^2\right)dx$

$= \frac{256\pi}{3}$

$2 = \frac{1}{8}(x-4)^2$

$16 = (x-4)^2$

$\pm 4 = x-4$

$x = 4 \pm 4$

$x = 0, 8$

29. **Free Response:**

Let T be the region bounded by the graphs of $y = \frac{4}{x+2}$ and $y = 2 - \frac{2}{5}x$.

a. Find the area of the region.

b. What is the volume of the solid formed when T is revolved about the y-axis?

c. What is the volume of the solid formed when T is revolved about the x-axis?

a) $\boxed{A = \int_0^3 \left[\left(2-\tfrac{2}{5}x\right)-\left(\tfrac{4}{x+2}\right)\right]dx \approx 0{,}53484\,u^2}$ (sketch)

b) $\boxed{V = 2\pi \int_0^3 x\left[\left(2-\tfrac{2}{5}x\right)-\left(\tfrac{4}{x+2}\right)\right]dx \approx 4{,}58877\,u^3}$ (sketch)

c) $\boxed{V = \pi \int_0^3 \left[\left(2-\tfrac{2}{5}x\right)^2-\left(\tfrac{4}{x+2}\right)^2\right]dx = \frac{36\pi}{25} = 4{,}52389\,u^3}$ (sketch)