

5.1 Worksheet (Odds, 28 and 30)

Verify the statement by showing that the derivative of the right hand side equals the integrand of the left hand side.

1.
$$\int \left(-\frac{6}{x^4}\right) dx = \frac{2}{x^3} + C$$

2.
$$\int \left(8x^3 + \frac{1}{2x^2}\right) dx = 2x^4 - \frac{1}{2x} + C$$

Find the general solution of the differential equation and check the result by differentiation.

3.
$$\frac{dy}{dt} = 9t^2$$

4.
$$\frac{dy}{dt} = 5$$

5.
$$\frac{dy}{dx} = x^{3/2}$$

6.
$$\frac{dy}{dx} = 2x^{-3}$$

Complete the table to find the indefinite integral.

Original Integral	Rewrite	Integrate	Simplify
7. $\int \sqrt[3]{x} dx$			
8. $\int \frac{1}{4x^2} dx$			
9. $\int \frac{1}{x\sqrt{x}} dx$			
10. $\int \frac{1}{(3x)^2} dx$			

Find the indefinite integral and check the result by differentiation.

11. $\int (3x^3 - 6x^2 + 2)dx$

12. $\int (x^2 + 7)dx$

13. $\int \frac{x+6}{\sqrt{x}} dx$

14. $\int \frac{x^4-3x^2+5}{x^4} dx$

15. $\int (x+1)(3x-2) dx$

16. $\int (4t^2+3)^2 dt$

17. $\int (2 \sin(x) - 5e^x) dx$

18. $\int (\sec(y))(\tan(y) - \sec(y)) dy$

19. $\int (\tan^2(y) + 1) dy$

20. $\int (4x - \csc^2(x)) dx$

21. $\int \left(x - \frac{5}{x}\right) dx$

22. $\int \left(\frac{4}{x} + \sec^2(x)\right) dx$

Find the particular solution that satisfies the differential equation and the initial condition.

23. $f'(x) = 6x, f(0) = 8$

24. $f'(x) = 10x - 12x^3, f(3) = 2$

25. $f''(x) = 2$, $f'(2) = 5$, $f(2) = 10$

26. $f''(x) = x^2$, $f'(0) = 8$, $f(0) = 4$

Use $a(t) = -32$ feet per second per second as the acceleration due to gravity. (Neglect air resistance.)

27. A ball is thrown vertically upward from a height of 6 feet with an initial velocity of 60 feet per second. How high will the ball go?

28. A balloon, rising vertically with a velocity of 16 feet per second, releases a sandbag at the instant it is 64 feet above the ground.

a. How many seconds after its release will the bag strike the ground?

b. At what velocity will it hit the ground?

29. **Multiple Choice:**

$$\int \sqrt{x}(10x - 3)dx =$$

a) $\frac{2}{5}x^{5/2} - \frac{2}{3}x^{3/2} + C$

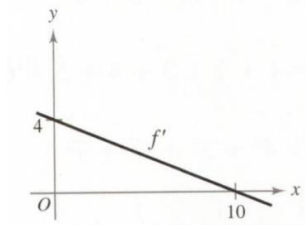
b) $2x^{5/2} - 2x^{3/2} + C$

c) $2x^{5/2} - x^{3/2} + C$

d) $4x^{5/2} - 2x^{3/2} + C$

30. **Multiple Choice:**

The graph of $f'(x)$ is shown. If $f(0) = 3$, what is the value of $f(10)$?



- a) 0
c) 20

- b) 4
d) 23

31. **Free Response:**

The rate of growth of a population P is given by $\frac{dP}{dt} = 20t^3 - 35t^{4/3}$, where t is time in years. The initial population is 8000.

a) Is the population always increasing? Justify your answer.

b) When is the population at its lowest point? Round your answer to one decimal place.

c) Find the population function. Estimate the population after 10 years.

5.2 Worksheet (Odds, 24, 26)

Find the sum and use the summation capabilities of your calculator to verify results.

1. $\sum_{i=1}^6 (3i + 2)$

2. $\sum_{k=3}^9 (k^2 + 1)$

3. $\sum_{k=0}^4 \frac{1}{k^2+1}$

4. $\sum_{j=4}^6 \left(\frac{3}{j}\right)$

5. $\sum_{k=1}^4 c$

6. $\sum_{i=1}^4 [(i-1)^2 + (i+1)^3]$

Use sigma notation to write the sum.

7. $\left[7\left(\frac{1}{6}\right) + 5\right] + \left[7\left(\frac{2}{6}\right) + 5\right] + \cdots + \left[7\left(\frac{6}{6}\right) + 5\right]$

8. $\left[1 - \left(\frac{1}{4}\right)^2\right] + \left[1 - \left(\frac{2}{4}\right)^2\right] + \cdots + \left[1 - \left(\frac{4}{4}\right)^2\right]$

9. $\left[\left(\frac{2}{n}\right)^3 - \frac{2}{n}\right]\left(\frac{2}{n}\right) + \cdots + \left[\left(\frac{2n}{n}\right)^3 - \frac{2n}{n}\right]\left(\frac{2}{n}\right)$

10. $\left[2\left(1 + \frac{3}{n}\right)^2\right]\left(\frac{3}{n}\right) + \cdots + \left[2\left(1 + \frac{3n}{n}\right)^2\right]\left(\frac{3}{n}\right)$

Use the properties of summation and summation formulas to evaluate the sum. Use your calculator to verify.

11. $\sum_{i=1}^{12} 7$

12. $\sum_{i=1}^{30} -18$

13. $\sum_{i=1}^{24} 4i$

14. $\sum_{i=1}^{16} (5i - 4)$

15. $\sum_{i=1}^{20} (i - 1)^2$

16. $\sum_{i=1}^{10} (i^2 - 1)$

17. $\sum_{i=1}^{15} i(i - 1)^2$

18. $\sum_{i=1}^{25} (i^3 - 2i)$

Use the limit process to find the area of the region bounded by the graph of the function and the x-axis over the given interval. Sketch the region.

19. $y = 25 - x^2$; $[1,4]$

20. $y = 4 - x^2$; $[-2,2]$

21. $y = x^2 - x^3; [-1,1]$

22. $y = 2x^3 - x^2; [1,2]$

23. **Multiple Choice:**

Which value best approximates the area of the region bounded by the graph of $f(x) = \sin\left(\frac{\pi x}{4}\right)$ and the x-axis over the interval $[0, 4]$?

- a) 2 b) 2.5 c) 3 d) π

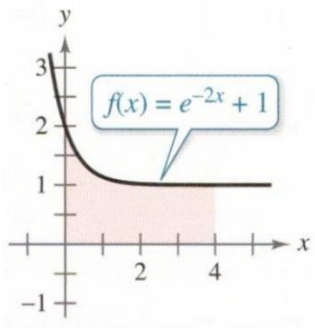
24. **Multiple Choice:**

$$\lim_{n \rightarrow \infty} \sum_{i=1}^n \left(\frac{1}{n}\right) \left[8\left(\frac{i}{n}\right) + 3\right] =$$

- a) 3 b) 4 c) 7 d) 8

25. **Multiple Choice:**

Find the Midpoint Rule approximation for the area of the shaded region. Use 8 subintervals of equal width.



- a) 4.425 b) 4.480 c) 6.719 d) 8.959

26. **Multiple Choice:**

Let f be a continuous function such that $f(-2) = 25$ and $f'(x) = 6x^2 - 7$. What is the value of $f(1)$?

- a) -10 b) 18 c) 22 d) 27

5.3 Worksheet (Odds, 2, 12, 14)

1. Given $\int_0^5 f(x)dx = 10$ and $\int_5^7 f(x)dx = 3$, evaluate:

a) $\int_0^7 f(x)dx$

b) $\int_5^0 f(x)dx$

c) $\int_5^5 f(x)dx$

d) $\int_0^5 3f(x)dx$

2. Given $\int_2^6 f(x)dx = 10$ and $\int_2^6 g(x)dx = -2$, evaluate:

a) $\int_2^6 [f(x) + g(x)]dx$

b) $\int_2^6 [g(x) - f(x)]dx$

c) $\int_2^6 2g(x)dx$

d) $\int_2^6 3f(x)dx$

3. Use the table of values to estimate $\int_0^6 f(x)dx$.

x	0	1	2	3	4	5	6
f(x)	-6	0	8	18	30	50	80

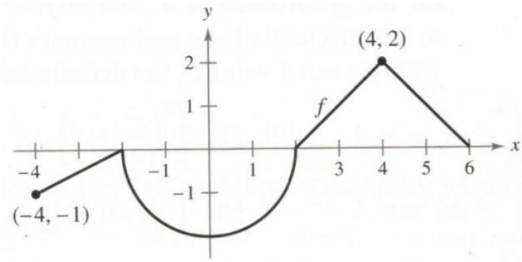
Use three equal subintervals and the a) LEFT ENDPOINTS, b) RIGHT ENDPOINTS, and c) MIDPOINTS. When f is an increasing function, how does each estimate compare with the actual value? Explain your reasoning.

4. Use the table of values to find the lower and upper estimates of $\int_0^{10} f(x)$.

Assume that f is a decreasing function.

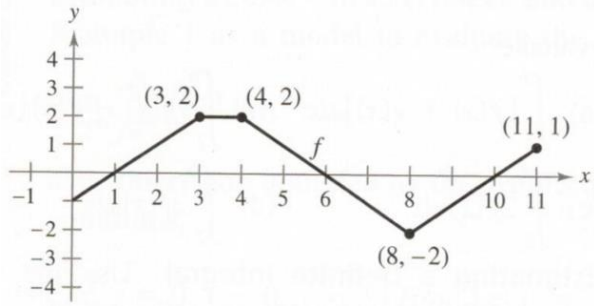
x	0	2	4	6	8	10
f(x)	32	24	12	-4	-20	-36

5. The graph of f consists of line segments and a semicircle, as shown in the figure. Evaluate each definite integral by using geometric formulas.



- a) $\int_0^2 f(x) dx$
- b) $\int_2^6 f(x) dx$
- c) $\int_{-4}^2 f(x) dx$
- d) $\int_{-4}^6 f(x) dx$
- e) $\int_{-4}^6 |f(x)| dx$
- f) $\int_{-4}^6 [f(x) + 2] dx$

6. The graph of f consists of line segments, as shown in the figure. Evaluate each definite integral by using geometric formulas.



a) $\int_0^1 -f(x)dx$

b) $\int_3^4 3f(x)dx$

c) $\int_0^7 f(x)dx$

d) $\int_5^{11} f(x)dx$

e) $\int_0^{11} f(x)dx$

f) $\int_4^{10} f(x)dx$

(7-10) Find possible values of a and b that make the statement true. If possible, use a graph to support your answer. (There may be more than one correct answer.)

7.
$$\int_{-2}^1 f(x) dx + \int_1^5 f(x) dx = \int_a^b f(x) dx$$

8.
$$\int_{-3}^3 f(x) dx + \int_3^6 f(x) dx - \int_a^b f(x) dx = \int_{-1}^6 f(x) dx$$

9.
$$\int_a^b \sin(x) dx < 0$$

10.
$$\int_a^b \cos(x) dx = 0$$

11. **Multiple Choice**

$$\int_{-3}^6 \left(-\frac{2}{3}x + 5\right) dx =$$

- a) 25 b) 32 c) 36 d) 45

12. **Multiple Choice**

The expression $\frac{1}{10} \left[\left(\frac{1}{10}\right)^2 + \left(\frac{2}{10}\right)^2 + \left(\frac{3}{10}\right)^2 + \dots + \left(\frac{10}{10}\right)^2 \right]$ is a Riemann sum approximation for

- a) $\int_0^1 x^2 dx$ b) $\frac{1}{10} \int_0^1 x^2 dx$ c) $\int_1^{10} x^2 dx$ d) $\int_0^{10} x^2 dx$

13. **Multiple Choice**

The table shows selected values for a continuous function f over the interval $[2, 8]$. Using the subintervals $[2, 3]$, $[3, 5]$, and $[5, 8]$, what is the trapezoidal approximation of $\int_2^8 f(x) dx$?

x	2	3	5	8
f(x)	8	22	72	142

- a) 268 b) 338 c) 430 d) 592

14. **Free Response**

Consider the function

$$f(x) = \begin{cases} x^3 - 6x^2 + 12x - 6, & x < 2 \\ x^2 - 4x, & x \geq 2 \end{cases}$$

a. Graph f over the interval $[0, 4]$.

b. Is f differentiable at $x = 2$? Justify your answer.

c. Is f integrable on the interval $[0, 4]$? If so, use your graph to approximate $\int_0^4 f(x)dx$. Show the work that leads to your answer.

