AP Calculus BC

Review for Unit 6 Test, Part 2

For #1-4, find the volume of the solid described. **Sketch and setup the integral, but do not evaluate the integral.**

#1) The solid whose base is the region enclosed by $y = x^2, \quad y = 2x \quad (\textit{in the first quadrant})$ and whose cross-sections are perpendicular to the y-axis and are squares.

#2) The solid whose base is the region enclosed by $y = x^2, \quad y = 2x \quad (\textit{in the first quadrant})$ and whose cross-sections are perpendicular to the y-axis and are semicircles. (assume b = diameter)

#3) The solid whose base is the region enclosed by $y = x^2, \quad y = 2x \quad (\textit{in the first quadrant})$ and whose cross-sections are perpendicular to the y-axis and are right (isosceles) triangles with a leg in the base region.

For #5-6, find the average value of the function on the given interval. **No sketch is required, but set up <u>and evaluate</u> the integral (by hand).**

#4) $f(x) = -x^4 + 2x^2 + 4; \quad [-2, 1]$

#5) $f(x) = 4x^{\frac{1}{2}}; \quad [0, 3]$

For #6-7, find the length of the curve described. **No sketch required, setup the integral, but do not evaluate the integral.**

#6) $f(x) = 2(x-1)^{\frac{3}{2}}; \quad [1, 5]$

#7) $f(x) = \dfrac{x^3}{6} + \dfrac{1}{2x}; \quad [1, 3]$

For #8-11, find the surface area of the surface of revolution described. **Sketch and setup the integral, but do not evaluate the integral.**

#8) $y = \sin x, \quad 0 \le x \le \pi, \quad \textit{about the } x-\textit{axis}$

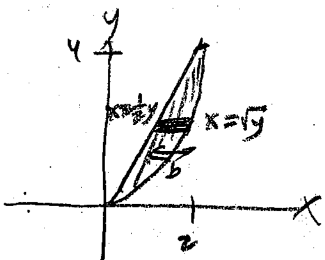
#9) $2y + x^2 = 1, \quad 0 \le x \le 1, \quad \textit{about the } y-\textit{axis}$

#10) $x - 1 = 2y^2, \quad 1 \le y \le 2, \quad \textit{about the } x-\textit{axis}$

#11) $x = \sqrt{2y - y^2}, \quad 0 \le y \le 1, \quad \textit{about the } y-\textit{axis}$

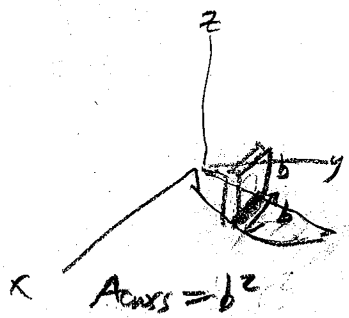
AP calc BC Unit 6 part 2 test review

①



$y = x^2$
$x = \sqrt{y}$
$y = 2x$
$x = \frac{1}{2}x$

$b = \sqrt{y} - \frac{1}{2}y$

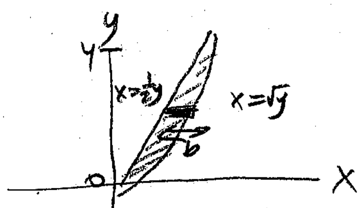


$A_{cross} = b^2$
$= (\sqrt{y} - \frac{1}{2}y)^2$

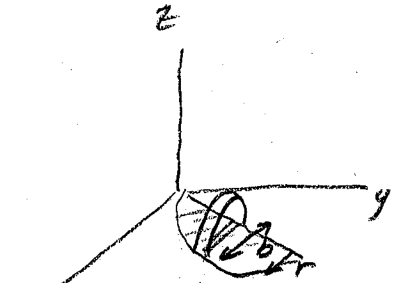
$V = \int A_{cross}\,dy$

$$\boxed{V = \int_0^4 (\sqrt{y} - \tfrac{1}{2}y)^2\,dy}$$

②



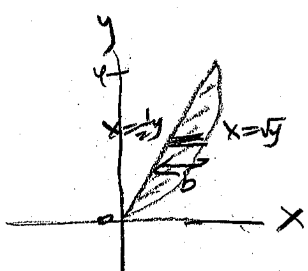
$b = \sqrt{y} - \frac{1}{2}y$



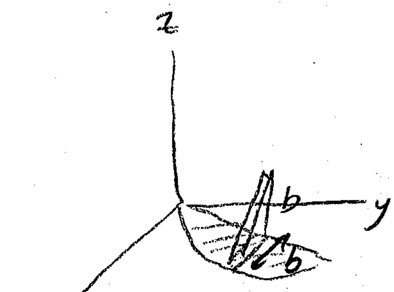
$r = \frac{1}{2}b$
$A_{cross} = \frac{1}{2}\pi r^2$
$= \frac{1}{2}\pi(\frac{1}{2}b)^2$
$= \frac{1}{2}\pi(\frac{1}{2}(\sqrt{y} - \frac{1}{2}y))^2$

$$\boxed{V = \int_0^4 \tfrac{1}{2}\pi\left[\tfrac{1}{2}(\sqrt{y} - \tfrac{1}{2}y)\right]^2 dy}$$

③



$b = \sqrt{y} - \frac{1}{2}y$



$A = \frac{1}{2}bh = \frac{1}{2}b^2$
$= \frac{1}{2}(\sqrt{y} - \frac{1}{2}y)^2$

$$\boxed{V = \int_0^4 \tfrac{1}{2}(\sqrt{y} - \tfrac{1}{2}y)^2\,dy}$$

(4)

$$AV = \frac{1}{1-(-2)} \int_{-2}^{1} (-x^4 + 2x^2 + 4)\,dx$$

$$= \frac{1}{3}\left[-\tfrac{1}{5}x^5 + \tfrac{2}{3}x^3 + 4x\right]_{-2}^{1}$$

$$= \boxed{\frac{1}{3}\left[\left(-\tfrac{1}{5}(1)^5 + \tfrac{2}{3}(1)^3 + 4(1)\right) - \left(-\tfrac{1}{5}(-2)^5 + \tfrac{2}{3}(-2)^3 + 4(-2)\right)\right]} = \boxed{\frac{19}{5}}$$

(5)

$$AV = \frac{1}{3-0}\int_0^3 4x^{1/2}\,dx$$

$$= \frac{1}{3}\left[4\left(\tfrac{2}{3}\right)x^{3/2}\right]_0^3 = \frac{1}{3}\left[\tfrac{8}{3}x^{3/2}\right]_0^3$$

$$= \boxed{\frac{1}{3}\left[\tfrac{8}{3}(3)^{3/2} - \tfrac{8}{3}(0)^{3/2}\right]} = \frac{1}{3}\cdot\frac{8}{3}\sqrt{3}\sqrt{3}\sqrt{3} = \boxed{\frac{8}{3}\sqrt{3}}$$

(6)

$$f(x) = 2(x-1)^{3/2}$$

$$f'(x) = 2\left(\tfrac{3}{2}\right)(x-1)^{1/2}(1)$$

$$f'(x) = 3(x-1)^{1/2}$$

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$$

$$\boxed{L = \int_1^5 \sqrt{1 + [3(x-1)^{1/2}]^2}\,dx}$$

(7)

$$f(x) = \tfrac{1}{6}x^3 + (2x)^{-1}$$

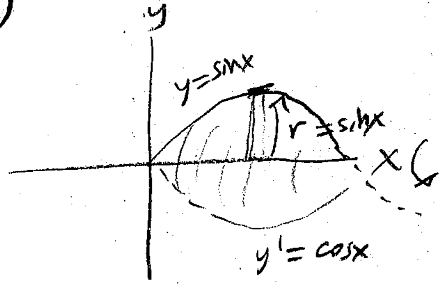
$$f'(x) = \tfrac{1}{2}x^2 - (2x)^{-2}(2)$$

$$f'(x) = \tfrac{1}{2}x^2 - \frac{2}{(2x)^2}$$

$$L = \int_a^b \sqrt{1 + [f'(x)]^2}\,dx$$

$$= \boxed{\int_1^3 \sqrt{1 + \left[\tfrac{1}{2}x^2 - \tfrac{2}{(2x)^2}\right]^2}\,dx}$$

(8)



$A_{surf} = \int_a^b 2\pi r \sqrt{1+[f'(x)]^2}\,dx$

$\boxed{A_{surf} = \int_0^{\pi} 2\pi(\sin(x))\sqrt{1+[\cos(x)]^2}\,dx}$

(9)

$2y + x^2 = 1$

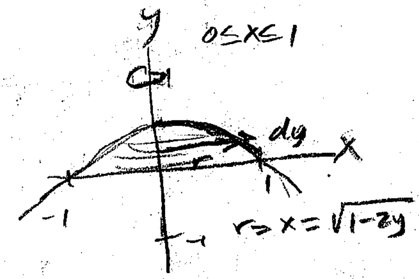
$2y = 1 - x^2$

$y = \frac{1}{2} - \frac{1}{2}x^2$

x ints: $y=0$

$\frac{1}{2} = \frac{1}{2}x^2$

$x = \pm 1$



$x^2 = 1-2y$

$x = \pm\sqrt{1-2y} = (1-2y)^{1/2}$

$x' = \frac{1}{2}(1-2y)^{-1/2}(-2) = \frac{-1}{\sqrt{1-2y}}$

$A_{surf} = \int_a^b 2\pi r\sqrt{1+[f'(y)]^2}\,dy = \boxed{\int_0^{1/2} 2\pi\sqrt{1-2y}\sqrt{1+\left[\frac{-1}{\sqrt{1-2y}}\right]^2}\,dy}$

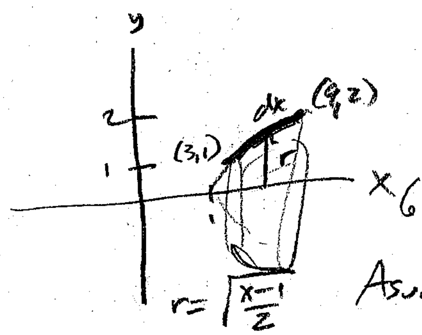
(10)

$x - 1 = 2y^2$

$x = 1 + 2y^2$

$1 \le y \le 2$

$y = \pm\sqrt{\frac{x-1}{2}}$



$y = \left(\frac{x-1}{2}\right)^{1/2}$

$y' = \frac{1}{2}\left(\frac{x-1}{2}\right)^{-1/2}\left(\frac{1}{2}\right) = \frac{1}{4}$

$y=1$: $x-1 = 2(1)^2$

$x - 1 = 2$

$x = 3$

$y=2$: $x-1 = 2(2)^2$

$x-1 = 8$

$x = 9$

$A_{surf} = \boxed{\int_3^9 2\pi\sqrt{\frac{x-1}{2}}\sqrt{1+\left[\frac{1}{4}\left(\frac{x-1}{2}\right)^{-1/2}\right]^2}\,dx}$

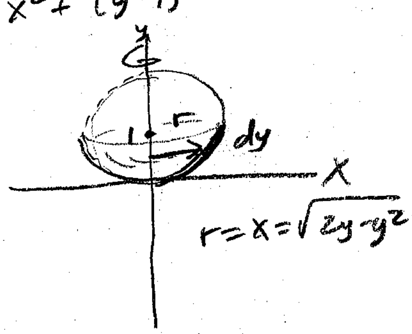
(11) $x = \sqrt{2y-y^2} = (2y-y^2)^{1/2}$, $x' = \frac{1}{2}(2y-y^2)^{-1/2}(2-2y)$

$x^2 = 2y - y^2$

$x^2 + y^2 - 2y = 0$

$x^2 + y^2 - 2y + 1 = 0 + 1$

$x^2 + (y-1)^2 = 1$



$A_{surf} = \int_a^b 2\pi r\sqrt{1+[x']^2}\,dy$

$= \boxed{\int_0^1 2\pi\sqrt{2y-y^2}\sqrt{1+\left[\frac{1}{2}(2y-y^2)^{-1/2}(2-2y)\right]^2}\,dy}$