Unit 4 Review

(1) $C(x) = 680 + 4x + 0.01x^2$

$x(p) = 6000 - 500p$

$500p = 6000 - x$

$p(x) = \frac{6000 - x}{500}$

$R(x) = x \cdot p = \frac{6000}{500}x - \frac{1}{500}x^2$

$P = R - C$

so $P' = R' - C'$ when $R' = C'$

$R'(x) = 12 - \frac{1}{250}x$

$C'(x) = 4 + 0.02x$

$12 - \frac{1}{250}x = 4 + .02x$

$.024x = 8$ $\boxed{x = \frac{8}{.024} = 333.333 \text{ units}}$

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(2) $f(x) = x^2 e^{-x}$ $[0,3]$

relative extrema at $f'(x) = 0$, DNE

$f'(x) = x^2(-e^{-x}) + e^{-x}(2x)$

$= xe^{-x}(2 - x)$

$= \frac{x(2-x)}{e^x} = 0$

at $x = 0$, $x = 2$

0 —— 2 —— 3

$f'(1)$: $\frac{(+)(+)}{(+)}$ $f'(\frac{5}{2})$: $\frac{(+)(-)}{(+)}$

x	$f(x)$
0	0
3	$9e^{-3} \approx .44808$
2	$4e^{-2} \approx 0.54134$

$(0,0)$ is absolute minimum

$(2, 4e^{-2})$ is relative and absolute maximum

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(3) $\lim_{x \to \infty} \frac{\ln(\ln x)}{\ln x}$ $\frac{\infty}{\infty}$

L'Hopital's Rule:

$= \lim_{x \to \infty} \frac{\frac{1}{\ln x} \cdot \frac{1}{x}}{\frac{1}{x}} = \lim_{x \to \infty} \frac{1}{\ln x} = \frac{1}{\infty} \boxed{= 0}$

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(4) this graph is f'

$(-\infty,-2)(-2,-1)(-1,0)(0,1)(1,2)(2,3)(3,4)(4,5)(5,\infty)$

$f'(x)$ $-$ incr $+$ incr $+$ decr $-$ decr $-$ incr $-$ decr $-$ incr $+$ incr $+$ decr

(a) f increasing where f' +:

increasing $(-2,0) \cup (4,\infty)$

decreasing $(-\infty,-2) \cup (0,2) \cup (2,4)$

(b) f local max/min when $f'(x) = 0$ and sign of f' changes

$x = -2$: $- \to +$ local min at $x = -2$

$x = 0$: $+ \to -$ local max at $x = 0$

$x = 4$: $- \to +$ local min at $x = 4$

(no extrema at $x = 2$ b/c no sign change)

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(5) $f(x) = \frac{x}{x+2}$ $[1,4]$

✓ f is continuous & differentiable on $(1,4)$ ($x = -2$) is not in the interval

$f(1) = \frac{1}{3}$ $f(4) = \frac{4}{6} = \frac{2}{3}$

$m = \frac{f(4) - f(1)}{4 - 1} = \frac{2/3 - 1/3}{3} = \frac{1/3}{3} = \frac{1}{9}$

$f'(x) = \frac{(x+2)(1) - (x)(1)}{(x+2)^2}$

$f'(x) = \frac{2}{(x+2)^2}$

so $f'(c) = \frac{2}{(c+2)^2} = \frac{1}{9}$

$(c+2)^2 = 18$

$c + 2 = \pm\sqrt{18}$

$c = -2 - \sqrt{18}$, $\boxed{-2 + \sqrt{18}}$

(not in $[1,4]$)

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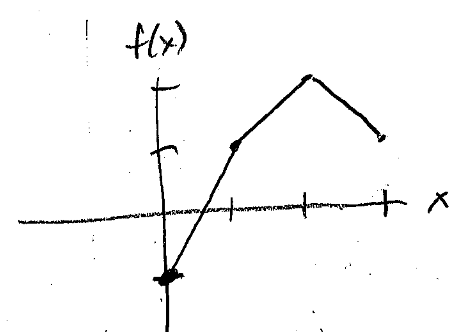
(6)



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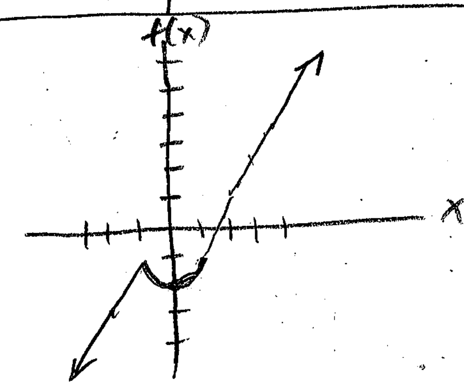


8. $y = x^4+8x^3-72x^2+4$

$y' = 4x^3+24x^2-144x$

$y'' = 12x^2+48x-144 = 0$

$12(x^2+4x-12) = 0$

$12(x+6)(x-2) = 0$

inflection: $x=-6$ $\quad x=2$

$\smile \quad -6 \quad \frown \quad 2 \quad \smile$

$f''(-7)$ $(+)(-)(-)$ $+$ $\qquad f''(0)$ $(+)(+)(-)$ $-$ $\qquad f''(3)$ $(+)(+)(+)$ $+$

concave up $(-\infty,-6)\cup(2,\infty)$

concave down $(-6,2)$ $\boxed{A}$

9. $\lim\limits_{x\to 1} x^{\frac{1}{1-x}} = 1^\infty$

$y = \lim\limits_{x\to 1} x^{\frac{1}{1-x}}$

$\ln y = \lim\limits_{x\to 1} \ln\left(x^{\frac{1}{1-x}}\right)$

$\ln y = \lim\limits_{x\to 1} \left(\frac{1}{1-x}\right)\ln x \quad \infty\cdot 0$

$\ln y = \lim\limits_{x\to 1} \frac{\ln x}{1-x} \quad \frac{0}{0}$

L'Hopital's Rule

$\ln y = \lim\limits_{x\to 1} \frac{\frac{1}{x}}{-1} = -1$

$\ln y = -1$, $y = \boxed{e^{-1}}$

10. $\lim\limits_{x\to 0} \frac{\ln(1-x)+x+\frac{1}{2}x^2}{x^3} = \frac{0}{0}$

L'Hopital's Rule

$\lim\limits_{x\to 0} \frac{\frac{1}{1-x}(-1)+1+x}{3x^2} \quad \frac{0}{0}$

$= \lim\limits_{x\to 0} \frac{-(1-x)^{-1}+1+x}{3x^2}$

L'Hopital's Rule

$\lim\limits_{x\to 0} \frac{(1-x)^{-2}(-1)+1}{6x} \quad \frac{0}{0}$

L'Hopital's Rule

$\lim\limits_{x\to 0} \frac{2(1-x)^{-3}(-1)}{6} = \frac{-2}{6} = \boxed{\frac{-1}{3}}$

11. $f(x) = \frac{x^2+5x-24}{x^2+10x+16}$

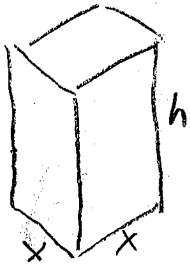
$\lim\limits_{x\to -8} f(x) = \frac{(-8)^2+5(-8)-24}{(-8)^2+10(-8)+16} = \frac{0}{0}$

factor or L'Hopital's twice

$\lim\limits_{x\to -8} \frac{(x+8)(x-3)}{(x+8)(x+2)} = \lim\limits_{x\to 8} \frac{x-3}{x+2} = \frac{-8-3}{-8+2} = \frac{-11}{-6} = \boxed{\frac{11}{6}}$

$\boxed{D}$

12.



max $V = x^2h \qquad A = 2x^2+4xh = 337.5$

$4xh = 337.5-2x^2$

$h = \frac{337.5-2x^2}{4x}$

$V = x^2\left(\frac{337.5-2x^2}{4x}\right)$

$V = \frac{337.5}{4}x - \frac{1}{2}x^3$

$V' = \frac{337.5}{4} - \frac{3}{2}x^2 = 0$

$\frac{3}{2}x^2 = \frac{337.5}{4}$

$x^2 = \frac{225}{4}$

$x = \pm\sqrt{\frac{225}{4}}$ (can't be negative)

$\boxed{x = \frac{\sqrt{225}}{\sqrt{4}} = \frac{15}{2}\text{ cm}}$

$h = \frac{337.5-2\left(\frac{15}{2}\right)^2}{4\left(\frac{15}{2}\right)} = \frac{15}{2}\text{ cm}$

$\boxed{h = \frac{15}{2}\text{ cm}}$

verify is a maximum

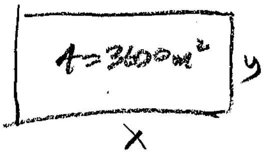
$V'' = -3x$

$V''\left(\frac{15}{2}\right)$ $(-)$ $\frown$

concave down

so $x = \frac{15}{2}$ is a relative max

(13)



$\min F = 2x + 2y \qquad A = xy = 3600$

$F = 2x + 2\left(\frac{3600}{x}\right) \leftarrow y = \frac{3600}{x}$

$F = 2x + \frac{7200}{x} = 2x + 7200x^{-1}$

$F' = 2 - 7200x^{-2} = 2 - \frac{7200}{x^2} = 0$

$2 = \frac{7200}{x^2}, \quad 2x^2 = 7200, \quad x^2 = 3600, \quad x = \pm 60$ (can't be neg)

verify min fencing:

$F'' = 14400x^{-3} = \frac{14400}{x^3}$

pos $x \rightarrow F''+$

concave up

so relative min

$$\boxed{\begin{aligned} x &= 60 \text{ m} \\ y &= \frac{3600}{60} = 60 \text{ m} \end{aligned}}$$

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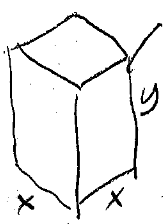
(14)



$V = 200 m^3$

$\min C = 10(\text{top/bottom area}) + 6(\text{sides area}) \qquad V = x^2 y = 200$

$C = 10(2x^2) + 6(4xy) \leftarrow y = \frac{200}{x^2}$

$C = 20x^2 + 24x\left(\frac{200}{x^2}\right)$

$C = 20x^2 + \frac{4800}{x} = 20x^2 + 4800x^{-1}$

$C' = 40x - 4800x^{-2} = 40x - \frac{4800}{x^2} = 0$

$40x = \frac{4800}{x^2}$

$x^3 = \frac{4800}{40} = 120$

$$\boxed{\begin{aligned} x &= \sqrt[3]{120} \approx 4.932 m \\ y &= \frac{200}{(\sqrt[3]{120})^2} \approx 8.221 m \end{aligned}}$$

verify min cost

$C'' = 40 + 9600x^{-3}$

$C''(\sqrt[3]{120}) = (+)$

concave up

so relative min cost