AP Calculus BC

Name: answers/solutions

5.1 Worksheet (Odds, 28 and 30)

Verify the statement by showing that the derivative of the right hand side equals the integrand of the left hand side.

1. $\int\left(-\frac{6}{x^4}\right)dx = \frac{2}{x^3} + C$

(show verification steps)

2. $\int\left(8x^3 + \frac{1}{2x^2}\right)dx = 2x^4 - \frac{1}{2x} + C$

$\frac{d}{dx}\left[2x^4 - (2x)^{-1} + C\right]$

$8x^3 + (2x)^{-2}(2) + 0$

$8x^3 + \frac{2}{(2x)^2}$

$8x^3 + \frac{2}{4x^2}$

$8x^3 + \frac{1}{2x^2}$ ✓

Find the general solution of the differential equation and check the result by differentiation.

3. $\frac{dy}{dt} = 9t^2$

$\boxed{y = 3t^3 + C}$

4. $\frac{dy}{dt} = 5$

$y = \int 5\,dt$

$\boxed{y = 5t + C}$

$\frac{d}{dt}[5t + C]$

$5 + 0$

5 ✓

5. $\frac{dy}{dx} = x^{3/2}$

$\boxed{y = \frac{2}{5}x^{5/2} + C}$

6. $\frac{dy}{dx} = 2x^{-3}$

$y = \int 2x^{-3}\,dx$

$y = 2\frac{x^{-2}}{-2} + C$

$\boxed{y = -x^{-2} + C}$

$\frac{d}{dx}\left[-x^{-2} + C\right]$

$2x^{-3} + 0$

$2x^{-3}$ ✓

Complete the table to find the indefinite integral.

Original Integral	Rewrite	Integrate	Simplify
7. $\int \sqrt[3]{x}\, dx$			$\frac{3}{4}x^{4/3}+C$
8. $\int \frac{1}{4x^2}\, dx$	$\frac{1}{4}\int x^{-2}dx$	$\frac{1}{4}\left[\frac{x^{-1}}{-1}\right]+C$	$\frac{-1}{4x}+C$
9. $\int \frac{1}{x\sqrt{x}}\, dx$			$\frac{-2}{\sqrt{x}}+C$
10. $\int \frac{1}{(3x)^2}\, dx$	$\frac{1}{9}\int x^{-2}dx$	$\frac{1}{9}\left[\frac{x^{-1}}{-1}\right]+C$	$\frac{-1}{9x}+C$

Find the indefinite integral and check the result by differentitation.

11. $\int(3x^3 - 6x^2 + 2)dx$

$\boxed{\frac{3}{4}x^4 - 2x^3 + 2x + C}$

12. $\int(x^2 + 7)dx$

$\boxed{\frac{1}{3}x^3 + 7x + C}$

$\frac{d}{dx}\left[\frac{1}{3}x^3 + 7x + C\right]$

$x^2 + 7 + 0$

$x^2 + 7$ ✓

13. $\int \frac{x+6}{\sqrt{x}}\,dx$

$\boxed{\frac{2}{3}x^{3/2} + 12x^{1/2} + c}$

14. $\int \frac{x^4-3x^2+5}{x^4}\,dx = \int (1 + 3x^{-2} + 5x^{-4})\,dx$

$= x + 3\left(\frac{x^{-1}}{-1}\right) + 5\left(\frac{x^{-3}}{-3}\right) + c$

$= \boxed{x - \frac{3}{x} - \frac{5}{3x^3} + c}$

$\frac{d}{dx}\left[x - 3x^{-1} - \frac{5}{3}x^{-3} + c\right]$

$1 + 3x^{-2} + 5x^{-4}$ ✓

15. $\int (x+1)(3x-2)\,dx$

$\boxed{x^3 + \frac{1}{2}x^2 - 2x + c}$

16. $\int (4t^2+3)^2\,dt = \int (4t^2+3)(4t^2+3)\,dt$

$= \int (16t^4 + 24t^2 + 9)\,dt$

$= \boxed{\frac{16}{5}t^5 + 8t^3 + 9t + C}$

$\frac{d}{dt}\left[\frac{16}{5}t^5 + 8t^3 + 9t + c\right]$

$16t^4 + 24t^2 + 9$ ✓

17. $\int (2\sin(x) - 5e^x)\,dx$

$\boxed{-2\cos x - 5e^x + c}$

18. $\int (\sec(y))(\tan(y) - \sec(y))\,dy$

$= \int \sec y \tan y\,dy - \int \sec^2 y\,dy$

$= \boxed{\sec y - \tan y + C}$

$\frac{d}{dy}\left[\sec y - \tan y + c\right]$

$\sec y \tan y - \sec^2 y + 0$

$\sec y(\tan y - \sec y)$ ✓

19. $\int(\tan^2(y)+1)dy$

$\boxed{\tan y + C}$

20. $\int(4x-\csc^2(x))dx$

$= 4\int x\,dx - \int \csc^2 x\,dx$

$= 4\left(\frac{x^2}{2}\right) - (-\cot x) + C$

$= \boxed{2x^2 + \cot x + C}$

21. $\int\left(x-\frac{5}{x}\right)dx$

$\boxed{\frac{1}{2}x^2 - 5\ln|x| + C}$

22. $\int\left(\frac{4}{x}+\sec^2(x)\right)dx$

$= 4\int \frac{1}{x}dx + \int \sec^2 x\,dx$

$= \boxed{4\ln|x| + \tan x + C}$

$\frac{d}{dx}[4\ln|x| + \tan x + C]$

$\frac{4}{x} + \sec^2 x$ ✓

Find the particular solution that satisfies the differential equation and the initial condition.

23. $f'(x)=6x,\ f(0)=8$

$\boxed{f(x) = 3x^2 + 8}$

24. $f'(x)=10x-12x^3,\ f(3)=2$

$f(x) = \int(10x - 12x^3)dx$

$f(x) = 5x^2 - 3x^4 + C$

$2 = 5(3)^2 - 3(3)^4 + C$

$C = 200$

$\boxed{f(x) = 5x^2 - 3x^4 + 200}$

25. $f''(x) = 2,\ f'(2) = 5,\ f(2) = 10$

$\boxed{f(x) = x^2 + x + 4}$

26. $f''(x) = x^2,\ f'(0) = 8,\ f(0) = 4$

$f'(x) = \int x^2 dx = \frac{1}{3}x^3 + C_1$

$8 = \frac{1}{3}(0)^3 + C_1 \rightarrow C_1 = 8$

$f'(x) = \frac{1}{3}x^3 + 8$

$f(x) = \int (\frac{1}{3}x^3 + 8) dx = \frac{1}{12}x^4 + 8x + C_2$

$4 = \frac{1}{12}(0)^4 + 8(0) + C_2 \rightarrow C_2 = 4$

$\boxed{f(x) = \frac{1}{12}x^4 + 8x + 4}$

Use $a(t) = -32$ feet per second per second as the acceleration due to gravity. (Neglect air resistance.)

27. A ball is thrown vertically upward from a height of 6 feet with an initial velocity of 60 feet per second. How high will the ball go?

$\boxed{62.25 \text{ ft}}$

28. A balloon, rising vertically with a velocity of 16 feet per second, releases a sandbag at the instant it is 64 feet above the ground.

a. How many seconds after its release will the bag strike the ground?

$a = -32$

$v = \int -32\, dt = -32t + C_1$

$16 = -32(0) + C_1 \rightarrow C_1 = 16$

$v = -32t + 16$

$s = \int (-32t + 16) dt = -16t^2 + 16t + C_2$

$64 = -16(0)^2 + 16(0) + C_2 \rightarrow C_2 = 64$

$s = -16t^2 + 16t + 64$

ground: $s = 0$

$-16t^2 + 16t + 64 = 0$

$-16(t^2 - t - 4) = 0$

$t = \frac{1 \pm \sqrt{1^2 - 4(1)(-4)}}{2(1)} = \frac{1 \pm \sqrt{17}}{2}$

$t = -1.56155,\ \boxed{2.56155 \text{ sec}}$

b. At what velocity will it hit the ground?

$v(2.56155) = -32(2.56155) + 16$

$= \boxed{-65.970 \text{ ft/sec}}$ (negative because the sandbag is moving downward)

29. **Multiple Choice:**

$\int \sqrt{x}(10x-3)dx =$

(show work)

a) $\frac{2}{5}x^{5/2} - \frac{2}{3}x^{\frac{3}{2}} + C$

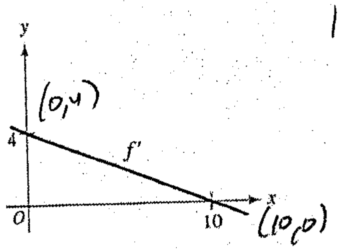
b) $2x^{5/2} - 2x^{\frac{3}{2}} + C$

c) $2x^{5/2} - x^{\frac{3}{2}} + C$

$\boxed{\text{d)} \quad 4x^{5/2} - 2x^{\frac{3}{2}} + C}$

30. **Multiple Choice:**

The graph of $f'(x)$ is shown. If $f(0) = 3$, what is the value of $f(10)$?



line: $m = \frac{4-0}{0-10} = \frac{-4}{10} = \frac{-2}{5}$

$(y-0) = -\frac{2}{5}(x-10)$

$y = -\frac{2}{5}x + 4$

so $f'(x) = -\frac{2}{5}x + 4$

$f(x) = \int(-\frac{2}{5}x+4)dx$

$= -\frac{1}{5}x^2 + 4x + C$

$3 = -\frac{1}{5}(0)^2 + 4(0) + C \rightarrow C = 3$

$f(x) = -\frac{1}{5}x^2 + 4x + 3$

$f(10) = -\frac{1}{5}(10)^2 + 4(10) + 3$

$= -20 + 40 + 3$

$= 23$

a) 0

b) 4

c) 20

$\boxed{\text{d)}}$ 23

31. **Free Response:**

The rate of growth of a population P is given by $\frac{dP}{dt} = 20t^3 - 35t^{4/3}$, where t is time in years. The initial population is 8000.

a) Is the population always increasing? Justify your answer.

The population is not always increasing
(show that there is at least one interval where $f'(x) < 0$)

b) When is the population at its lowest point? Round your answer to one decimal place.

$\boxed{t = 1.399 \text{ years}}$

c) Find the population function. Estimate the population after 10 years.

$\boxed{P(t) = 5t^4 - 15t^{7/3} + 8000}$

$\boxed{P(10) = 54{,}768 \text{ people}}$

AP Calculus BC

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5.2 Worksheet (Odds, 24, 26)

Find the sum and use the summation capabilities of your calculator to verify results.

1. $\sum_{i=1}^{6}(3i+2)$

$\boxed{75}$

2. $\sum_{k=3}^{9}(k^2+1)$

$10+17+26+37+50+65+82$

$\boxed{287}$

3. $\sum_{k=0}^{4}\frac{1}{k^2+1}$

$\boxed{\frac{158}{85}}$

4. $\sum_{j=4}^{6}\left(\frac{3}{j}\right)$

$\frac{3}{4}+\frac{3}{5}+\frac{3}{6}$

$\boxed{\frac{37}{20}}$

5. $\sum_{k=1}^{4}c$

$\boxed{4c}$

6. $\sum_{i=1}^{4}[(i-1)^2+(i+1)^3]$

$(0^2+2^3)+(1^2+3^3)+(2^2+4^3)+(3^2+5^3)$

$8 + 28 + 68 + 134$

$\boxed{238}$

Use sigma notation to write the sum.

7. $\left[7\left(\frac{1}{6}\right)+5\right]+\left[7\left(\frac{2}{6}\right)+5\right]+\cdots+\left[7\left(\frac{6}{6}\right)+5\right]$

$\boxed{\sum_{i=1}^{6}(7i+5)}$

8. $\left[1-\left(\frac{1}{4}\right)^2\right]+\left[1-\left(\frac{2}{4}\right)^2\right]+\cdots+\left[1-\left(\frac{4}{4}\right)^2\right]$

$\boxed{\sum_{i=1}^{4}\left(1-\left(\frac{i}{4}\right)^2\right)}$

9. $\left[\left(\frac{2}{n}\right)^3-\frac{2}{n}\right]\left(\frac{2}{n}\right)+\cdots+\left[\left(\frac{2n}{n}\right)^3-\frac{2n}{n}\right]\left(\frac{2}{n}\right)$

$\boxed{\sum_{i=1}^{n}\left[\left(\frac{2i}{n}\right)^3-\frac{2i}{n}\right]\left(\frac{2}{n}\right)}$

10. $\left[2\left(1+\frac{3}{n}\right)^2\right]\left(\frac{3}{n}\right)+\cdots+\left[2\left(1+\frac{3n}{n}\right)^2\right]\left(\frac{3}{n}\right)$

$\boxed{\sum_{i=1}^{n}\left[2\left(1+\frac{3i}{n}\right)^2\right]\left(\frac{3}{n}\right)}$

Use the properties of summation and summation formulas to evaluate the sum. Use your calculator to verify.

11. $\sum_{i=1}^{12} 7 = \boxed{84}$

12. $\sum_{i=1}^{30} -18 = 30(-18) = \boxed{-540}$

13. $\sum_{i=1}^{24} 4i = \boxed{1200}$

14. $\sum_{i=1}^{16} (5i-4) = 5\sum_{i=1}^{16} i - \sum_{i=1}^{16} 4$

$= 5\frac{(16)(16+1)}{2} - 4(16) = 680 - 64 = \boxed{616}$

15. $\sum_{i=1}^{20} (i-1)^2 = \boxed{2470}$

16. $\sum_{i=1}^{10} (i^2-1) = \sum_{i=1}^{10} i^2 - \sum_{i=1}^{10} 1$

$= \frac{(10)(10+1)(2(10)+1)}{6} - (1)(10)$

$= 385 - 10 = \boxed{375}$

17. $\sum_{i=1}^{15} i(i-1)^2 = \boxed{12040}$

18. $\sum_{i=1}^{25} (i^3-2i) = \sum_{i=1}^{25} i^3 - 2\sum_{i=1}^{25} i$

$= \frac{(25)^2(25+1)^2}{4} - 2\frac{(25)(25+1)}{2}$

$= 105625 - 650 = \boxed{104975}$

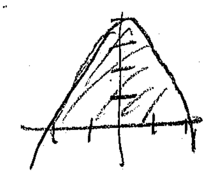
Use the limit process to find the area of the region bounded by the graph of the function and the x-axis over the given interval. Sketch the region.

19. $y = 25 - x^2$; $[1,4]$

$\boxed{54 \text{ units}^2}$

(and sketch)

20. $y = 4 - x^2$; $[-2,2]$



$\Delta x = \frac{2-(-2)}{n} = \frac{4}{n}$

$x_i = -2 + i\left(\frac{4}{n}\right)$

$A = \lim_{n\to\infty} \sum_{i=1}^{n} f(x_i)\Delta x$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[4 - \left(-2 + i\left(\frac{4}{n}\right)\right)^2\right]\left(\frac{4}{n}\right)$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[4 - \left(-2 + \frac{4}{n}i\right)\left(-2 + \frac{4}{n}i\right)\right]\left(\frac{4}{n}\right)$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[4 - \left(4 - \frac{16}{n}i + \frac{16}{n^2}i^2\right)\right]\left(\frac{4}{n}\right)$

$= \lim_{n\to\infty} \left(\frac{4}{n}\right)\sum_{i=1}^{n} \left(\frac{16}{n}i - \frac{16}{n^2}i^2\right)$

$= \lim_{n\to\infty} \left(\frac{4}{n}\right)\left[\frac{16}{n}\frac{(n)(n+1)}{2} - \frac{16}{n^2}\frac{(n)(n+1)(2n+1)}{6}\right]$

continued....

5.2 #20 continued...

$$= \lim_{n \to \infty} \left(\frac{4}{n}\right) \sum_{i=1}^{n} \left[\frac{16}{n} \frac{n(n+1)}{2} - \frac{16}{n^2} \frac{n(n+1)(2n+1)}{6}\right]$$

$$= \lim_{n \to \infty} \left(32 \frac{n+1}{n} - \frac{32}{3} \frac{2n^2+3n+1}{n^2}\right)$$

$$= 32\left(\frac{1}{1}\right) - \frac{32}{3}\left(\frac{2}{1}\right)$$

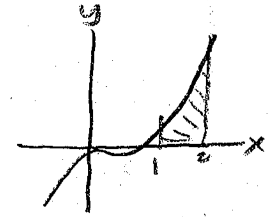
$$= \boxed{\frac{32}{3} \text{ units}^2}$$

21. $y = x^2 - x^3;\quad [-1,1]$

$\boxed{\frac{2}{3}\ \text{unit}^2}$

(and a sketch)

22. $y = 2x^3 - x^2;\quad [1,2]$



$\Delta x = \frac{2-1}{n} = \frac{1}{n}$

$x_i = 1 + i\left(\frac{1}{n}\right)$

$A = \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} f(x_i)\,\Delta x$

$= \lim\limits_{n\to\infty} \sum\limits_{i=1}^{n} \left[2\left(1+i\left(\frac{1}{n}\right)\right)^3 - \left(1+i\left(\frac{1}{n}\right)\right)^2\right]\left(\frac{1}{n}\right)$

expanding ...

$\left(1+i\left(\frac{1}{n}\right)\right)^2 = \left(1+i\left(\frac{1}{n}\right)\right)\left(1+i\left(\frac{1}{n}\right)\right)$

$= 1 + \frac{2}{n}i + \frac{1}{n^2}i^2$

$\left(1+i\left(\frac{1}{n}\right)\right)^3$ by binomial theorem...

continued ...

23. **Multiple Choice:**

Which value best approximates the area of the region bounded by the graph of $f(x) = \sin\left(\frac{\pi x}{4}\right)$ and the x-axis over the interval [0, 4]?

a) 2 $\qquad$ $\boxed{\text{b) } 2.5}$ $\qquad$ c) 3 $\qquad$ d) π

(suggest using a 4 interval midpoint Riemann Sum)

24. **Multiple Choice:**

$$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\frac{1}{n}\right)\left[8\left(\frac{i}{n}\right) + 3\right] =$$

a) 3 $\qquad$ b) 4 $\qquad$ $\boxed{\text{c) } 7}$ $\qquad$ d) 8

$= \lim\limits_{n\to\infty} \left(\frac{1}{n}\right) \sum\limits_{i=1}^{n} \left(\frac{8}{n}i + 3\right) = \lim\limits_{n\to\infty} \left(\frac{1}{n}\right)\left[\frac{8}{n}\frac{n(n+1)}{2} + 3n\right]$

$= \lim\limits_{n\to\infty} \left(4\,\frac{n+1}{n} + 3\right) = 4\left(\frac{1}{1}\right) + 3 = 7$

5.2 #22 continued...

$$\begin{matrix} 1 \\ 1\ 1 \\ 1\ 2\ 1 \\ 1\ 3\ 3\ 1 \end{matrix}$$

$(1+i(\frac{1}{n}))^3$

$= 1(1)^3(\frac{1}{n}i)^0 + 3(1)^2(\frac{1}{n}i)^1 + 3(1)^1(\frac{1}{n}i)^2 + 1(1)^0(\frac{1}{n}i)^3$

$= (1)(1)(1) + 3(1)(\frac{1}{n}i) + 3(1)(\frac{1}{n^2}i^2) + (1)(1)(\frac{1}{n^3}i^3)$

$= 1 + \frac{3}{n}i + \frac{3}{n^2}i^2 + \frac{1}{n^3}i^3$

so $A = \lim_{n\to\infty} \sum_{i=1}^{n} \left[2(1+i(\frac{1}{n}))^3 - (1+i(\frac{1}{n}))^2\right](\frac{1}{n})$

$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[2(1+\frac{3}{n}i+\frac{3}{n^2}i^2+\frac{1}{n^3}i^3) - (1+\frac{2}{n}i+\frac{1}{n^2}i^2)\right](\frac{1}{n})$

$= \lim_{n\to\infty} (\frac{1}{n}) \sum_{i=1}^{n} (2 + \frac{6}{n}i + \frac{6}{n^2}i^2 + \frac{2}{n^3}i^3 - 1 - \frac{2}{n}i - \frac{1}{n^2}i^2)$

$= \lim_{n\to\infty} (\frac{1}{n}) \sum_{i=1}^{n} (1 + \frac{4}{n}i + \frac{5}{n^2}i^2 + \frac{2}{n^3}i^3)$

$= \lim_{n\to\infty} (\frac{1}{n}) [n + \frac{4}{n}\frac{(n)(n+1)}{2} + \frac{5}{n^2}\frac{(n)(n+1)(2n+1)}{6} + \frac{2}{n^3}\frac{(n^2)(n+1)^2}{4})$

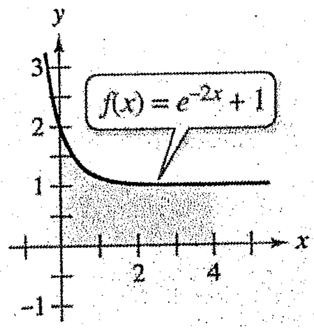
$= \lim_{n\to\infty} (1 + 2\frac{n+1}{n} + \frac{5}{6}\frac{2n^2+3n+1}{n^2} + \frac{1}{2}\frac{n^2+2n+1}{n^2})$

$= 1 + 2(\frac{1}{1}) + \frac{5}{6}(\frac{2}{1}) + \frac{1}{2}(\frac{1}{1})$

$= \boxed{\frac{31}{6} \text{ units}^2}$

25. **Multiple Choice:**

Find the Midpoint Rule approximation for the area of the shaded region. Use 8 subintervals of equal width.



a) 4.425 b) 4.480 c) 6.719 d) 8.959

(Show Riemann Sum work)

26. **Multiple Choice:**

Let f be a continuous function such that $f(-2) = 25$ and $f'(x) = 6x^2 - 7$. What is the value of $f(1)$?

a) -10 b) 18 c) 22 d) 27

$f(x) = \int (6x^2 - 7)dx = 2x^3 - 7x + C$

$25 = 2(-2)^3 - 7(-2) + C$

$25 = -2 + C \rightarrow C = 27$

$f(x) = 2x^3 - 7x + 27$

$f(1) = 2(1)^3 - 7(1) + 27 = 22$

AP Calculus BC

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5.3 Worksheet (Odds, 2, 12, 14)

1. Given $\int_0^5 f(x)dx = 10$ and $\int_5^7 f(x)dx = 3$, evaluate:

a) $\int_0^7 f(x)dx = \boxed{13}$

b) $\int_5^0 f(x)dx = \boxed{-10}$

c) $\int_5^5 f(x)dx = \boxed{0}$

d) $\int_0^5 3f(x)dx = \boxed{30}$

2. Given $\int_2^6 f(x)dx = 10$ and $\int_2^6 g(x)dx = -2$, evaluate:

a) $\int_2^6 [f(x) + g(x)]dx$

$= \int_2^6 f(x)dx + \int_2^6 g(x)dx = 10 + (-2) = \boxed{8}$

b) $\int_2^6 [g(x) - f(x)]dx$

$= \int_2^6 g(x)dx - \int_2^6 f(x)dx = (-2) - (10) = \boxed{-12}$

c) $\int_2^6 2g(x)dx = 2\int_2^6 g(x)dx$

$= 2(-2) = \boxed{-4}$

d) $\int_2^6 3f(x)dx = 3\int_2^6 f(x)dx$

$= 3(10) = \boxed{30}$

3. Use the table of values to estimate $\int_0^6 f(x)dx$.

x	0	1	2	3	4	5	6
f(x)	-6	0	8	18	30	50	80

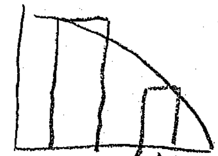
Use three equal subintervals and the a) LEFT ENDPOINTS, b) RIGHT ENDPOINTS, and c) MIDPOINTS. When f is an increasing function, how does each estimate compare with the actual value? Explain your reasoning.

Left: 64, underestimation

Right: 236, overestimation

midpoint: 136, slight underestimation

f(x) is decreasing and concave down



4. Use the table of values to find the lower and upper estimates of $\int_0^{10} f(x)$.

left over- right under-estimates

Assume that f is a decreasing function.

x	0	2	4	6	8	10
f(x)	32	24	12	-4	-20	-36

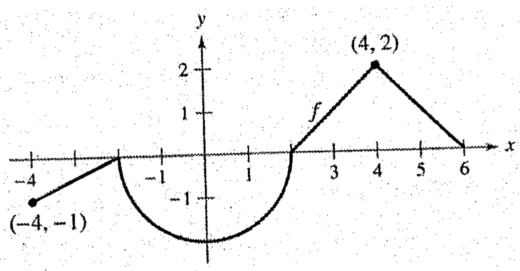
Lower estimate (Right endpoint)

interval	x_i	$f(x_i) \cdot \Delta x$	= area
(0,2)	2	24 · 2	= 48
(2,4)	4	12 · 2	= 24
(4,6)	6	−4 · 2	= −8
(6,8)	8	−20 · 2	= −40
(8,10)	10	−36 · 2	= −72
			$\boxed{-48}$

Upper estimate (Left endpoint)

interval	x_i	$f(x_i) \cdot \Delta x$	= area
(0,2)	0	32 · 2	= 64
(2,4)	2	24 · 2	= 48
(4,6)	4	12 · 2	= 24
(6,8)	6	−4 · 2	= −8
(8,10)	8	−20 · 2	= −40
			$\boxed{88}$

5. The graph of f consists of line segments and a semicircle, as shown in the figure. Evaluate each definite integral by using geometric formulas.



a) $\int_0^2 f(x)dx = \boxed{-\pi}$

b) $\int_2^6 f(x)dx = \boxed{4}$

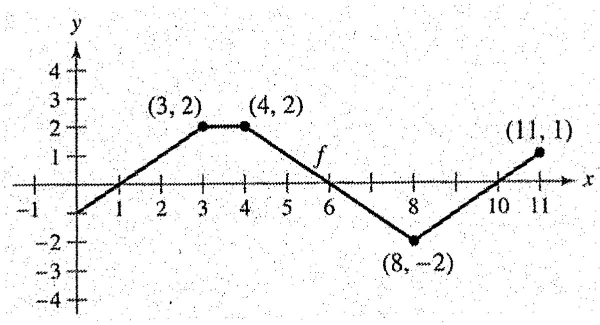
c) $\int_{-4}^2 f(x)dx = \boxed{-1-2\pi}$

d) $\int_{-4}^6 f(x)dx = \boxed{3-2\pi}$

e) $\int_{-4}^6 |f(x)|\, dx = \boxed{5+2\pi}$

f) $\int_{-4}^6 [f(x)+2]\, dx = \boxed{23-2\pi}$

6. The graph of f consists of line segments, as shown in the figure. Evaluate each definite integral by using geometric formulas.



a) $\int_0^1 -f(x)dx = -\frac{1}{2}(1)(1) = \boxed{-\frac{1}{2}}$

b) $\int_3^4 3f(x)dx = 3\int_3^4 f(x)dx = 3(1)(2) = \boxed{6}$

c) $\int_0^7 f(x)dx = \int_0^1 f(x)dx + \int_1^3 f(x)dx + \int_3^4 f(x)dx + \int_4^6 f(x)dx + \int_6^7 f(x)dx = -\frac{1}{2}(1)(1) + \frac{1}{2}(2)(2) + (1)(2) + \frac{1}{2}(2)(2) + (-\frac{1}{2})(1)(1) = \boxed{5}$

d) $\int_5^{11} f(x)dx = \int_5^6 f(x)dx + \int_6^{10} f(x)dx + \int_{10}^{11} f(x)dx = \frac{1}{2}(1)(1) + (-\frac{1}{2})(4)(2) + \frac{1}{2}(1)(1) = \boxed{-3}$

e) $\int_0^{11} f(x)dx = \int_0^1 f(x)dx + \int_1^3 f(x)dx + \int_3^4 f(x)dx + \int_4^6 f(x)dx + \int_6^{10} f(x)dx + \int_{10}^{11} f(x)dx = -\frac{1}{2}(1)(1) + \frac{1}{2}(2)(2) + (1)(2) + \frac{1}{2}(2)(2) + (-\frac{1}{2})(4)(2) + \frac{1}{2}(1)($
$= \boxed{2}$

f) $\int_4^{10} f(x)dx$
$= \int_4^6 f(x)dx + \int_6^{10} f(x)dx = \frac{1}{2}(2)(2) + (-\frac{1}{2})(4)(2) = \boxed{-2}$

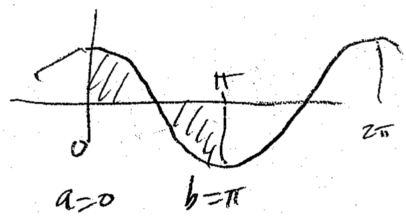
(7-10) Find possible values of a and b that make the statement true. If possible, use a graph to support your answer. (There may be more than one correct answer.)

7. $\int_{-2}^{1} f(x)dx + \int_1^5 f(x)dx = \int_a^b f(x)dx$ $\boxed{a = -2 \quad b = 5}$

8. $\int_{-3}^{3} f(x)dx + \int_3^6 f(x)dx - \int_a^b f(x)dx = \int_{-1}^{6} f(x)dx$ $\boxed{a = -3 \quad b = -1}$

9. $\int_a^b \sin(x)\,dx < 0$ $\boxed{a = \pi \quad b = 2\pi}$

10. $\int_a^b \cos(x)\,dx = 0$



$\boxed{\begin{matrix} a=0 \\ b=\pi \end{matrix}}$

11. **Multiple Choice**

$$\int_{-3}^{6}\left(-\frac{2}{3}x+5\right)dx =$$

a) 25 b) 32 $\boxed{\text{c) } 36}$ d) 45

(use a trapezoid to find area geometrically)

12. **Multiple Choice** $\frac{1}{10} = \Delta x$

The expression $\frac{1}{10}\left[\left(\frac{1}{10}\right)^2+\left(\frac{2}{10}\right)^2+\left(\frac{3}{10}\right)^2+\cdots+\left(\frac{10}{10}\right)^2\right]$ is a Reimann sum approximation for

a) $\boxed{\int_0^1 x^2dx}$ b) $\frac{1}{10}\int_0^1 x^2dx$ c) $\int_1^{10} x^2dx$ d) $\int_0^{10} x^2dx$

$\frac{1}{10} = \Delta x$

integrand: $0+\left(\frac{i}{10}\right)^2 \to f(x)=x^2$

10 terms, each $\Delta x = \frac{1}{10}$, start at $x=0$, end at $x=10\left(\frac{1}{10}\right)=1$, so $\int_0^1 x^2dx$

13. **Multiple Choice**

The table shows selected values for a continuous function f over the interval [2, 8]. Using the subintervals [2, 3], [3, 5], and [5, 8], what is the trapezoidal approximation of $\int_2^8 f(x)dx$?

x	2	3	5	8
f(x)	8	22	72	142

a) 268 b) 338 $\boxed{\text{c) } 430}$ d) 592

(note: the intervals' Δx is different for each interval)

14. **Free Response**

Consider the function

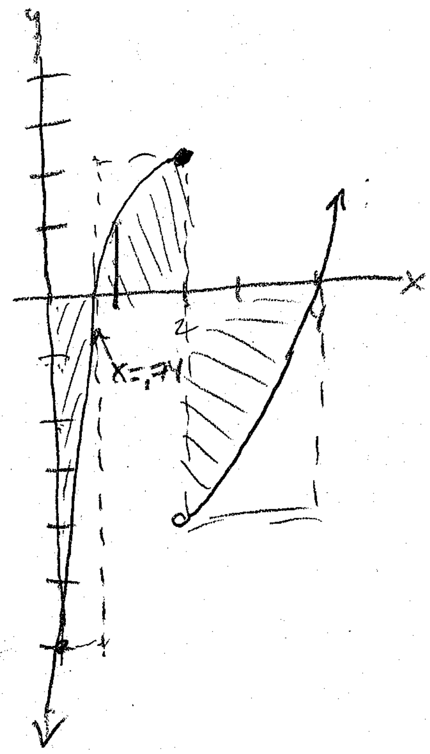
$$f(x) = \begin{cases} x^3 - 6x^2 + 12x - 6, & x < 2 \\ x^2 - 4x, & x \geq 2 \end{cases}$$

a. Graph f over the interval [0, 4].

b. Is f differentiable at x = 2? Justify your answer.

c. Is f integrable on the interval [0, 4]? If so, use your graph to approximate $\int_0^4 f(x)dx$. Show the work that leads to your answer.

(a)



(b) $\boxed{\text{NO}}$ due to the discontinuity at x=2

(c) yes, split at discontinuity;

$$\int_0^4 f(x)dx = \int_0^2 f(x)dx + \int_2^4 f(x)dx$$

$$= \int_0^{0.74} f(x)dx + \int_{0.74}^{2} f(x)dx + \int_2^4 f(x)dx$$

$$\approx \underset{\text{triangle}}{\tfrac{1}{2}(1)(-6)} + \overbrace{\underset{\text{triangle}}{\tfrac{1}{2}(1-.74)(1)} + \underset{\text{trapezoid}}{\tfrac{1}{2}(1+2)(1)}} + \underset{\text{triangle}}{\tfrac{1}{2}(2)(-4)}$$

$$= -3 + (.37 + 1.5) - 4$$

$$= \boxed{-5.13}$$