

5.4 Worksheet (Odds, 40, 44, 46)

Evaluate the definite integral. Use your calculator to verify your result.

1. $\int_0^2 6x \, dx$

2. $\int_{-3}^1 8 \, dt$

3. $\int_{-1}^0 (2x - 1) \, dx$

4. $\int_{-1}^2 (7 - 3t) \, dt$

5. $\int_{-1}^1 (t^2 - 2) \, dt$

6. $\int_1^2 (6x^2 - 3x) \, dx$

7. $\int_0^1 (2t - 1)^2 \, dt$

8. $\int_1^3 (4x^3 - 3x^2) \, dx$

9. $\int_1^2 \left(\frac{3}{x^2} - 1 \right) \, dx$

10. $\int_{-2}^{-1} \left(u - \frac{1}{u^2} \right) \, du$

11. $\int_1^4 \frac{u-2}{\sqrt{u}} du$

12. $\int_{-8}^8 x^{1/3} dx$

13. $\int_{-\pi/6}^{\pi/6} \sec^2(x) dx$

14. $\int_{\pi/4}^{\pi/2} (2 - \csc^2(x)) dx$

15. $\int_{-\pi/3}^{\pi/3} 4 \sec(\theta) \tan(\theta) d\theta$

16. $\int_{-\pi/2}^{\pi/2} (2t + \cos(t)) dt$

17. $\int_0^2 (2^x + 6) dx$

18. $\int_0^3 (t - 5^t) dt$

Find the area of the region bounded by the graphs of the equations.

19. $y = 5x^2 + 2$, $x = 0$, $x = 2$, $y = 0$

20. $y = x^3 + x$, $x = 2$, $y = 0$

21. $y = 1 + \sqrt[3]{x}$, $x = 0$, $x = 8$, $y = 0$

22. $y = -x^2 + 4x$, $y = 0$

Use the Second Fundamental Theorem of Calculus to find $F'(x)$.

23. $F(x) = \int_{-2}^x (t^2 - 2t) dt$

24. $F(x) = \int_1^x \frac{t^2}{t^2+1} dt$

25. $F(x) = \int_{-1}^x \sqrt{t^4 + 1} dt$

26. $F(x) = \int_1^x \sqrt[4]{t} dt$

Find $F'(x)$.

27. $F(x) = \int_x^{x+2} (4t + 1) dt$

28. $F(x) = \int_{-x}^x t^3 dt$

29. $F(x) = \int_0^{\sin(x)} \sqrt{t} dt$

30. $F(x) = \int_2^{x^2} \frac{1}{t^3} dt$

31. $F(x) = \int_0^{x^3} \sin(t^2) dt$

32. $F(x) = \int_0^{x^2} \sin(\theta^2) d\theta$

Find the average value over the given interval and all values of x for which the function equals its average value.

33. $f(x) = 4 - x^2, [-2,2]$

34. $f(x) = \frac{4(x^2+1)}{x^2}, [1,3]$

35. $f(x) = 2e^x, [-1,1]$

36. $f(x) = \frac{1}{2x}, [1,4]$

37. $f(x) = \sin(x)$, $[0, \pi]$

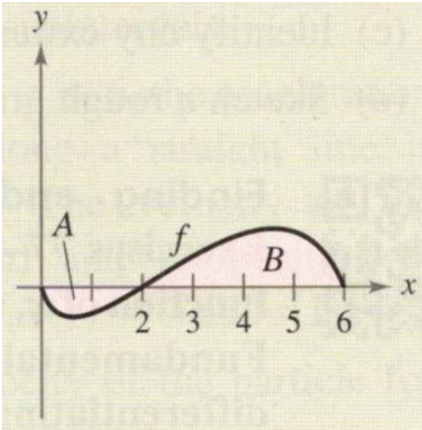
38. $f(x) = \cos(x)$, $\left[0, \frac{\pi}{2}\right]$

39. The force F (in Newtons) of a hydraulic cylinder in a press is directly proportional to the square of $\sec(x)$, where x is the distance (in meters) that the cylinder is extended in its cycle. The domain of F is $\left[0, \frac{\pi}{3}\right]$, and $F(0) = 500$ N.

a. Find F as a function of x .

b. Find the average force exerted by the press over the interval $\left[0, \frac{\pi}{3}\right]$.

40. The graph of f is shown in the figure. The shaded region A has an area of 1.5, and $\int_0^6 f(x) dx = 3.5$. Use this information to answer the questions.



- a. $\int_0^2 f(x) dx =$
- b. $\int_2^6 f(x) dx =$
- c. $\int_0^6 |f(x)| dx =$
- d. $\int_0^2 -2f(x) dx =$
- e. $\int_0^6 [2 + f(x)] dx =$
- f. The average value of f over the interval $[0, 6]$ is _____ .

A particle is moving along a straight line. Its velocity function, given in feet per second, is given below. Find the displacement and total distance travelled over the given interval.

41. $v(t) = t^3 - 10t^2 + 27t - 18, 1 \leq t \leq 7$

A particle is moving along a straight line. Its velocity function, given in feet per second, is given below. Find the displacement and total distance travelled over the given interval.

42. $v(t) = t^3 - 8t^2 + 15t, \quad 0 \leq t \leq 5$

43. Water flows from a storage tank at a rate of $(500 - 5t)$ liters per minute. Find the amount of water that flows out of the tank during the first 18 minutes.

44. **Multiple Choice:**

If $f(x) = x^3$ has an average value of 12 on the interval $[0, k]$, then $k =$

- a) $\sqrt[3]{12}$ b) $\sqrt[4]{48}$ c) $\sqrt[3]{24}$ d) $\sqrt[3]{48}$

45. **Multiple Choice:**

$$\frac{d}{dx} \left[\int_0^{x^2} e^{t^2} dt \right] =$$

- a) 0 b) $2e^{x^4}$ c) $2xe^{x^4}$ d) e^{x^4}

46. Free Response:

The velocity $v(t)$ (in feet per second) of a high-speed rail train is positive over $0 \leq t \leq 60$ seconds. The velocities at a time t are given as ordered pairs $(t, v(t))$: $(0,0)$, $(10,45)$, $(20,105)$, $(30,140)$, $(40,165)$, $(50,195)$, $(60,210)$.

a) Estimate the acceleration of the train at $t = 25$ seconds. Indicate units of measure.

b) Use a Left Riemann Sum with three subintervals of equal length to approximate $\int_{20}^{50} v(t) dt$. Using correct units, explain the meaning of the integral in the context of the problem.

c) Evaluate $\int_{20}^{50} v'(t) dt$. Using correct units, explain the meaning of the integral in the context of this problem.

d) Estimate the average velocity of the train over the 60-second time period of time using a Midpoint Riemann Sum with 3 subintervals.

47. Free Response:

For $0 \leq t \leq 6$, the acceleration of a particle moving along a straight line is given by $a(t) = 2t - 6$. The velocity of the particle is given by $v(t)$ and its position is given by $s(t)$. When $t = 1$, $v(1) = 3$, and $s(1) = \frac{4}{3}$.

- a) Find the average velocity of the particle for the time period $0 \leq t \leq 6$.
- b) When is the particle moving to the left? Explain.
- c) Find the total distance travelled by the particle from time $t = 0$ to $t = 6$.
- d) Find the time t at which the particle is farthest to the left. Explain.

5.5 Worksheet (Odds, 32, 34, 36, 38)

Find the indefinite integral and check the result by differentiation.

1. $\int 6(1 + 6x)^4 dx$

2. $\int (2x)(x^2 - 9)^3 dx$

3. $\int -2x\sqrt{25 - x^2} dx$

4. $\int -8x\sqrt[3]{3 - 4x^2} dx$

5. $\int x^3(x^4 + 3)^2 dx$

6. $\int x^2(6 - x^3)^5 dx$

7. $\int x^2(2x^3 - 1)^4 dx$

8. $\int x(5x^2 + 4)^3 dx$

9. $\int \frac{x}{\sqrt{1-x^2}} dx$

10. $\int \frac{x^3}{\sqrt{1+x^4}} dx$

11. $\int \left(1 + \frac{1}{t}\right)^3 \left(\frac{1}{t^2}\right) dt$

12. $\int \left[x^2 + \frac{1}{(3x)^2}\right] dx$

Find the indefinite integral.

13. $\int \pi \sin(\pi x) dx$

14. $\int \sin(4x) dx$

15. $\int \frac{1}{\theta^2} \cos\left(\frac{1}{\theta}\right) d\theta$

16. $\int x \sin(x^2) dx$

17. $\int \frac{5-e^x}{e^{2x}} dx$

18. $\int \frac{e^{2x}+2e^x+1}{e^x} dx$

19. $\int e^{-x} \sec^2(e^{-x}) dx$

20. $\int \ln(e^{2x-1}) dx$

Evaluate the definite integral. Use your calculator to verify.

21. $\int_1^2 2x^2 \sqrt{x^3 + 1} dx$

22. $\int_0^1 x \sqrt{1 - x^2} dx$

23. $\int_0^4 \frac{1}{\sqrt{2x+1}} dx$

24. $\int_0^2 \frac{x}{\sqrt{1+2x^2}} dx$

25. $\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})^2} dx$

26. $\int_1^5 \frac{x}{\sqrt{2x-1}} dx$

27. $\int_1^3 \frac{e^{3/x}}{x^2} dx$

28. $\int_0^{\sqrt{2}} x e^{-x^2/2} dx$

Write the integral as the sum of the integral of an odd function and the integral of an even function. Use this simplification to evaluate the integral.

29. $\int_{-3}^3 (x^3 + 4x^2 - 3x - 6) dx$

30. $\int_{-\pi/2}^{\pi/2} (\sin(4x) + \cos(4x)) dx$

You are asked to find one of the integrals. Which one would you choose? Explain.

31. $\int \sqrt{x^3 + 1} dx$ or $\int x^2 \sqrt{x^3 + 1} dx$

32. $\int \tan(3x) \sec^2(3x) dx$ or $\int \tan(3x) dx$

Rewriting the integral:

33. Show that $\int_0^1 x^2(1-x)^5 dx = \int_0^1 x^5(1-x)^2 dx$

34. $\int_0^1 x^a(1-x)^b dx = \int_0^1 x^b(1-x)^a dx$

35. **Multiple Choice:**

$$\int x\sqrt{16-3x^2} dx =$$

a) $\frac{2}{3}(16-3x^2)^{3/2} + C$

b) $-\frac{1}{4}(16-3x^2)^{\frac{3}{2}} + C$

c) $-\frac{1}{9}(16-3x^2)^{3/2} + C$

d) $\frac{1}{9}(16-3x^2)^{3/2} + C$

36. **Multiple Choice:**

What is the average value of $h(x) = (\sec^2(x))(1 + 2 \tan(x))^3$ on the interval $\left[0, \frac{\pi}{4}\right]$?

- a) $\frac{40}{\pi}$ b) 10 c) $\frac{81}{2\pi}$ d) $\frac{80}{\pi}$

37. **Multiple Choice:**

A chemical is leaking from a storage area at a rate of $f(t) = 1600e^{-0.12t}$ gallons per hour. How many gallons of the chemical leak out of the storage area from time $t = 0$ hours to $t = 12$ hours?

- a) 379 gallons b) 1221 gallons c) 3159 gallons d) 10174 gallons

38. Free Response:

The function f is defined by $f(x) = \sqrt{100 - x^2}$ for $-10 \leq x \leq 10$.

- Find $f'(x)$.
- Find an equation of the tangent line to the graph of f at $x = -6$.
- Let g be the function defined by

$$g(x) = \begin{cases} f(x), & -10 \leq x \leq -6 \\ -\frac{1}{2}x + 5, & -6 < x \leq 10 \end{cases}$$

Is g continuous at $x = -6$? Use the definition of continuity to explain your answer.

- Find the value of $\int_0^{10} x\sqrt{100 - x^2} dx$.

5.6 Worksheet (Odds, 20, 22)

Find the indefinite integral.

1. $\int \frac{dx}{\sqrt{9-x^2}}$

2. $\int \frac{dx}{\sqrt{1-4x^2}}$

3. $\int \frac{1}{x\sqrt{4x^2-1}} dx$

5. $\int \frac{12}{1+9x^2} dx$

7. $\int \frac{t}{t^4+25} dt$

8. $\int \frac{1}{x\sqrt{x^4-4}} dx$

9. $\int \frac{e^{2x}}{4+e^{4x}} dx$

10. $\int \frac{2}{x\sqrt{9x^2-25}} dx$

11. $\int \frac{1}{\sqrt{x}\sqrt{1-x}} dx$

12. $\int \frac{3}{2\sqrt{x}(1+x)} dx$

13. $\int \frac{x-3}{x^2+1} dx$

14. $\int \frac{x^2+3}{x\sqrt{x^2-4}} dx$

Find or evaluate the integral by completing the square.

15. $\int \frac{2x}{x^2+6x+13} dx$

16. $\int \frac{2x-5}{x^2+2x+2} dx$

17. $\int \frac{1}{\sqrt{-x^2-4x}} dx$

18. $\int \frac{2}{\sqrt{-x^2+4x}} dx$

19. **Multiple Choice:**

Which integral has a value of π ?

I. $\int_{5/2}^5 \frac{2}{\sqrt{5x-x^2}} dx$

II. $\int_{\pi}^{2\pi} \frac{\pi}{2} \sin\left(\frac{x}{2}\right) dx$

III. $\int_3^8 \frac{\pi}{2\sqrt{x+1}} dx$

a) I and II only

b) I and III only

c) II and III only

d) I, II and III

20. **Multiple Choice:**

Which of the following is the solution of the differential equation?

$$y' = \frac{4}{\sqrt{16-x^2}}$$

a) $y = \arcsin\left(\frac{x}{4}\right) + C$

b) $y = 4 \arcsin\left(\frac{x}{4}\right) + C$

c) $y = \frac{1}{4} \arcsin\left(\frac{x}{16}\right) + C$

d) $y = 16 \arcsin\left(\frac{x}{4}\right) + C$

21. **Free Response:**

Let $f(x) = \arccos(x)$ and let $g(x) = x^2$. Define $h(x) = f(g(x))$.

- a. At what values of x does $h(x)$ have a relative maximum?
- b. Write, but do not evaluate, an expression that can determine the area of the region bounded by the graphs of $h(x)$ and the horizontal line $y = \frac{\pi}{3}$.
- c. Evaluate $\frac{d}{dx} \left[f^{-1} \left(\frac{\pi}{3} \right) \right]$.

22. Free Response:

The function f is defined by

$$f(x) = \frac{1}{\sqrt{4-x^2}}$$

for $-2 < x < 2$. Let g be the function defined by

$$g(x) = \begin{cases} f(x), & -2 < x \leq 0 \\ x + \frac{1}{2}, & 0 < x \leq 2 \end{cases}$$

- Is g continuous at $x = 0$? Use the definition of continuity to explain your answer.
- What is the area of the region bounded by f , the x -axis, the y -axis, and the line $x = 1$?
- Find the value of $\int_{-1}^1 g(x) dx$.