Unit 5 Review

① $\int 2x\,dx$

$2\int x\,dx$

$2\cdot\frac{1}{2}x^2+C$

$\boxed{x^2+C}$

② $\int \frac{10}{x}\,dx$

$10\int \frac{1}{x}\,dx$

$\boxed{10\ln|x|+C}$

③ $\int \frac{3}{x^2}\,dx$

$3\int x^{-2}\,dx$

$3(-1)x^{-1}+C$

$\boxed{\frac{-3}{x}+C}$

④ $\int \frac{x^2+7x}{x}\,dx$

$\int \frac{x^2}{x}\,dx+\int \frac{7x}{x}\,dx$

$\int x\,dx+\int 7\,dx$

$\boxed{\frac{1}{2}x^2+7x+C}$

⑤ $\int_{-2}^{-1} \frac{dx}{(2x+3)^4}$

$u=2x+3$

$\frac{du}{dx}=2$

$du=2\,dx$

$dx=\frac{1}{2}du$

$x=-2$: $u=2(-2)+3=-1$

$x=-1$: $u=2(-1)+3=1$

these are now u values

$\frac{1}{2}\int_{-1}^{1} u^{-4}\,du$

$\frac{1}{2}\left[\frac{u^{-3}}{-3}\right]_{-1}^{1}$

$-\frac{1}{6}\left[\frac{1}{u^3}\right]_{-1}^{1}$

$-\frac{1}{6}\left(\left[\frac{1}{(1)^3}\right]-\left[\frac{1}{(-1)^3}\right]\right)$

$-\frac{1}{6}(1-(-1))$

$=-\frac{1}{6}(2)$

$=\boxed{-\frac{1}{3}}$

*notes:

- could also replace u with $2x+3$ and use original limits
- this integral is technically undefined because $\frac{1}{(2x+3)^4}$ is undefined at $x=-3/2$ which is in $[-2,-1]$ (ignore this ☺)

⑥ $\int t^{10}(t-10)\,dt$

$\int (t^{11}-10t^{10})\,dt$

$\int t^{11}\,dt-10\int t^{10}\,dt$

$\frac{t^{12}}{12}-10\frac{t^{11}}{11}+C$

$\boxed{\frac{1}{12}t^{12}-\frac{10}{11}t^{11}+C}$

(7) $\int x^2\sqrt{x^3+7}\,dx$

$u=x^3+7$

$\frac{du}{dx}=3x^2$

$du=3x^2dx$

$x^2dx=\frac{1}{3}du$

$\int (x^3+7)^{1/2}x^2dx$

$\int u^{1/2}(\frac{1}{3}du)$

$\frac{1}{3}\int u^{1/2}du$

$\frac{1}{3}\left[\frac{2}{3}u^{3/2}\right]+c$

$\boxed{\frac{2}{9}(x^3+7)^{3/2}+C}$

(8) $\int_0^4 x\sqrt{16-3x}\,dx$

$u=16-3x$

$\frac{du}{dx}=-3$

$du=-3dx$

$dx=-\frac{1}{3}du$

$3x=16-u$

$x=\frac{16-u}{3}$

$x=0$: $u=16$

$x=4$: $u=16-3(4)$, $u=4$

$\int (16-3x)^{1/2}x\,dx$

$\int_{16}^{4} u^{1/2}\left(\frac{16-u}{3}\right)\left(-\frac{1}{3}du\right)$

$-\frac{1}{9}\int_{16}^{4} u^{1/2}(16-u)\,du$

$-\frac{1}{9}\int (16u^{1/2}-u^{1/2}u^{1})\,du$

$-\frac{1}{9}\left(\int_{16}^{4}16u^{1/2}du-\int_{16}^{4}u^{3/2}du\right)$

$-\frac{1}{9}\left(\left[16\left(\frac{2}{3}\right)u^{3/2}\right]_{16}^{4}-\left[\frac{2}{5}u^{5/2}\right]_{16}^{4}\right)$

$\boxed{-\frac{1}{9}\left(\left[\frac{32}{3}(4)^{3/2}-\frac{32}{3}(16)^{3/2}\right]-\left[\frac{2}{5}(4)^{5/2}-\frac{2}{5}(16)^{5/2}\right]\right)}$

$\boxed{\frac{3008}{135}}\simeq 22.28148$

(9) $\int \frac{x}{(x-5)^3}\,dx$

$u=x-5$

$\frac{du}{dx}=1$

$du=dx$

$x=u+5$

$\int (x-5)^{-3}x\,dx$

$\int u^{-3}(u+5)\,du$

$\int (u^{-3}u+5u^{-3})\,du$

$\int (u^{-2}+5u^{-3})\,du$

$\frac{u^{-1}}{-1}+5\frac{u^{-2}}{-2}+c$

$-\frac{1}{u}-\frac{5}{2}\frac{1}{u^2}+c$

$\boxed{-\frac{1}{(x-5)}-\frac{5}{2}\frac{1}{(x-5)^2}+C}$

(10) $\int \frac{(\ln x)^2}{x}\,dx$

$u=\ln x$

$\frac{du}{dx}=\frac{1}{x}$

$du=\frac{1}{x}dx$

$\int (\ln x)^2\frac{1}{x}dx$

$\int u^2\,du$

$\frac{1}{3}u^3+C$

$\boxed{\frac{1}{3}(\ln x)^3+C}$

(11) $\int \frac{e^{2y}}{e^{2y}+1}\,dy$

$u = e^{2y}+1$

$\frac{du}{dy} = 2e^{2y}$

$du = 2e^{2y}dy$

$\frac{1}{2}du = e^{2y}dy$

$\int \frac{1}{u} e^{2y}\,dy$

$\int \frac{1}{u}\frac{1}{2}\,du$

$\frac{1}{2}\int \frac{1}{u}\,du$

$\frac{1}{2}\ln|u| + c$

$\boxed{\frac{1}{2}\ln|e^{2y}+1| + C}$

(12) $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$

$u = \sqrt{x} = x^{1/2}$

$\frac{du}{dx} = \frac{1}{2}x^{-1/2} = \frac{1}{2}\frac{1}{\sqrt{x}}$

$du = \frac{1}{2}\frac{1}{\sqrt{x}}dx$

$\frac{1}{\sqrt{x}}dx = 2du$

$\int e^{u}\frac{1}{\sqrt{x}}\,dx$

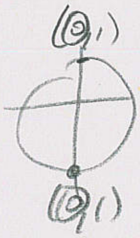
$\int e^{u}(2du)$

$2\int e^{u}du$

$2e^{u} + C$

$\boxed{2e^{\sqrt{x}} + C}$

(13) $\int_{-\frac{\pi}{2}}^{\pi/2} \sin x\,dx$

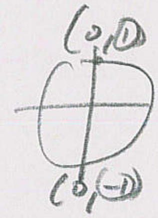


$\left[-\cos x\right]_{-\pi/2}^{\pi/2}$

$\left[-\cos(\pi/2)\right] - \left[-\cos(-\frac{\pi}{2})\right]$

$[-(0)] + (0) = \boxed{0}$

(14) $\int_{-\pi/2}^{\pi/2} \cos x\,dx$



$\left[\sin x\right]_{-\pi/2}^{\pi/2}$

$\left[\sin(\pi/2)\right] - \left[\sin(-\pi/2)\right]$

$1 - (-1) = \boxed{2}$

(15) $\int \csc^2(3t)\,dt$

$u = 3t$

$\frac{du}{dt} = 3$

$du = 3dt$

$dt = \frac{1}{3}du$

$= \int (\csc(3t))^2\,dt$

$\int \csc^2 u \,\frac{1}{3}du$

$\frac{1}{3}\int \csc^2 u\,du$

$\frac{1}{3}\left[-\cot u\right] + C$

$\boxed{-\frac{1}{3}\cot(3t) + C}$

(16) $\int \cot(7t)\,dt$

$\int \frac{\cos(7t)}{\sin(7t)}\,dt$

$u = \sin(7t)$

$\frac{du}{dt} = \cos(7t)\cdot 7$

$du = 7\cos(7t)\,dt$

$\frac{1}{7}du = \cos(7t)\,dt$

$\int \frac{1}{u}\cos(7t)\,dt$

$\int \frac{1}{u}\left(\frac{1}{7}du\right)$

$\frac{1}{7}\int \frac{1}{u}\,du$

$\frac{1}{7}\ln|u| + C$

$\boxed{\frac{1}{7}\ln|\sin(7t)| + C}$

17) $\int \frac{\cos x}{1+\sin^2 x}\,dx$ $u = 1 + (\sin x)^2$

$\frac{du}{dx} = 2(\sin x)\cdot\cos x$

Not working

$\int \frac{1}{1+u^2}\cos x\,dx$ try $u = \sin x$

$\int \frac{1}{1+u^2}\,du$ $\frac{du}{dx} = \cos x$

$\arctan u + C$ $du = \cos x\,dx$

$\boxed{\arctan(\sin x) + C}$

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18) $\int_{\pi/6}^{2\pi/3} \sin^2(\theta)\cos(\theta)\,d\theta$ $u = \sin\theta$

$\frac{du}{d\theta} = \cos\theta$

$(\frac{\sqrt{3}}{2}, \frac{1}{2})$ $du = \cos\theta\,d\theta$

$\int_{1/2}^{\sqrt{3}/2} u^2\,du$

$\theta = \pi/6,\ u = \sin \pi/6$

$u = 1/2$

$(-\frac{1}{2}, \frac{\sqrt{3}}{2})$

$\left[\frac{1}{3}u^3\right]_{1/2}^{\sqrt{3}/2}$

$\theta = 2\pi/3,\ u = \sin 2\pi/3$

$u = \sqrt{3}/2$

$\boxed{\frac{1}{3}\left(\frac{\sqrt{3}}{2}\right)^3 - \frac{1}{3}\left(\frac{1}{2}\right)^3} = \frac{\sqrt{3}}{8} - \frac{1}{24} = 0.17484$

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19) $\int_0^{\pi/4} (1+\tan t)^3 \sec^2 t\,dt$

$u = 1 + \tan t$

$\int_1^2 u^3\,du$ $\frac{du}{dt} = \sec^2 t$

$du = \sec^2 t\,dt$

$\left[\frac{1}{4}\cdot u^4\right]_1^2$ $t = 0,\ u = 1 + \tan 0$

$u = 1 + 0$

$u = 1$

$\frac{1}{4}(2)^4 - \frac{1}{4}(1)^4$ $t = \pi/4,\ u = 1 + \tan \pi/4$

$u = 1 + 1$

$\frac{16}{4} - \frac{1}{4}$ $u = 2$

$\boxed{\frac{15}{4}}$

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20) $\int \frac{\cos x}{\sqrt{1+\sin x}}\,dx$ $u = 1 + \sin x$

$\frac{du}{dx} = \cos x$

$\int \frac{1}{\sqrt{1+\sin x}}\cos x\,dx$ $du = \cos x\,dx$

$\int u^{-1/2}\,du$

$\frac{u^{1/2}}{(1/2)} + C$

$\boxed{2(1+\sin x)^{1/2} + C} = \boxed{2\sqrt{1+\sin x} + C}$

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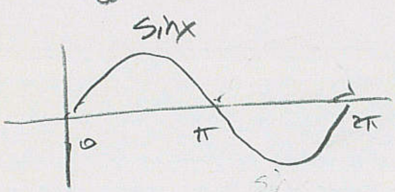
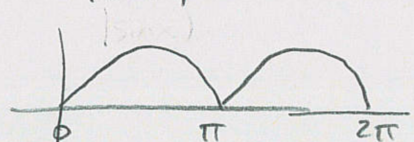
21) $\int_0^{2\pi} |\sin x|\,dx$





$[0, \pi]$ $[\pi, 2\pi]$

$|\sin x| = \sin x$ $|\sin x| = -\sin x$

$(-1,0)$ $(1,0)$

so $\int_0^{2\pi} |\sin x|\,dx = \int_0^{\pi} \sin x\,dx - \int_{\pi}^{2\pi} \sin x\,dx$

$= \left[-\cos x\right]_0^{\pi} - \left[-\cos x\right]_{\pi}^{2\pi}$

$= -\cos\pi - (-\cos 0) - ((-\cos 2\pi) - (-\cos\pi))$

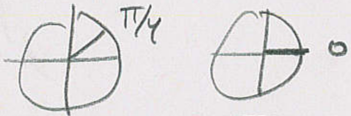
$= -(-1) + (1) - (-1 + (-1))$

$= 1 + 1 - (0) = \boxed{2}$

(22) $\displaystyle\int_0^1 \frac{1}{x^2+1}\,dx$

$\left[\arctan x\right]_0^1$

$\arctan(1) - \arctan(0)$



$\frac{\pi}{4} - 0 = \boxed{\frac{\pi}{4}}$

(23) $\displaystyle\int \frac{x^3}{\sqrt{x^2+1}}\,dx$

$u = x^2+1 \Rightarrow x^2 = u-1$

$\frac{du}{dx} = 2x$

$du = 2x\,dx$

$dx = \frac{1}{2x}\,du$

$\displaystyle\int \frac{1}{\sqrt{x^2+1}} x^3\,dx$

$\displaystyle\int \frac{1}{\sqrt{u}} x^3 \frac{1}{2x}\,du$

$\displaystyle\int u^{-1/2}\,\frac{1}{2}\,x^2\,du$

$\displaystyle\frac{1}{2}\int u^{-1/2}(u-1)\,du = \frac{1}{2}\int \left(u^{-1/2}u - u^{-1/2}\right)du$

$\displaystyle = \frac{1}{2}\int \left(u^{1/2} - u^{-1/2}\right)du$

$= \frac{1}{2}\cdot\frac{2}{3}u^{3/2} - \cancel{\frac{1}{2}}\cancel{2}u^{1/2} + C$

$= \boxed{\frac{1}{3}(x^2+1)^{3/2} - (x^2+1)^{1/2} + C}$

(24) $\displaystyle\int \frac{x}{\sqrt{1-x^4}}\,dx$

$u = x^2$

$\frac{du}{dx} = 2x$

$du = 2\underline{x\,dx}$

$\frac{1}{2}du = x\,dx$

$\displaystyle\int \frac{1}{\sqrt{1-u^2}}\,x\,dx$

$\displaystyle\int \frac{1}{\sqrt{1-u^2}}\,\frac{1}{2}\,du$

$\displaystyle\frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du$

$\frac{1}{2}\arcsin(u) + C$

$\boxed{\frac{1}{2}\arcsin(x^2) + C}$

(25) $\displaystyle\int \frac{1}{\sqrt{1-4x^2}}\,dx$

$u = 2x$

$\frac{du}{dx} = 2$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$\displaystyle\int \frac{1}{\sqrt{1-u^2}}\,\frac{1}{2}\,du$

$\displaystyle\frac{1}{2}\int \frac{1}{\sqrt{1-u^2}}\,du$

$\frac{1}{2}\arcsin(u) + C$

$\boxed{\frac{1}{2}\arcsin(2x) + C}$

(26) $\displaystyle\int_{-1}^{1} \frac{x+x^3+x^5}{1+x^2+x^4}\,dx$

$\displaystyle\int_{-1}^{1} \frac{x(1+x^2+x^4)}{1+x^2+x^4}\,dx$

$\displaystyle\int_{-1}^{1} x\,dx = \left[\frac{1}{2}x^2\right]_{-1}^{1} = \frac{1}{2}(1)^2 - \frac{1}{2}(-1)^2 = \frac{1}{2} - \frac{1}{2} = \boxed{0}$

(27) $\frac{d}{dx}\left[\int_{2x}^{3x+1} \sin(t^4)\,dt\right]$

$$\int_{2x}^{3x+1} \sin(t^4)\,dt = \int_{2x}^{0} \sin(t^4)\,dt + \int_{0}^{3x+1} \sin(t^4)\,dt$$

and $\frac{d}{dx}[f+g] = \frac{d}{dx}[f] + \frac{d}{dx}[g]$

so $\frac{d}{dx}\left[\int_{2x}^{3x+1} \sin(t^4)\,dt\right] = \frac{d}{dx}\int_{2x}^{0} \sin(t^4)\,dt + \frac{d}{dx}\int_{0}^{3x+1} \sin(t^4)\,dt$

↑ this one has constant on wrong limit, swap and make negative

$$-\frac{d}{dx}\int_{0}^{2x} \sin(t^4)\,dt + \frac{d}{dx}\int_{0}^{3x+1} \sin(t^4)\,dt$$

now, by Fundamental Theorem of Calculus:

$$-\underbrace{\sin((2x)^4)\cdot(2)}_{\text{chain rule}} + \underbrace{\sin((3x+1)^4)\cdot(3)}_{\text{chain rule}}$$

$$\boxed{-2\sin((2x)^4) + 3\sin((3x+1)^4)}$$

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(28) $\frac{d}{dt}\left[\int_{\sqrt{x}}^{x} \frac{e^t}{t}\,dt\right]$ Same approach as #27

$$\frac{d}{dt}\int_{\sqrt{x}}^{0} \frac{e^t}{t}\,dt + \frac{d}{dt}\int_{0}^{x} \frac{e^t}{t}\,dt$$

$$-\frac{d}{dt}\int_{0}^{\sqrt{x}} \frac{e^t}{t}\,dt + \frac{d}{dt}\int_{0}^{x} \frac{e^t}{t}\,dt$$

$$-\frac{e^{\sqrt{x}}}{\sqrt{x}}\cdot\underbrace{\left(\tfrac{1}{2}x^{-1/2}\right)}_{\text{chain Rule}} + \frac{e^x}{x}\underbrace{(1)}_{\text{chain Rule}}$$

chain Rule
$u = \sqrt{x} = x^{1/2}$
$\frac{du}{dx} = \frac{1}{2}x^{-1/2}$

$$\boxed{-\frac{e^{\sqrt{x}}}{\sqrt{x}}\left(\tfrac{1}{2}x^{-1/2}\right) + \frac{e^x}{x}}$$

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$$F(x) = \int_0^x (t^3 + 5t)\,dt$$

$$F(3) = \int_0^3 (t^3 + 5t)\,dt$$

$$= \left[\frac{1}{4}t^4 + \frac{5}{2}t^2\right]_0^3$$

$$= \boxed{\left[\frac{1}{4}(3)^4 + \frac{5}{2}(3)^2\right] - \left[\frac{1}{4}(0)^4 + \frac{5}{2}(0)^2\right]}$$

$$= \boxed{\frac{171}{4} = 42.75}$$