AP Calculus BC

Name: answers / solutions

5.4 Worksheet (Odds, 40, 44, 46)

Evaluate the definite integral. Use your calculator to verify your result.

1. $\int_0^2 6x\,dx$ $\boxed{12}$

2. $\int_{-3}^1 8\,dt = \left[8t\right]_{-3}^{1}$

$= (8(1) - (8(-3)) = 8 + 24 = \boxed{32}$

3. $\int_{-1}^0 (2x-1)dx$

$\boxed{-2}$

4. $\int_{-1}^2 (7-3t)dt = \left[7t - \frac{3}{2}t^2\right]_{-1}^{2}$

$= \left[7(2) - \frac{3}{2}(2)^2\right] - \left[7(-1) - \frac{3}{2}(-1)^2\right]$

$= 8 - \left(-\frac{17}{2}\right) = \boxed{\frac{33}{2}}$

5. $\int_{-1}^1 (t^2-2)dt$

$\boxed{\frac{-10}{3}}$

6. $\int_1^2 (6x^2-3x)dx = \left[2x^3 - \frac{3}{2}x^2\right]_1^2$

$\left[2(2)^3 - \frac{3}{2}(2)^2\right] - \left[2(1)^3 - \frac{3}{2}(1)^2\right]$

$= 10 - \frac{1}{2} = \boxed{\frac{19}{2}}$

7. $\int_0^1 (2t-1)^2 dt$

$\boxed{\frac{1}{3}}$

8. $\int_1^3 (4x^3-3x^2)dx$

$= \left[x^4 - x^3\right]_1^3$

$= \left[(3)^4 - (3)^3\right] - \left[(1)^4 - (1)^3\right]$

$= 54 - 0 = \boxed{54}$

9. $\int_1^2 \left(\frac{3}{x^2} - 1\right) dx$

$\boxed{\frac{1}{2}}$

10. $\int_{-2}^{-1} \left(u - \frac{1}{u^2}\right) du$

$= \int_{-2}^{-1} (u - u^{-2})du$

$= \left[\frac{1}{2}u^2 + \frac{1}{u}\right]_{-2}^{-1}$

$= \left[\frac{1}{2}(-1)^2 + \frac{1}{(-1)}\right] - \left[\frac{1}{2}(-2)^2 + \frac{1}{(-2)}\right]$

$= -\frac{1}{2} - \frac{3}{2}$

$= \boxed{-2}$

11. $\int_1^4 \frac{u-2}{\sqrt{u}}\,du$

$\boxed{\frac{2}{3}}$

12. $\int_{-8}^{8} x^{1/3}\,dx$

$= \left(\frac{3}{4}x^{4/3}\right]_{-8}^{8}$

$= \left[\frac{3}{4}\left(\sqrt[3]{8}\right)^4\right] - \left[\frac{3}{4}\left(\sqrt[3]{-8}\right)^4\right]$

$= 12 \quad - 12$

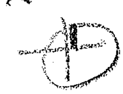
$= \boxed{0}$

13. $\int_{-\pi/6}^{\pi/6} \sec^2(x)\,dx$

$\boxed{\frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3} \approx 1.1547}$

14. $\int_{\pi/4}^{\pi/2} (2 - \csc^2(x))\,dx = \int_{\pi/4}^{\pi/2} 2\,dx - \int_{\pi/4}^{\pi/2} \csc^2 x\,dx$

$\left[2x\right]_{\pi/4}^{\pi/2} - \left[(-\cot x)\right]_{\pi/4}^{\pi/2}$



$\left[2\left(\frac{\pi}{2}\right) - 2\left(\frac{\pi}{4}\right)\right] + \left[\frac{\cos(\pi/2)}{\sin(\pi/2)} - \frac{\cos(\pi/4)}{\sin(\pi/4)}\right]$

$\frac{\pi}{2} + \left(\frac{0}{1} - \frac{(\sqrt{2}/2)}{\sqrt{2}/2}\right)$

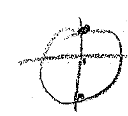
$\boxed{\frac{\pi}{2} - 1 \approx 0.5708}$

15. $\int_{-\pi/3}^{\pi/3} 4\sec(\theta)\tan(\theta)\,d\theta$

$\boxed{0}$

16. $\int_{-\pi/2}^{\pi/2} (2t + \cos(t))\,dt$

$\left[t^2 + \sin t\right]_{-\pi/2}^{\pi/2}$



$\left[\left(\frac{\pi}{2}\right)^2 + \sin\left(\frac{\pi}{2}\right)\right] - \left[\left(-\frac{\pi}{2}\right)^2 + \sin\left(-\frac{\pi}{2}\right)\right]$

$\left(\frac{\pi^2}{4} + 1\right) - \left(\frac{\pi^2}{4} - 1\right)$

$= \boxed{2}$

17. $\int_0^2 (2^x + 6)dx$

$$\boxed{\frac{3}{\ln 2} + 12 \approx 16.32808}$$

18. $\int_0^3 (t - 5^t)dt$

$$\left[\frac{1}{2}t^2 - \frac{5^t}{\ln 5}\right]_0^3$$

$$\left[\frac{1}{2}(3)^2 - \frac{5^{(3)}}{\ln 5}\right] - \left(\frac{1}{2}(0)^2 - \frac{5^{(0)}}{\ln 5}\right]$$

$$\left[\frac{9}{2} - \frac{125}{\ln 5}\right) - \left[0 - \frac{1}{\ln 5}\right)$$

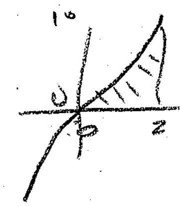
$$\boxed{\frac{9}{2} - \frac{124}{\ln 5} \approx -72.5455}$$

Find the area of the region bounded by the graphs of the equations.

19. $y = 5x^2 + 2, \quad x = 0, \quad x = 2, \quad y = 0$

$$\boxed{\frac{52}{3}\ \text{unit}^2}$$

20. $y = x^3 + x, \quad x = 2, \quad y = 0$



$$A = \int_0^2 (x^3 + x)dx$$

$$= \left[\frac{1}{4}x^4 + \frac{1}{2}x^2\right]_0^2$$

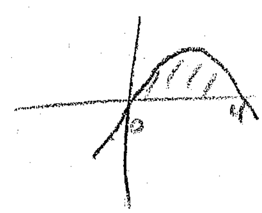
$$= \left[\frac{1}{4}(2)^4 + \frac{1}{2}(2)^2\right] - \left[\frac{1}{4}(0)^4 + \frac{1}{2}(0)^2\right]$$

$$= 6 - 0 = \boxed{6\ \text{units}^2}$$

21. $y = 1 + \sqrt[3]{x}, \quad x = 0, \quad x = 8, \quad y = 0$

$$\boxed{20\ \text{units}^2}$$

22. $y = -x^2 + 4x, \quad y = 0$



$$A = \int_0^4 (-x^2 + 4x)dx$$

$$= \left[-\frac{1}{3}x^3 + 2x^2\right]_0^4$$

$$= \left[-\frac{1}{3}(4)^3 + 2(4)^2\right] - [0]$$

$$= \boxed{\frac{32}{3}\ \text{units}^2}$$

Use the Second Fundamental Theorem of Calculus to find $F'(x)$.

23. $F(x) = \int_{-2}^{x} (t^2 - 2t)dt$

$\boxed{x^2 - 2x}$

24. $F(x) = \int_{1}^{x} \frac{t^2}{t^2+1}\, dt$

$F'(x) = \frac{d}{dx}\left[\int_1^x \frac{t^2}{t^2+1}\, dt\right]$

$= \boxed{\frac{x^2}{x^2+1}}$

25. $F(x) = \int_{-1}^{x} \sqrt{t^4+1}\, dt$

$\boxed{\sqrt{x^4+1}}$

26. $F(x) = \int_{1}^{x} \sqrt[4]{t}\, dt$

$F'(x) = \frac{d}{dx}\left[\int_1^x \sqrt[4]{t}\, dt\right]$

$= \boxed{\sqrt[4]{x}}$

Find $F'(x)$.

27. $F(x) = \int_{x}^{x+2} (4t+1)dt$

$\boxed{8}$

28. $F(x) = \int_{-x}^{x} t^3\, dt$

$F'(x) = \frac{d}{dx}\left[\int_{-x}^{0} t^3 dt + \int_0^x t^3 dt\right]$ (0 = any constant between $-x$ & x)

$= \frac{d}{dx}\left[-\int_0^{-x} t^3 dt + \int_0^x t^3 dt\right]$

$= -\frac{d}{dx}\left[\int_0^{-x} t^3 dt\right] + \frac{d}{dx}\left[\int_0^x t^3 dt\right]$

$= -\left[(-x)^3 \frac{d}{dx}[-x]\right] + (x)^3$

$= -((-x^3)(-1)) + x^3 = -x^3 + x^3 = \boxed{0}$

29. $F(x) = \int_{0}^{\sin(x)} \sqrt{t}\, dt$

$\boxed{\sqrt{\sin x}\,(\cos(x))}$

30. $F(x) = \int_{2}^{x^2} \frac{1}{t^3}\, dt$

$F'(x) = \frac{d}{dx}\left[\int_2^{x^2} \frac{1}{t^3} dt\right]$

$= \frac{1}{(x^2)^3} \frac{d}{dx}[x^2]$

$= \frac{1}{x^6} 2x = \frac{2x}{x^6} = \boxed{\frac{2}{x^5}}$

31. $F(x) = \int_0^{x^3} \sin(t^2)\ dt$

$$\boxed{3x^2 \sin(x^6)}$$

32. $F(x) = \int_0^{x^2} \sin(\theta^2)\ d\theta$

$$F'(x) = \frac{d}{dx}\left[\int_0^{x^2} \sin(\theta^2)\,d\theta\right]$$
$$= \sin[(x^2)^2]\frac{d}{dx}[x^2]$$
$$= \sin(x^4)(2x)$$
$$= \boxed{2x\sin(x^4)}$$

Find the average value over the given interval and all values of x for which the function equals its average value.

33. $f(x) = 4 - x^2,\ [-2,2]$

$$\boxed{Av = \frac{8}{3}}$$

$$\boxed{x = \frac{2}{\sqrt{3}},\ x = -\frac{2}{\sqrt{3}}}$$

34. $f(x) = \frac{4(x^2+1)}{x^2},\quad [1,3]$

$$Av = \frac{1}{3-1}\int_1^3 (4+4\cdot x^{-2})dx$$
$$= \frac{1}{2}\left[4x - \frac{4}{x}\right]_1^3$$
$$= \frac{1}{2}\left(\left[4(3) - \frac{4}{(3)}\right] - \left[4(1) - \frac{4}{(1)}\right]\right)$$
$$= \frac{1}{2}\left(12 - \frac{4}{3} - 4 + 4\right) = \boxed{\frac{16}{3}}$$
$$f(x) = \frac{16}{3}$$
$$4 + \frac{4}{x^2} = \frac{16}{3}$$
$$\frac{4}{x^2} = \frac{4}{3},\ x^2 = \frac{4}{(4/3)} = 3,\ x = \pm\sqrt{3}$$
$$\boxed{x = \sqrt{3}}$$
$(-\sqrt{3}$ not in interval$)$

35. $f(x) = 2e^x,\quad [-1,1]$

$$\boxed{Av = e - \frac{1}{e} \approx 2.3504}$$

$$\boxed{x = \ln\left(\frac{1}{2}\left(e - \frac{1}{e}\right)\right) \approx 0.1614}$$

36. $f(x) = \frac{1}{2x},\quad [1,4]$

$$Av = \frac{1}{4-1}\int_1^4 \frac{1}{2}\frac{1}{x}dx$$
$$= \frac{1}{6}\int_1^4 \frac{1}{x}dx = \frac{1}{6}\left[\ln|x|\right]_1^4$$
$$= \frac{1}{6}(\ln 4 - \ln 1) = \boxed{\frac{1}{6}\ln 4 \approx 0.2310}$$
$$f(x) = \frac{1}{6}\ln 4$$
$$\frac{1}{2x} = \frac{\ln 4}{6}$$
$$2x\ln 4 = 6$$
$$\boxed{x = \frac{6}{2\ln 4} \approx 2.164}$$
$$\left(\text{or } \frac{3}{\ln 4}\right)$$

37. $f(x) = \sin(x), \quad [0, \pi]$

$$\boxed{Av = \frac{2}{\pi}}$$

$$\boxed{\begin{array}{l} x = \sin^{-1}\left(\frac{2}{\pi}\right) \simeq 0.6901 \\ \text{also } x = \pi - 0.6901 = 2.451 \end{array}}$$

38. $f(x) = \cos(x), \quad \left[0, \frac{\pi}{2}\right]$

$$Av = \frac{1}{\pi/2 - 0} \int_0^{\pi/2} \cos x \, dx$$

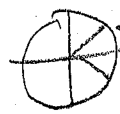
$$= \frac{2}{\pi} \left[\sin x\right]_0^{\pi/2}$$

$$= \frac{2}{\pi} \left[\sin \pi/2 - \sin 0\right] = \frac{2}{\pi}(1 - 0) = \boxed{\frac{2}{\pi}}$$

$$f(x) = \frac{2}{\pi}$$

$$\cos x = \frac{2}{\pi}$$

$$\boxed{x = \cos^{-1}\left(\frac{2}{\pi}\right) \simeq 0.88069}$$



0.88069

this angle also has $\cos\theta = \frac{2}{\pi}$ but is not in the interval

39. The force F (in Newtons) of a hydraulic cylinder in a press is directly proportional to the square of $\sec(x)$, where x is the distance (in meters) that the cylinder is extended in its cycle. The domain of F is $\left[0, \frac{\pi}{3}\right]$, and $F(0) = 500\ N$.

a. Find F as a function of x.

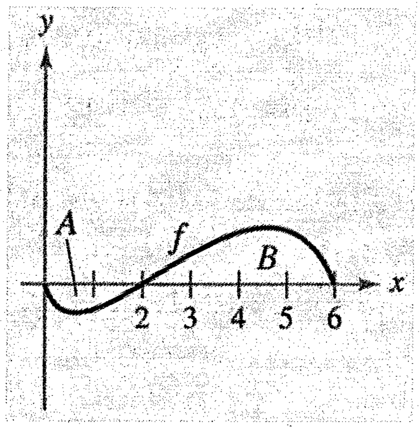
b. Find the average force exerted by the press over the interval $\left[0, \frac{\pi}{3}\right]$.

(a) $\boxed{F(x) = 500 \sec^2(x)}$

(b) $\boxed{AV = \frac{1500\sqrt{3}}{\pi} \simeq 826.9933\ N}$

40. The graph of f is shown in the figure. The shaded region A has an area of 1.5, and $\int_0^6 f(x)\,dx = 3.5$. Use this information to answer the questions.

$\int_0^2 f(x)\,dx = -1.5$



a. $\int_0^2 f(x)\,dx = \boxed{-1.5}$

b. $\int_2^6 f(x)\,dx = \int_0^6 f(x)dx - \int_0^2 f(x)dx = 3.5 - (-1.5) = \boxed{5}$

c. $\int_0^6 |f(x)|\,dx = \int_0^2 |f(x)|dx + \int_2^6 |f(x)|dx = |-1.5| + |5| = 1.5 + 5 = \boxed{6.5}$

d. $\int_0^2 -2f(x)\,dx = -2\int_0^2 f(x)dx = -2(-1.5) = \boxed{3}$

e. $\int_0^6 [2 + f(x)]\,dx = \int_0^6 2dx\,dx + \int_0^6 f(x)dx = [2x]_0^6 + 3.5 = 2(6-0) + 3.5 = \boxed{15.5}$

f. The average value of f over the interval [0, 6] is ________ .

$Av = \frac{1}{6-0}\int_0^6 f(x)dx = \frac{1}{6}(3.5) = \boxed{\frac{7}{12}}$

A particle is moving along a straight line. Its velocity function, given in feet per second, is given below. Find the displacement and total distance travelled over the given interval.

41. $v(t) = t^3 - 10t^2 + 27t - 18,\ \ 1 \le t \le 7$

displacement $= \boxed{0 \text{ ft}}$

total distance travelled $= \boxed{31.5 \text{ ft}}$

A particle is moving along a straight line. Its velocity function, given in feet per second, is given below. Find the displacement and total distance travelled over the given interval.

42. $v(t) = t^3 - 8t^2 + 15t, \quad 0 \le t \le 5$

$s(t) = \int (t^3 - 8t^2 + 15t)\,dt$

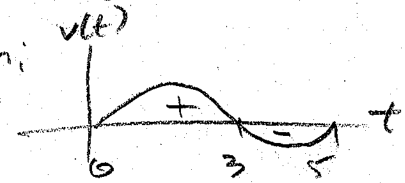
$s(t) = \frac{1}{4}t^4 - \frac{8}{3}t^3 + \frac{15}{2}t^2 + c$

displacement $= s(5) - s(0) = \left(\frac{125}{12} + c\right) - (0 + c) = \boxed{\frac{125}{12} \approx 10.417 \text{ ft}}$

by calculator

total distance travelled $= \int_0^5 |v(t)|\,dt$

by calculator graph:



total distance traveled $= \int_0^3 (t^3 - 8t^2 + 15t)\,dt - \int_3^5 (t^3 - 8t^2 + 15t)\,dt$

(math 9) (math 9)

$\frac{64}{4} - \left(-\frac{16}{3}\right) = \boxed{\frac{253}{12} \approx 21.083 \text{ ft}}$

43. Water flows from a storage tank at a rate of (500 – 5t) liters per minute. Find the amount of water that flows out of the tank during the first 18 minutes.

$\boxed{8190 \text{ liters}}$

44. **Multiple Choice:**

If $f(x) = x^3$ has an average value of 12 on the interval [0, k], then k =

a) $\sqrt[3]{12}$ b) $\sqrt[4]{48}$ c) $\sqrt[3]{24}$ $\boxed{\text{d) } \sqrt[3]{48}}$

$$Av = \frac{1}{k-0}\int_0^k x^3 dx = 12$$

$$\frac{1}{k}\left[\frac{1}{4}x^4\right]_0^k = 12$$

$$\frac{1}{k}\left[\frac{1}{4}k^4 - 0\right] = 12$$

$$\frac{1}{4}k^3 = 12$$

$$k^3 = 48, \quad k = \sqrt[3]{48}$$

45. **Multiple Choice:**

$$\frac{d}{dx}\left[\int_0^{x^2} e^{t^2}\,dt\right] =$$

a) 0 b) $2e^{x^4}$ $\boxed{\text{c) } 2xe^{x^4}}$ d) e^{x^4}

(don't forget the chain rule)

46. **Free Response:**

The velocity $v(t)$ (in feet per second) of a high-speed rail train is positive over $0 \le t \le 60$ seconds. The velocities at a time t are given as ordered pairs $(t, v(t))$: $(0,0), (10,45), (20,105), (30,140), (40,165), (50,195), (60,210)$.

a) Estimate the acceleration of the train at t = 25 seconds. Indicate units of measure.

b) Use a Left Riemann Sum with three subintervals of equal length to approximate $\int_{20}^{50} v(t)\ dt$. Using correct units, explain the meaning of the integral in the context of the problem.

c) Evaluate $\int_{20}^{50} v'(t)\ dt$. Using correct units, explain the meaning of the integral in the context of this problem.

d) Estimate the average velocity of the train over the 60-second time period of time using a Midpoint Riemann Sum with 3 subintervals.

(a) $a(25) \approx \frac{v(30)-v(20)}{30-20} = \frac{140-105}{10} = \boxed{3.5\ ft/sec^2}$

(b)

interval	x_i	$f(x_i) \cdot \Delta x$	= area
[20,30)	20	105 · 10	= 1050
[30,40)	30	140 · 10	= 1400
[40,50)	40	165 · 10	= 1650
			$\boxed{4100\ ft}$

This is the approximate displacement of the train from t = 20 to t = 30 seconds.

(c) $\int_{20}^{50} v'(t)dt = v(50) - v(20)$

$= 195 - 105$

$= \boxed{90\ ft/sec}$

This is the amount the velocity of the train increased from t = 20 to t = 50 seconds.

(d) $Av = \frac{1}{60-0}\underbrace{\int_0^{60} v(t)dt}_{\text{Riemann Sum for this}}$

interval	x_i	$f(x_i) \cdot \Delta x$	= area
[0,20)	10	45 · 20	= 900
(20,40)	30	140 · 20	= 2800
[40,60)	50	195 · 20	= 3900
			$\boxed{7600}$

(total distance traveled)

$Av = \frac{7600}{60-0} = \boxed{126.667\ ft/sec}$

47. **Free Response:**

For $0 \leq t \leq 6$, the acceleration of a particle moving along a straight line is given by $a(t) = 2t - 6$. The velocity of the particle is given by $v(t)$ and its position is given by $s(t)$. When $t = 1$, $v(1) = 3$, and $s(1) = \frac{4}{3}$.

a) Find the average velocity of the particle for the time period $0 \leq t \leq 6$.

b) When is the particle moving to the left? Explain.

c) Find the total distance travelled by the particle from time $t = 0$ to $t = 6$.

d) Find the time t at which the particle is farthest to the left. Explain.

(a) $\boxed{0}$

(b) The partical is moving to the left when $v(t) < 0$ which occurs from $t = 2$ to $t = 4$.

(c) $\boxed{\frac{44}{3}}$

(d) $\boxed{t = 0}$

AP Calculus BC

Name: ______________________

5.5 Worksheet (Odds, 32, 34, 36, 38)

Find the indefinite integral and check the result by differentiation.

1. $\int 6(1+6x)^4\,dx$

$\boxed{\frac{1}{5}(1+6x)^5+C}$

2. $\int (2x)(x^2-9)^3\,dx$

$u=x^2-9$

$\frac{du}{dx}=2x$

$du=2x\,dx$

$\int u^3\,du$

$\frac{1}{4}u^4+C$

$\boxed{\frac{1}{4}(x^2-9)^4+C}$

$\frac{d}{dx}\left[\frac{1}{4}(x^2-9)^4+C\right]=(x^2-9)^3(2x)$ ✓

3. $\int -2x\sqrt{25-x^2}\,dx$

$\boxed{\frac{2}{3}(25-x^2)^{3/2}+C}$

4. $\int -8x\sqrt[3]{3-4x^2}\,dx$

$u=3-4x^2$

$\frac{du}{dx}=-8x$

$du=-8x\,dx$

$\int u^{1/3}\,du$

$\frac{u^{4/3}}{(4/3)}+C$

$\frac{3}{4}u^{4/3}+C=\boxed{\frac{3}{4}(3-4x^2)^{4/3}+C}$

$\frac{d}{dx}\left[\frac{3}{4}(3-4x^2)^{4/3}+C\right]=(3-4x^2)^{1/3}(-8x)$ ✓

5. $\int x^3(x^4+3)^2\,dx$

$\boxed{\frac{1}{12}(x^4+3)^3+C}$

6. $\int x^2(6-x^3)^5\,dx$

$u=6-x^3$

$\frac{du}{dx}=-3x^2$

$du=-3x^2\,dx$

$x^2\,dx=-\frac{1}{3}du$

$-\frac{1}{3}\int u^5\,du$

$-\frac{1}{3}\left(\frac{1}{6}u^6\right)+C$

$\boxed{-\frac{1}{18}(6-x^3)^6+C}$

$\frac{d}{dx}\left[-\frac{1}{18}(6-x^3)^6+C\right]=-\frac{1}{18}(6)(6-x^3)^5(-3x^2)$

$=x^2(6-x^3)^5$ ✓

7. $\int x^2(2x^3-1)^4\,dx$

$\boxed{\frac{1}{30}(2x^3-1)^5+C}$

8. $\int x(5x^2+4)^3\,dx$

$u=5x^2+4$

$\frac{du}{dx}=10x$

$du=10x\,dx$

$x\,dx=\frac{1}{10}\,du$

$\frac{1}{10}\int u^3\,du$

$\frac{1}{10}\left(\frac{1}{4}u^4\right)+C$

$\boxed{\frac{1}{40}(5x^2+4)^4+C}$

$\frac{d}{dx}\left[\frac{1}{40}(5x^2+4)^4+C\right]=\frac{1}{40}(4)(5x^2+4)^3(10x)$

$=x(5x^2+4)^3$ ✓

9. $\int \frac{x}{\sqrt{1-x^2}}\,dx$

$\boxed{-\sqrt{1-x^2}+C}$

10. $\int \frac{x^3}{\sqrt{1+x^4}}\,dx$

$u=1+x^4$

$\frac{du}{dx}=4x^3$

$du=4x^3\,dx$

$x^3\,dx=\frac{1}{4}\,du$

$\frac{1}{4}\int u^{-1/2}\,du$

$\frac{1}{4}\left[\frac{u^{1/2}}{(1/2)}\right]+c$

$\frac{1}{2}(1+x^4)^{1/2}+C$

$=\boxed{\frac{1}{2}\sqrt{1+x^4}+C}$

$\frac{d}{dx}\left[\frac{1}{2}(1+x^4)^{1/2}+C\right]=\frac{1}{2}\left(\frac{1}{2}\right)(1+x^4)^{-1/2}4x^3$

$=x^3(1+x^4)^{-1/2}=\frac{x^3}{\sqrt{1+x^4}}$ ✓

11. $\int\left(1+\frac{1}{t}\right)^3\left(\frac{1}{t^2}\right)dt$

$\boxed{-\frac{1}{4}\left(1+\frac{1}{t}\right)^4+C}$

12. $\int\left[x^2+\frac{1}{(3x)^2}\right]dx$

x^2 & $\frac{1}{(3x)^2}$ are always positive so

$\left|x^2+\frac{1}{(3x)^2}\right|=x^2+\frac{1}{(3x)^2}$

$u=3x$

$\frac{du}{dx}=3$

$du=3dx$

$dx=\frac{1}{3}du$

$\int\left(x^2+\frac{1}{(3x)^2}\right)dx$

$\int x^2dx+\frac{1}{3}\int u^{-2}\,du$

$\frac{1}{3}x^3+\frac{1}{3}\left(\frac{u^{-1}}{-1}\right)+c$

$\boxed{\frac{1}{3}x^3-\frac{1}{9x}+C}$

$\frac{d}{dx}\left[\frac{1}{3}x^3-\frac{1}{9}x^{-1}+C\right]=x^2+\frac{1}{9}x^{-2}$

$=x^2+\frac{1}{(3x)^2}$ ✓

Find the indefinite integral.

13. $\int \pi \sin(\pi x)\, dx$

$\boxed{-\cos(\pi x)+c}$

14. $\int \sin(4x)\, dx$

$u=4x$

$\frac{du}{dx}=4$

$du=4dx$

$dx=\frac{1}{4}du$

$\frac{1}{4}\int \sin u\, du$

$\frac{1}{4}(-\cos u)+c$

$\boxed{-\frac{1}{4}\cos(4x)+c}$

$\frac{d}{dx}[-\frac{1}{4}\cos(4x)+c] = -\frac{1}{4}(\sin(4x)\cdot 4)$

$= \sin(4x)$ ✓

15. $\int \frac{1}{\theta^2}\cos\left(\frac{1}{\theta}\right) d\theta$

$\boxed{-\sin\left(\frac{1}{\theta}\right)+c}$

16. $\int x\sin(x^2)\, dx$

$u=x^2$

$\frac{du}{dx}=2x$

$du=2x\,dx$

$x\,dx=\frac{1}{2}du$

$\frac{1}{2}\int \sin u\, du$

$\frac{1}{2}(-\cos u)+c$

$\boxed{-\frac{1}{2}\cos(x^2)+c}$

$\frac{d}{dx}[-\frac{1}{2}\cos(x^2)+c)$

$= -\frac{1}{2}(-\sin(x^2)2x) = x\sin(x^2)$ ✓

17. $\int \frac{5-e^x}{e^{2x}}\, dx$

$\boxed{-\frac{5}{2}e^{(-2x)}+e^{(-x)}+c}$

18. $\int \frac{e^{2x}+2e^x+1}{e^x}\, dx = \int \frac{e^{2x}}{e^x}dx + \int \frac{2e^x}{e^x}dx + \int \frac{1}{e^x}dx$

$= \int e^x dx + \int 2dx + \int e^{-x}dx$

$u=-x$

$\frac{du}{dx}=-1$

$du=-dx$

$= e^x + 2x - \int e^u du$

$-e^u$

$\boxed{= e^x+2x-e^{-x}+c}$

$\frac{d}{dx}[e^x+2x-e^{-x}+c]$

$= e^x+2+e^{-x}$ ✓

19. $\int e^{-x} \sec^2(e^{-x})\, dx$

$\boxed{-\tan(e^{-x})+c}$

20. $\int \ln(e^{2x-1})\, dx$

$= \int (2x-1)dx$

$= \boxed{x^2-x+c}$

$\frac{d}{dx}[x^2-x+c] = 2x-1$

$= \ln(e^{(2x-1)})$ ✓

Evaluate the definite integral. Use your calculator to verify.

21. $\int_1^2 2x^2\sqrt{x^3+1}\, dx$

$\boxed{\frac{4}{9}(27-\sqrt{8}) \approx 10.7429}$

22. $\int_0^1 x\sqrt{1-x^2}\, dx$

$u = 1-x^2$

$\frac{du}{dx} = -2x$

$du = -2x\,dx$

$x\,dx = -\frac{1}{2}du$

x	$\to u$
0	$\to 1$
1	$\to 0$

$-\frac{1}{2}\int_1^0 u^{1/2}\, du$

$-\frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_1^0$

$-\frac{1}{3}\left[0^{3/2}-1^{3/2}\right]$

$\boxed{\frac{1}{3}}$

23. $\int_0^4 \frac{1}{\sqrt{2x+1}}\, dx$

$\boxed{2}$

24. $\int_0^2 \frac{x}{\sqrt{1+2x^2}}\, dx$

$u = 1+2x^2$

$\frac{du}{dx} = 4x$

$du = 4x\,dx$

$x\,dx = \frac{1}{4}du$

x	$\to u$
0	$\to 1$
2	$\to 9$

$\frac{1}{4}\int_1^9 u^{-1/2}\, du$

$\frac{1}{4}\left[2u^{1/2}\right]_1^9$

$\frac{1}{2}\left[\sqrt{9}-\sqrt{1}\right]$

$\frac{1}{2}(3-1)$

$\boxed{1}$

25. $\int_1^9 \frac{1}{\sqrt{x}(1+\sqrt{x})^2}\,dx$

$\boxed{\frac{1}{2}}$

26. $\int_1^5 \frac{x}{\sqrt{2x-1}}\,dx$

$2x = u+1$

$x = \frac{1}{2}(u+1)$

$u = 2x-1$

$\frac{du}{dx} = 2$

$du = 2dx$

$dx = \frac{1}{2}dx$

$x \to u$

$1 \to 1$

$5 \to 9$

$\frac{1}{2}\int_1^9 u^{-1/2}\left(\frac{1}{2}(u+1)\right)du$

$\frac{1}{4}\int_1^9 \left(u^{1/2} + u^{-1/2}\right)du$

$\frac{1}{4}\left[\frac{2}{3}u^{3/2} + 2u^{1/2}\right]_1^9$

$\frac{1}{4}\left[\left(\frac{2}{3}(\sqrt{9})^3 + 2\sqrt{9}\right) - \left(\frac{2}{3}1^{3/2} + 2\sqrt{1}\right)\right]$

$= \frac{1}{4}(18+6) - \left(\frac{2}{3}+2\right)] = \frac{1}{4}\left(\frac{64}{3}\right) = \boxed{\frac{16}{3}}$

27. $\int_1^3 \frac{e^{3/x}}{x^2}\,dx$

$\boxed{-\frac{1}{3}e(1-e^2) \simeq 5.78085}$

28. $\int_0^{\sqrt{2}} xe^{-x^2/2}\,dx$

$u = -\frac{1}{2}x^2$

$\frac{du}{dx} = -x$

$du = -x\,dx$

$x\,dx = -du$

$x \to u$

$0 \to 0$

$\sqrt{2} \to -1$

$-\int_0^{-1} e^u\,du$

$-\left[e^u\right]_0^{-1}$

$-\left[e^{-1} - e^0\right]$

$-\left(\frac{1}{e} - 1\right)$

$= \boxed{1 - \frac{1}{e} \approx 0.63212}$

Write the integral as the sum of the integral of an odd function and the integral of an even function. Use this simplification to evaluate the integral.

29. $\int_{-3}^3 (x^3 + 4x^2 - 3x - 6)dx$

$\boxed{36}$

30. $\int_{-\pi/2}^{\pi/2} (\sin(4x) + \cos(4x))dx$

$\int_{-\pi/2}^{\pi/2} \sin(4x)dx + \int_{-\pi/2}^{\pi/2} \cos(4x)dx$

$\left[\cos(4x)\right]_{-\pi/2}^{\pi/2} + \left(-\sin(4x)\right]_{-\pi/2}^{\pi/2}$

$\left(\cos(2\pi) - \cos(-2\pi)\right) - \left(\sin(2\pi) - \sin(-2\pi)\right)$

$0 \quad - \quad 0 \quad = \boxed{0}$

(the odd function term will always be zero)

(the even function term will always be double the value of just one limit of integration plugged in)

You are asked to find one of the integrals. Which one would you choose? Explain.

31. $\int \sqrt{x^3+1}\,dx$ or $\int x^2\sqrt{x^3+1}\,dx$

32. $\int \tan(3x)\sec^2(3x)\,dx$ or $\int \tan(3x)\,dx$

because if $u = 3x$

$\frac{du}{dx} = 3$ and you can always

fix up the constants,

Rewriting the integral:

33. Show that $\int_0^1 x^2(1-x)^5\,dx = \int_0^1 x^5(1-x)^2\,dx$

(show steps)

34. $\int_0^1 x^a(1-x)^b\,dx = \int_0^1 x^b(1-x)^a\,dx$

$-\int_1^0 (1-u)^a (u)^b\,du$

$= \int_0^1 u^b(1-u)^a\,du$

just change letter back...

$= \int_0^1 x^b(1-x)^a\,dx$ ✓

$u = 1-x$, $x = 1-u$

$\frac{du}{dx} = -1$

$du = -dx$

x	$\to u$
0	$\to 1$
1	$\to 0$

35. **Multiple Choice:**

$$\int x\sqrt{16-3x^2}\,dx =$$

a) $\frac{2}{3}(16-3x^2)^{3/2} + C$

b) $-\frac{1}{4}(16-3x^2)^{\frac{3}{2}} + C$

c) $\boxed{-\frac{1}{9}(16-3x^2)^{3/2} + C}$

d) $\frac{1}{9}(16-3x^2)^{3/2} + C$

36. **Multiple Choice:**

What is the average value of $h(x) = (\sec^2(x))(1 + 2\tan(x))^3$ on the interval $\left[0, \frac{\pi}{4}\right]$?

$\boxed{\text{a) } \frac{40}{\pi}}$ b) 10 c) $\frac{81}{2\pi}$ d) $\frac{80}{\pi}$

$$Av = \frac{1}{\frac{\pi}{4} - 0} \int_0^{\pi/4} \sec^2 x\,(1 + 2\tan x)^3\,dx$$

$$\frac{4}{\pi} \int_0^{\pi/4} \sec^2 x (1 + 2\tan x)^3\,dx$$

$$\frac{4}{\pi} \frac{1}{2} \int_1^3 u^3\,du$$

$$\frac{2}{\pi}\left[\frac{1}{4} u^4\right]_1^3$$

$$\frac{1}{2\pi}\left[3^4 - 1^4\right] = \frac{40}{\pi}$$

$u = 1 + 2\tan x$

$\frac{du}{dx} = 2\sec^2 x$

$du = 2\sec^2 x\,dx$

$\sec^2 x\,dx = \frac{1}{2}du$

$x \Rightarrow u$

$0 \Rightarrow 1 + 2\tan 0 = 1$

$\frac{\pi}{4} \Rightarrow 1 + 2\tan\frac{\pi}{4} = 3$

37. **Multiple Choice:**

A chemical is leaking from a storage area at a rate of $f(t) = 1600e^{-0.12t}$ gallons per hour. How many gallons of the chemical leak out of the storage area from time t = 0 hours to t = 12 hours?

a) 379 gallons b) 1221 gallons c) 3159 gallons $\boxed{\text{d) 10174 gallons}}$

38. **Free Response:**

The function f is defined by $f(x) = \sqrt{100 - x^2}$ for $-10 \le x \le 10$.

a. Find $f'(x)$.

b. Find an equation of the tangent line to the graph of f at x = -6.

c. Let g be the function defined by

$$g(x) = \begin{cases} f(x), & -10 \le x \le -6 \\ -\frac{1}{2}x + 5, & -6 < x \le 10 \end{cases}$$

Is g continuous at x = -6? Use the definition of continuity to explain your answer.

d. Find the value of $\int_0^{10} x\sqrt{100 - x^2}\, dx$.

(a) $f'(x) = \frac{1}{2}(100-x^2)^{-1/2}(-2x) = \boxed{\dfrac{-x}{\sqrt{100-x^2}}}$

(b) $m = f'(-6) = \dfrac{-(-6)}{\sqrt{100-(-6)^2}} = \dfrac{3}{4}$, $y = f(-6) = \sqrt{100-(-6)^2} = \sqrt{64} = 8$

$\boxed{(y-8) = \frac{3}{4}(x+6)}$

(c) 1) $g(-6)$ exist? yes, $g(-6) = f(-6) = 8$ ✓

2) $\lim_{x\to -6} g(x)$ exist?

LH: $\lim_{x\to -6^-} \sqrt{100-x^2}$, $\sqrt{100-(-6)^2}$, 8

RH: $\lim_{x\to -6^+} \left(-\frac{1}{2}x+5\right)$, $-\frac{1}{2}(-6)+5$, 8

$=$, yes $\lim_{x\to -6} g(x) = 8$

3) $\lim_{x\to -6} g(x) \stackrel{?}{=} g(-6)$

$8 = 8$ yes

$\boxed{\text{so } g(x) \text{ is continuous at } x = -6}$

(d) $\int_0^{10} x\sqrt{100-x^2}\, dx$

$u = 100 - x^2$

$\frac{du}{dx} = -2x$

$du = -2x\,dx$

$x\,dx = -\frac{1}{2}du$

x	$\to u$
0	$\to 100$
10	$\to 0$

$-\frac{1}{2}\int_{100}^{0} u^{1/2}\, du$

$-\frac{1}{2}\left[\frac{2}{3}u^{3/2}\right]_{100}^{0}$

$-\frac{1}{3}\left[0^{3/2} - 100^{3/2}\right]$

$-\frac{1}{3}\left[-(\sqrt{100})^3\right] = \boxed{\dfrac{1000}{3}}$

AP Calculus BC

Name: ______________________

5.6 Worksheet (Odds, 20, 22)

Find the indefinite integral.

1. $\int \frac{dx}{\sqrt{9-x^2}}$

$\boxed{\arcsin\left(\frac{x}{3}\right)+C}$

2. $\int \frac{dx}{\sqrt{1-4x^2}}$

$\int \frac{du}{\sqrt{a^2-u^2}} = \arcsin\left(\frac{u}{a}\right)+C$

$a=1$ $\quad u=2x$

$\frac{du}{dx}=2$

$du=2dx$

$dx=\frac{1}{2}du$

$\frac{1}{2}\int \frac{du}{\sqrt{a^2-u^2}}$

$\frac{1}{2}\arcsin\left(\frac{2x}{1}\right)+C$

$\boxed{\frac{1}{2}\arcsin(2x)+C}$

3. $\int \frac{1}{x\sqrt{4x^2-1}}dx$

$\boxed{\operatorname{arcsec}(|2x|)+C}$

5. $\int \frac{12}{1+9x^2}dx$

$\int \frac{du}{a^2+u^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+C$

$a=1$ $\quad u=3x$

$\frac{du}{dx}=3$

$du=3dx$

$dx=\frac{1}{3}du$

$\int \frac{12}{1+u^2}\frac{1}{3}du$

$4\int \frac{1}{a^2+u^2}du$

$4\left(\frac{1}{1}\right)\arctan\left(\frac{3x}{1}\right)+C$

$\boxed{4\arctan(3x)+C}$

7. $\int \frac{t}{t^4+25}dt$

$\boxed{\frac{1}{10}\arctan\left(\frac{1}{5}t^2\right)+C}$

8. $\int \frac{1}{x\sqrt{x^4-4}}dx$

$\left(u^2=x^4 \longrightarrow u=x^2\right)$

$\int \frac{du}{u\sqrt{u^2-a^2}} = \frac{1}{a}\operatorname{arcsec}\left(\frac{|u|}{a}\right)+C$

(add this)

$\int \left(\frac{x}{x}\right)\frac{1}{x\sqrt{x^4-4}}dx$

$a=2$ $\quad u=x^2$

$\frac{du}{dx}=2x$

$du=2xdx$

$xdx=\frac{1}{2}du$

$\int \frac{x\,dx}{x^2\sqrt{x^4-4}}$

$\frac{1}{2}\int \frac{du}{u\sqrt{u^2-a^2}}$

$\frac{1}{2}\left(\frac{1}{2}\right)\operatorname{arcsec}\left(\frac{|x^2|}{2}\right)+C$

$\boxed{\frac{1}{4}\operatorname{arcsec}\left(\frac{1}{2}x^2\right)+C}$

(don't need | |, x^2 is always positive)

9. $\int \frac{e^{2x}}{4+e^{4x}}\,dx$

$\boxed{\frac{1}{4}\arctan\left(\frac{1}{2}e^{2x}\right)+c}$

10. $\int \frac{2}{x\sqrt{9x^2-25}}\,dx$

$\int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\operatorname{arcsec}\left(\frac{|u|}{a}\right)+$

$\int \frac{2}{x\sqrt{u^2-a^2}}\,\frac{1}{3}dx$

$a=5$ $u=3x$

$du=3\,dx$

$dx=\frac{1}{3}du$

$2\int \frac{1}{u\sqrt{u^2-a^2}}\,du$

$2\left(\frac{1}{5}\right)\operatorname{arcsec}\left(\frac{|3x|}{5}\right)+c$

$\boxed{\frac{2}{5}\operatorname{arcsec}\left(\frac{1}{5}|3x|\right)+C}$

11. $\int \frac{1}{\sqrt{x}\sqrt{1-x}}\,dx$

$\boxed{2\arcsin(\sqrt{x})+c}$

12. $\int \frac{3}{2\sqrt{x}(1+x)}\,dx$

$\int \frac{1}{a^2+u^2}\,du = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+c$

$a=1$ $u=\sqrt{x}=x^{1/2}$

$\frac{du}{dx}=\frac{1}{2}x^{-1/2}=\frac{1}{2\sqrt{x}}$

$du=\frac{1}{2}\frac{1}{\sqrt{x}}dx$

$\frac{1}{\sqrt{x}}dx=2\,du$

$=\frac{3}{2}\int \frac{1}{(1+x)}\frac{1}{\sqrt{x}}\,dx$

$=\frac{3}{2}(2)\int \frac{1}{a^2+u^2}\,du$

$=3\left(\frac{1}{1}\right)\arctan\left(\frac{\sqrt{x}}{1}\right)+c$

$=\boxed{3\arctan(\sqrt{x})+c}$

13. $\int \frac{x-3}{x^2+1}\,dx$

$\boxed{\frac{1}{2}\ln|x^2+1|-3\arctan(x)+c}$

14. $\int \frac{x^2+3}{x\sqrt{x^2-4}}\,dx$

$\int \frac{1}{u\sqrt{u^2-a^2}}\,du = \frac{1}{a}\operatorname{arcsec}\left(\frac{|u|}{a}\right)+$

$=\int \frac{x^2}{x\sqrt{x^2-4}}\,dx + 3\int \frac{1}{x\sqrt{x^2-4}}\,dx$

$a=2$, $u=x$

$du=dx$

$=\int \frac{x\,dx}{\sqrt{x^2-4}}$

u-sub: $u=x^2-4$

$\frac{du}{dx}=2x$

$du=2x\,dx$

$x\,dx=\frac{1}{2}du$

$\frac{1}{2}\int u^{-1/2}\,du$

$\frac{1}{2}(2)\,u^{1/2}$

$3\int \frac{1}{u\sqrt{u^2-a^2}}\,du$

$3\left(\frac{1}{2}\right)\operatorname{arcsec}\left(\frac{|x|}{2}\right)+c$

$\boxed{\sqrt{x^2-4}+\frac{3}{2}\operatorname{arcsec}\left(\frac{1}{2}|x|\right)+C}$

Find or evaluate the integral by completing the square.

15. $\int \frac{2x}{x^2+6x+13}\,dx$

$\boxed{\ln|x^2+6x+13| - 3\arctan\left(\frac{x+3}{2}\right) + C}$

16. $\int \frac{2x-5}{x^2+2x+2}\,dx$

u-sub: $u = x^2+2x+2$

$\frac{du}{dx} = 2x+2$

$du = (2x+2)dx$

$= \int \frac{2x+2-7}{x^2+2x+2}\,dx$

$\int \frac{2x+2}{x^2+2x+2}\,dx - 7\int \frac{1}{x^2+2x+2}\,dx$

complete the square for $\int \frac{1}{u^2+a^2} = \frac{1}{a}\arctan\left(\frac{u}{a}\right)+C$

usub:

$u = x^2+2x+2$

$\frac{du}{dx} = 2x+2$

$du = (2x+2)dx$

$\int \frac{1}{u}\,du$

$\ln|u|$

$\ln|x^2+2x+2|$

$x^2+2x+1+2-1$

$(x+1)^2+1$

$-7\int \frac{1}{(x+1)^2+1}\,dx$ $\quad a=1 \quad u=x+1$

$\frac{du}{dx}=1$

$du=dx$

$-7\int \frac{1}{u^2+a^2}\,du$

$-7\left(\frac{1}{1}\arctan\left(\frac{x+1}{1}\right)\right)+C$

$\boxed{\ln|x^2+2x+2| - 7\arctan(x+1) + C}$

17. $\int \frac{1}{\sqrt{-x^2-4x}}\,dx$

$\boxed{\arcsin\left(\frac{x+2}{2}\right) + C}$

18. $\int \frac{2}{\sqrt{-x^2+4x}}\,dx$

Complete the square for $\int \frac{1}{\sqrt{a^2-u^2}}\,du = \arcsin\left(\frac{u}{a}\right)+C$

$-x^2+4x$

$-(x^2-4x+4)+4$

$-(x-2)^2+4$

$2\int \frac{1}{\sqrt{4-(x-2)^2}}\,dx$ $\quad a=2 \quad u=x-2$

$du=dx$

$2\int \frac{1}{\sqrt{a^2-u^2}}\,du$

$\boxed{2\arcsin\left(\frac{x-2}{2}\right) + C}$

19. **Multiple Choice:**

Which integral has a value of π?

$I. \int_{5/2}^{5} \frac{2}{\sqrt{5x-x^2}} dx$ $\quad$ $II. \int_{\pi}^{2\pi} \frac{\pi}{2} \sin\left(\frac{x}{2}\right) dx$ $\quad$ $III. \int_{3}^{8} \frac{\pi}{2\sqrt{x+1}} dx$

a) I and II only $\quad$ b) I and III only $\quad$ c) II and III only $\quad$ $\boxed{\text{d) I, II and III}}$

20. **Multiple Choice:**

Which of the following is the solution of the differential equation?

$$y' = \frac{4}{\sqrt{16 - x^2}}$$

a) $y = \arcsin\left(\frac{x}{4}\right) + C$ $\quad$ $\boxed{\text{b) } y = 4\arcsin\left(\frac{x}{4}\right) + C}$

c) $y = \frac{1}{4}\arcsin\left(\frac{x}{16}\right) + C$ $\quad$ d) $y = 16\arcsin\left(\frac{x}{4}\right) + C$

$$y = \int \frac{4}{\sqrt{16-x^2}}\, dx = 4\int \frac{1}{\sqrt{16-x^2}}$$

$a = 4$ $\quad$ $u = x$

$du = dx$

$$= 4\int \frac{1}{\sqrt{a^2-u^2}}\, du$$

$$= 4 \arcsin\left(\frac{x}{4}\right) + C$$

21. **Free Response:**

Let $f(x) = \arccos(x)$ and let $g(x) = x^2$. Define $h(x) = f(g(x))$.

a. At what values of x does $h(x)$ have a relative maximum?

b. Write, but do not evaluate, an expression that can determine the area of the region bounded by the graphs of $h(x)$ and the horizontal line $y = \frac{\pi}{3}$.

c. Evaluate $\frac{d}{dx}\left[f^{-1}\left(\frac{\pi}{3}\right)\right]$.

(a) $\boxed{x=0}$

(b) $\boxed{\int_{-\frac{\sqrt{2}}{2}}^{\frac{\sqrt{2}}{2}} \left(\arccos(x^2) - \frac{\pi}{3}\right)dx}$

(c) $\boxed{-\frac{\sqrt{3}}{2}}$

22. **Free Response:**

The function f is defined by

$$f(x) = \frac{1}{\sqrt{4 - x^2}}$$

for -2 < x < 2. Let g be the function defined by

$$g(x) = \begin{cases} f(x), & -2 < x \leq 0 \\ x + \frac{1}{2} & 0 < x \leq 2 \end{cases}$$

a. Is g continuous at x = 0? Use the definition of continuity to explain your answer.

b. What is the area of the region bounded by f, the x-axis, the y-axis, and the line x = 1?

c. Find the value of $\int_{-1}^{1} g(x)dx$.

(a) 1) g(0) exist? $g(0) = f(0) = \frac{1}{\sqrt{4-0^2}} = \frac{1}{\sqrt{4}} = \frac{1}{2}$ ✓

2) $\lim_{x \to 0} g(x)$ exist?

LH: $\lim_{x \to 0^-} \frac{1}{\sqrt{4-x^2}}$, $\frac{1}{\sqrt{4-0^2}}$, $\frac{1}{2}$

RH: $\lim_{x \to 0^+} (x+\frac{1}{2})$, $0 + \frac{1}{2}$, $\frac{1}{2}$

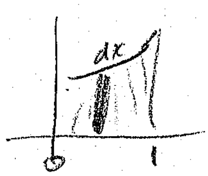
=, so $\lim_{x \to 0} g(x) = \frac{1}{2}$ ✓

3) $\lim_{x \to 0} g(x) \stackrel{?}{=} g(0)$

$\frac{1}{2} = \frac{1}{2}$

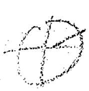
yes, $\boxed{\text{so } g(x) \text{ is continuous at } x=0}$

(b)



$A = \int_0^1 \frac{1}{\sqrt{4-x^2}} dx$ $a=2$, $u=x$, $du=dx$ $x \to u$: $0 \to 0$, $1 \to 1$

$\int_0^1 \frac{1}{\sqrt{a^2-u^2}} du$



$\left[\arcsin\left(\frac{x}{2}\right)\right]_0^1 = \arcsin(\frac{1}{2}) - \arcsin(0)$

$\frac{\pi}{6} - 0 = \boxed{\frac{\pi}{6}}$

(c) $\int_{-1}^{1} g(x)dx = \int_{-1}^{0} \frac{1}{\sqrt{4-x^2}} dx + \int_0^1 (x+\frac{1}{2})dx$

$a=2$ $u=x$ $du=dx$

$\int \frac{1}{\sqrt{a^2-u^2}} du$

$\left[\arcsin\left(\frac{x}{2}\right)\right]_{-1}^{0}$

$\arcsin(0) - \arcsin(-\frac{1}{2})$

$0 - (-\frac{\pi}{6})$

$\left(\frac{1}{2}x^2 + \frac{1}{2}x\right]_0^1$

$\left(\frac{1}{2}(1)^2 + \frac{1}{2}(1)\right) - (0)$

$+ \frac{1}{2} + \frac{1}{2}$

$= \boxed{\frac{\pi}{6} + 1}$